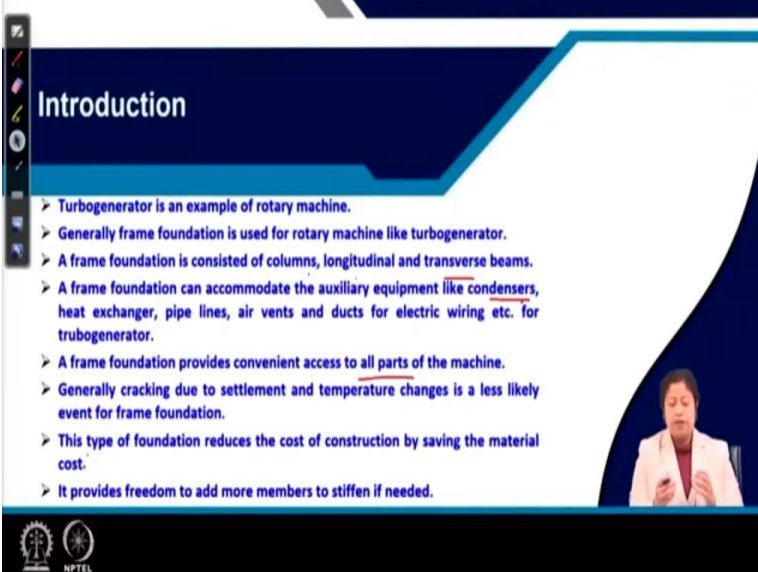


Soil Dynamics
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Lecture 47

Analysis of Machine Foundations (For Rotary Machines- Part I)

Hello everyone, today we will start our new topic in the course soil dynamics, where we will discuss the analysis of rotary machines foundation; that means this foundation will support rotary machines.

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The slide is titled "Introduction" and contains the following bullet points:

- Turbogenerator is an example of rotary machine.
- Generally frame foundation is used for rotary machine like turbogenerator.
- A frame foundation is consisted of columns, longitudinal and transverse beams.
- A frame foundation can accommodate the auxiliary equipment like condensers, heat exchanger, pipe lines, air vents and ducts for electric wiring etc. for trubogenerator.
- A frame foundation provides convenient access to all parts of the machine.
- Generally cracking due to settlement and temperature changes is a less likely event for frame foundation.
- This type of foundation reduces the cost of construction by saving the material cost.
- It provides freedom to add more members to stiffen if needed.

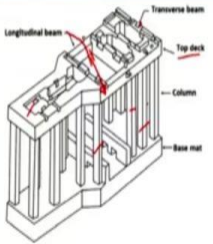
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So, typical example of rotary machine is turbogenerator. Generally, for this we use frame foundation; and this frame foundation is consisted of a column or series of columns, longitudinal and transverse beams. A frame foundation can accommodate the auxiliary equipment like you can say condensers, then heat exchanger, then pipelines, air vents et-cetera for turbogenerator. Because, just if we will construct a foundation for the turbogenerator that is not enough, we have to provide space, we have to accommodate these auxiliary components of the turbogenerators.

A frame Foundation provides convenient access to all parts of the machines as well. Generally, cracking due to settlement and temperature changes is a less likely event for frame foundation. This type of foundation that means frame foundation reduces the cost of construction by saving the material cost. Also, it provides freedom to add more members to stiffen if it is required.


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Introduction



- A frame foundation is consisted of columns, longitudinal and transverse beams.
- The transverse beams may be often eccentric with respect to the centre lines of the columns
- It generally have varying cross-section due to several opening in the top deck and haunches at the junction with columns.
- For good performance of the turbogenerator, an appropriate design of the foundations for all possible combinations of static and dynamic loads is essential.

Fig. 47.1 A typical frame foundation for turbogenerator unit



Now, in this figure, you can see a typical frame foundation which can be used for turbogenerator you need. So, what are the different components that we need to see here? You can see base mat which supports these columns, a series of columns you can see here; this is also columns, this is also columns. Now, the columns supports top deck, and it will be better to say columns supports transverse and longitudinal beams on which these top deck is resting.

So, as I said, a frame foundation is consisted of columns that you can say see here, then longitudinal and transverse beam. So, these are longitudinal beams, this one is also longitudinal beams and transverse beam which you can see here, and also this one.

The transverse beams may be often eccentric with respect to the center lines of the columns. It generally have varying cross-section due to several openings in that top deck and hunches at the junction with columns. For good performance of the turbogenerator and appropriate design of the foundations for all possible combinations of load static and dynamic loads is essential. We will see what are the different combinations of loading.

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Essential Features

- The entire foundation should be separated from the main building so that the transfer of vibrations is isolated from the top deck of the foundation to the building floor of the machine room. A clear gap must be provided all around.
- Other footings placed near to the machine foundation should be checked for non-uniform stresses imposed by the adjacent footings. The Pressure-bulbs under the adjacent footings should not interfere significantly with each other.
- All the junctions of beams and columns of the foundation should be provided with adequate haunches in order to increase the general rigidity of the frame foundation.
- The cross-sectional height of the cantilever elements at the embedment point should not be less than 60% to 75% of its span, being susceptible to excessive local vibrations.
- The transverse beams should have their axes vertically below the bearings to avoid torsion.
- For the same reason the axes of columns and transverse beams should lie in the same vertical plane.
- The upper platform should be as rigid as possible in its plane.

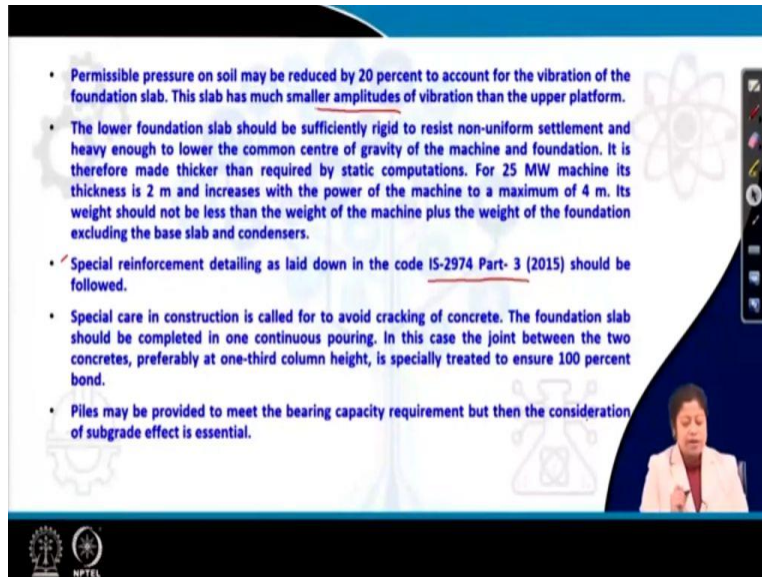
But, before that we will discuss few essential features. So, the first feature is the entire foundation should be separated from the beam building, so that the transfer of the vibrations is isolated from the top deck of the foundation to the building floor of the machine or machine room.

Now, you need to ensure a clear gap also all around the foundation frame. We have already discussed that when we were discussing the machine foundation for reciprocating machine that, if we will provide air gap that will act as vibration isolation. So, in this case also we need to ensure a clear gap.

Now, other footings which are placed near to the machine foundation should be checked for non-uniform stress imposed by the adjusting footings. The pressure-bulbs under the adjacent footings should not interfere significantly with each other. All the junctions of the beams and columns of the foundation should be provided with adequate hunches in order to increase the general rigidity of the frame foundation. Now, the cross-sectional height of the cantilever elements at the embedment point should not be less than 60 to 70 percent of its span, being susceptible to excessive local vibrations.

The transverse beams should have their axes vertically below the bearings to avoid torsion. For the same reason, the axis of columns and transverse beams should lie in the same vertical plane. The upper platform should be as rigid as possible in its plane.

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The slide contains the following text:

- Permissible pressure on soil may be reduced by 20 percent to account for the vibration of the foundation slab. This slab has much smaller amplitudes of vibration than the upper platform.
- The lower foundation slab should be sufficiently rigid to resist non-uniform settlement and heavy enough to lower the common centre of gravity of the machine and foundation. It is therefore made thicker than required by static computations. For 25 MW machine its thickness is 2 m and increases with the power of the machine to a maximum of 4 m. Its weight should not be less than the weight of the machine plus the weight of the foundation excluding the base slab and condensers.
- Special reinforcement detailing as laid down in the code IS-2974 Part- 3 (2015) should be followed.
- Special care in construction is called for to avoid cracking of concrete. The foundation slab should be completed in one continuous pouring. In this case the joint between the two concretes, preferably at one-third column height, is specially treated to ensure 100 percent bond.
- Piles may be provided to meet the bearing capacity requirement but then the consideration of subgrade effect is essential.

The slide also features a small video inset of a woman in the bottom right corner and logos for IIT Bombay and NPTEL at the bottom left.

Now, the permissible pressure on soil may be reduced by 20% percent to account for the vibration of the foundation slab. This slab has much smaller amplitude of vibration than the upper platform. The lower foundation slab should be sufficiently rigid to resist non-uniform settlement and heavy enough to the lower, sorry and heavy enough to lower the common center of gravity of the machine and foundation.

It is therefore made thicker than required by static computations. There are some other features among which we need to take care of this one. Special reinforcement detailing as laid down in the code IS-2974 Part-3 published in 2015 should be followed.

Now, special care in construction is called to is called for to avoid cracking of concrete. The foundation slab should be completed in one continuous pouring. In this case the joint between the two concretes preferably at one-third column height, is specifically treated to ensure special care in construction is called for to avoid cracking of concrete. We may need to provide piles to meet the bearing capacity requirement. If we see that the bearing capacity requirement is not fulfilled by the frame foundation alone, then we need to use pile also. And in that case, the consideration of subgrade effect is essential.

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- As far as possible the foundation should be dimensioned such that the centre of gravity of the foundation with the machine should be in vertical alignment with that of the base area in contact with the soil.
- The ground-water table should be as low as possible and deeper by at least one-fourth of the width of foundation below the base plane. This limits the vibrations propagation, ground-water being a good conductor to wave transmission.
- Soil-profile and characteristics of soil up to at least thrice the width of the turbine foundation or till hard stratum is reached or up to pile depth, if piles are provided, should be investigated.

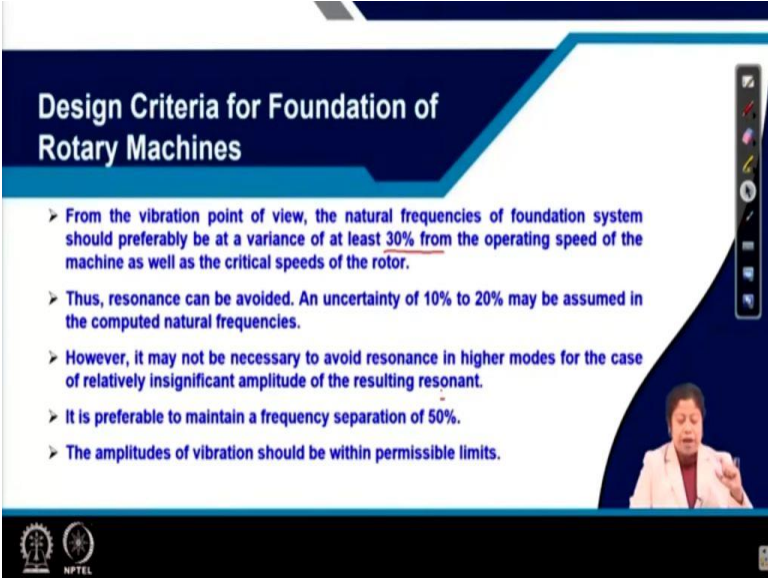
As far as possible, the foundation should be dimensioned such that the center of gravity of the foundation with the machine should be in the vertical alignment with that of the base area in contact to the soil. The groundwater table should be as low as possible and deeper at least one-fourth of the width of the foundation below the base plane.

So, if the width is suppose 4 meter, then we need to ensure that the ground, the distance between the foundation base and the groundwater table should be one-fourth of 4 meter that we need. The groundwater table should be as low as possible and deeper by at least one-fourth of the width of foundation below the base plane.

These limits, the vibrations propagation, groundwater table being a good conductor to wave transmission. That is the reason we need to ensure these minimum gap of one-fourth of the width the width of the foundation. So, what does it mean basically? It means suppose the width of the foundation is 4 meter, then the distance from the distance of the groundwater table from the base of the foundation should be at least 1 meter.

Now, soil-profile and characteristics of soil up to at least thrice the width of the turbine foundation or till hard stratum is reached, or up to pile depth if piles are used, should be investigated. So, soil investigation is another essential feature.

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Design Criteria for Foundation of Rotary Machines

- From the vibration point of view, the natural frequencies of foundation system should preferably be at a variance of at least 30% from the operating speed of the machine as well as the critical speeds of the rotor.
- Thus, resonance can be avoided. An uncertainty of 10% to 20% may be assumed in the computed natural frequencies.
- However, it may not be necessary to avoid resonance in higher modes for the case of relatively insignificant amplitude of the resulting resonant.
- It is preferable to maintain a frequency separation of 50%.
- The amplitudes of vibration should be within permissible limits.

The slide includes a small video inset of a woman in a white jacket speaking, and logos for NPTEL and other institutions at the bottom.

Now, let us see the design criteria for foundation of rotary machines. From the vibration point of view, the natural frequencies of foundation system should preferably be at a variance of at least 30 percent from the operating speed of the machines as well as the critical speeds of the rotor.

So, the resonance can be avoided. And uncertainty of 10 to 20 percent may be assumed in the computed natural frequencies; so, we need to consider also some extent of uncertainties. However, it may not be necessary to avoid resonance in higher modes for the case of relatively insignificant amplitude of resulting resonant. It is preferably, it is preferable to maintain a frequency; it is preferable to maintain a frequency separation of 50 percent.

The amplitudes of vibration should be within permissible limits. So, if you recall in the first class of analysis of machine foundation, we have discussed a few design criteria where the permissible limits of for the vibration was discussed. So, the amplitude of vibration permissible amplitude if I say, for vertical vibration I can write on the board.

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The image shows two screenshots from a video lecture. The top screenshot is a whiteboard with handwritten notes. The bottom screenshot is a presentation slide titled 'Design Criteria for Foundation of Rotary Machines' with a list of bullet points.

For vertical vibration

Speed	1500 - 3000 rpm	A_p	0.4 - 0.6 mm
	> 3000 rpm	A_p	0.2 - 0.3 mm

For horizontal vibration

Speed	1500 - 3000 rpm	A_p	0.7 - 0.9 mm
	> 3000 rpm	A_p	0.3 - 0.4 mm

Design Criteria for Foundation of Rotary Machines

- From the vibration point of view, the natural frequencies of foundation system should preferably be at a variance of at least 30% from the operating speed of the machine as well as the critical speeds of the rotor.
- Thus, resonance can be avoided. An uncertainty of 10% to 20% may be assumed in the computed natural frequencies.
- However, it may not be necessary to avoid resonance in higher modes for the case of relatively insignificant amplitude of the resulting resonant.
- It is preferable to maintain a frequency separation of 50%.
- The amplitudes of vibration should be within permissible limits.

So, for vertical vibration if I will write, for vertical vibration if the motor speed is in between 1500 to 3000; that means one point, sorry that means 1500 to 3000 RPM; then, the permissible amplitude varies from 0.4 to 0.6 in millimeter. Now, if the speed is more than 3000 RPM, that time permissible amplitude which is represented by A_p varies from 0.2 to 0.3 in millimeter. Likewise, we need to check the same for horizontal vibration. So, the permissible values we need to check when the speed is between 1500 to 3000 rpm; this permissible amplitude may vary from 0.7 to 0.9 millimeter.

If speed is more than 3000 rpm, then permissible amplitude varies from 0.3 to 0.4 millimeter; or we can take it 0.4 to 0.5 also, depending upon some kind of adjustment. So, the amplitudes of vibration should be within the same permissible limits.

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Load Calculation for Rotary Machine Foundations

- Following loads are required to calculate for rotary machine foundations:
 - ✓ Dead Load: It is the total weight of the foundation and the machine. ✓
 - ✓ Operation Load: It considers frictional forces, power torque, thermal elongation forces, vacuum in the condenser, piping forces etc. Generally, manufacturers of the machine supply this load.
 - ✓ However, the load due to vacuum in condenser may also be calculated as:

$$P_c = A(p_a - p_v) \quad \dots (1)$$
 where, P_c = Condenser vacuum load
 A = Cross-sectional area of the connecting tie between the condenser and turbine
 p_a = Atmospheric pressure
 p_v = Vacuum pressure

Now, how do we do the load calculation? For load calculation for rotary machine foundations, we need to consider four combinations of loading. So, before knowing all these four combinations of loading, first we need to know what are the different loads. First one dead load; it is the total weight of the foundation and the machine together.

Next is operation load, which considers frictional forces, power torque, thermal elongation forces, vacuum in the condenser, piping forces et-cetera. Generally, manufacturers of the machines supply this load. However, the load due to vacuum in condenser may also be calculated as load P_c is equal to A times p_a minus p_v , small p_a and small p_v within the bracket.

So, what is A here? What is p_a and p_v ? So, A is the cross sectional area of the connecting time between the condenser and the turbine; whereas, p_a is the condenser vacuum load. Sorry, p_v is the condenser vacuum load, small p_a is the pressure at atmospheric level, or we can call it as atmospheric pressure; whereas p_v is the vacuum pressure. So, if we know the vacuum pressure from that we can calculate the condenser vacuum load using equation-1.

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Fig. 47.2 Torque generated during operation of multi-stage turbine-generator unit

- $T_A = \frac{105 P_A}{N}$ kN m where, P_A is power transferred by coupling A in kW and N is operating speed in rpm
- $T_B = \frac{105(P_B - P_A)}{N}$ kN m where, P_B is power transferred by coupling B in kW.
- $T_C = \frac{105(P_C - P_B)}{N}$ kN m where, P_C is power transferred by coupling C in kW.
- $T_g = \frac{105 P_c}{N}$ kN m

Now, in this figure, you can see the torque generated during operation of a multi-stage turbine generator unit. So, here using the expansion given, we can find out the torque generated at different stages of the turbine generator unit. So, T_A can be calculated if we know P_A which is the power transferred by the coupling A in kilo watt; and capital N which represents the speed of the operating speed in rpm. Likewise, we can calculate T_B , T_C and T_g in kilo newton meter, if we know the power transferred by coupling B and C as well.

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- ✓ Normal Machine Unbalanced Load: the resultant unbalanced forces due to the two masses at any time cancel out, but there is a resulting moment M given by:

$$M = m_r e \omega^2 l \quad \dots (2)$$
 where, l = Distance between the mass centre of gravities of rotors.
- ✓ The vertical and horizontal components of the moment M (Refer Fig. 47.3a) are M_v and M_h respectively, and are expressed as:

$$M_h = m_r e \omega^2 l \cos \omega t \quad \dots (3a)$$

$$M_v = m_r e \omega^2 l \sin \omega t \quad \dots (3b)$$
- ✓ Referring Fig. 47.3b the unbalanced force is:

$$F = 2m_r e \omega^2 \quad \dots (4)$$
- ✓ If the number of rotors on a common shaft is more than two then the aforesaid approach will be followed to calculate the moments and combined unbalanced forces.

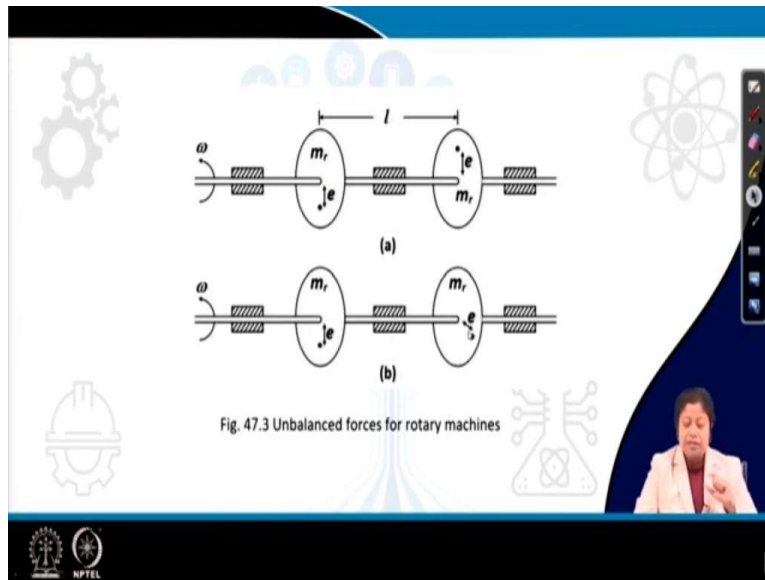


Fig. 47.3 Unbalanced forces for rotary machines

Next load is normal machine unbalanced load. The resultant unbalanced forces due to the two masses at any time can be canceled out. However, the resulting moment should always be there and how do we calculate that? So, we can calculate the resulting moment in using this equation, M is equal to m_r times e times ω^2 times l . So, what is l here? l is the distance between the mass center of gravity's of rotors; the vertical and horizontal components of the movement M , if we consider vertical component as capital MV and horizontal component as capital MH respectively.

Then, we can express MV and MH as, MH means m_r times e ω^2 times l cosine ωt . Likewise, MV means m_r times e ω^2 times l sine ωt . Here let us see the figure. So, this is mass rotating mass m_r ; it maintains an eccentricity e from the axis of the shaft; and you can see the operating speed is ω in this case.

So, now, next is the calculation of the unbalanced force for position b. So, here if we need to calculate the unbalanced force, what will be that unbalanced force is equal to you can see here; it is 2 times m_r times e ω^2 . Already we know what is m_r , what is e and what is ω ; so I am not repeating that.

If the number of rotors of on a common shaft is more than two, then that aforesaid approach that means, whatever written in equation 2 to 4 will be followed to calculate the moments and combined unbalanced forces.

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✓ Short Circuit Load: The moment imposed on the foundation by the short circuit condition can be calculated as (Ref. IS 1893: 1954):

$$M_{sc} = 10 r W_r \quad \dots (5)$$

where, W_r = capacity of turbogenerator unit in MW.
 r = radius of the rotor in m.

✓ Loss of Blade Unbalance or Bearing failure Load: Due to the breakage of one blade or bucket the unbalanced force is increased. This additional unbalanced force will depend on the weight of the bucket, the distance of its centre of gravity from the axis of rotation and operational speed.

✓ Seismic Load: This load can be computed by using the following Equation (6):

$$F_s = \alpha_h I \beta C S W \quad \dots (6)$$

F_s = Horizontal seismic force, α_h = Seismic zone coefficient, I = Importance factor
 β = Soil-foundation factor C = Numerical base shear coefficient
 S = Numerical site structure response coefficient
 W = Vertical load due to weight of all permanent components.

Next load is short circuit load. The moment imposed on the foundation by the short circuit condition can be calculated using IS code 198; sorry using IS code 1893 published in 1954. So, what it is said? Moment due to short circuit condition is equal to 10 times of r , times W_r . So, here W_r is the capacity of the turbogenerator in megawatt, r is the radius of the rotor in meter. So, if we know that capacity of the turbogenerator unit and if we do the radius of the rotor, from that we can calculate the magnitude of the moment that is imposed on the foundation due to short circuit condition.

Next, load is due to loss of blade unbalance or bearing failure load. So, due to the breakage of one blade or bucket the unbalanced force is increased. This additional unbalanced force depends upon the weight of the bucket; the distance of its CG from the axis of rotation and operational speed. So, these additional unbalanced force as I said, it depends upon it is just a repetition of the previous line.

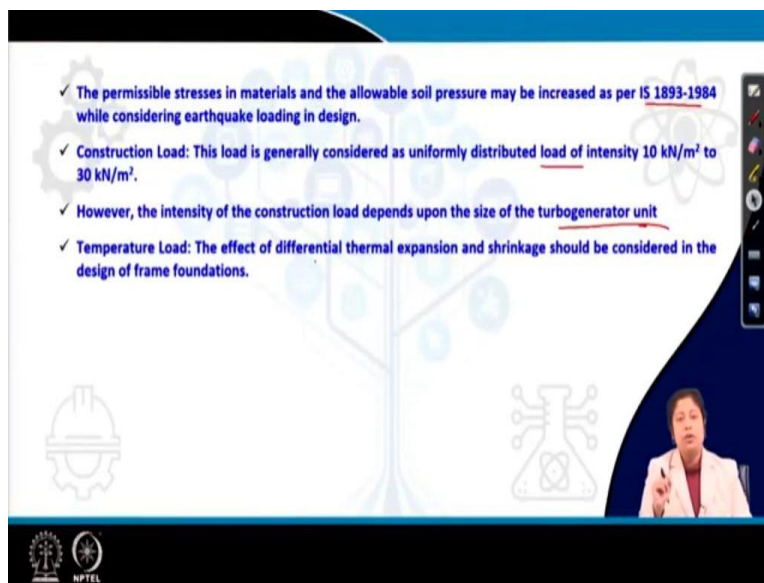
Next, so, next is seismic load; this load can be computed by using the equation shown here. So, you can see seismic force F_s is equal to alpha times h, times I, times beta, times C, times S, times W. So, we need to know what are the meaning of these symbols which I read just now.

So, next load is seismic load. So, seismic load can be computed by using the equation-6, this one. So, what it is saying? F_s which is representing the seismic load is equal to alpha h times I, times beta, times C, times S, times capital W. So, we need to note here what are the meanings of these

different symbols? F_s already we know; this is the seismic load. Now, α_h is seismic zone coefficient; so, it depends upon the seismic zone. I is an importance factor, so it may also vary; β is soil-foundation factor, whereas, C is numerical base shear coefficient.

Capital S is numerical site structure response coefficient, and W is vertical load due to the weight of all the permanent components. So, depending upon the total vertical load because of the permanent components, and also considering the site soil-foundation factor and other factors; we can calculate the seismic load using equation-6.

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- ✓ The permissible stresses in materials and the allowable soil pressure may be increased as per IS 1893-1984 while considering earthquake loading in design.
- ✓ Construction Load: This load is generally considered as uniformly distributed load of intensity 10 kN/m² to 30 kN/m².
- ✓ However, the intensity of the construction load depends upon the size of the turbogenerator unit
- ✓ Temperature Load: The effect of differential thermal expansion and shrinkage should be considered in the design of frame foundations.

Now, in the presence of seismic load, what is happening? The permissible stresses in materials and the allowable soil pressure may be increased as recommended in IS 1893, while considering earthquake loading in design; so, we need to take care of this point as well.

Now, construction load: this load is generally considered as a uniformly distributed load of intensity 10 kilo Newton per square meter to 30 kilo Newton per square meter. However, the intensity of the construction load depends upon the size of the turbogenerator unit. So, the effect of differential thermal expansion and shrinkage should be considered while designing the frame foundations. So, we need to consider temperature load as well.

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Design Load for Rotary Machine Foundations

➤ Following four load combinations are required to check for rotary machine foundations:

- ✓ Dead Load + Operation Load + Normal Machine Unbalanced Load + Temperature Load in Foundation
- ✓ Dead Load + Operation Load + Normal Machine Unbalanced Load + Temperature Load in Foundation + Short Circuit Load
- ✓ Dead Load + Operation Load + Temperature Load in Foundation + Loss of Blade Unbalance or Bearing failure Load
- ✓ Dead Load + Operation Load + Normal Machine Unbalanced Load + Temperature Load in Foundation + Earthquake Load

The slide features a dark blue header with the title, a white main area with a list of four load combinations, and a small video inset of a presenter in the bottom right corner. The NPTEL logo is visible in the bottom left corner.

Now, how to calculate the design load for rotary machine foundations? So, we need to consider four load combinations which are state, which I will now show you. So, the first one is the combination of dead load, operation load, normal machine unbalanced load, and temperature load together. So, how many loads we are considering here? 4 different loads are considered in first case; let us see the second case. Here we are considering five cases, five loads. What are those five loads? Dead load, operation load, normal machine unbalanced load, temperature load and short circuit load as well.

In third case, we also consider the four different loads; these loads are dead load, operation load, temperature load in foundation, loss of blade unbalance or bearing failure load. So, third case considers four different loads. Now, the fourth case which considers five different which consider five different loads, dead load, then operation load, normal machine unbalanced load, then temperature load, and earthquake load.

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Methodology of Analysis and Design

- For frame foundation, the frequencies and amplitudes of vibration of frame are required to check and then design the members of frame from structural considerations.
- The dynamic analysis of the frame foundation are classified into following groups :
 - ✓ Two-dimensional analysis- Resonance method (Rausch, 1959) , Amplitude method (Barkan, 1962) and Combined method (Major, 1980)
 - ✓ Three-dimensional analysis

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Now, considering all these four load combinations, our next target is to learn the methodology of analysis and design of machine foundation. So, for frame foundation, the frequencies and amplitudes of vibration of frame are required to check, and then the design the members of the frame from structural considerations.

The dynamic analysis of the frame or frame foundation can be classified into following groups; it should be groups. What are the different groups? We can do two-dimensional analysis; under two dimensional analysis, there are three different methods. First one you can see resonance method, second one amplitude method, and third one is the combined method. Another one is three-dimensional analysis.

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Two Dimensional Method

Assumptions:

- The difference between the deformations of individual frame columns is insignificant.
- The deformation of the longitudinal and transverse beams is almost identical.
- The torsional resistance of the longitudinal beams is insignificant in relation to the deformation of the transverse beams.
- The vertical vibrations of the frames can be determined for each frame individually.
- The weight transmitted from the longitudinal beam can be considered as a load supported by the column head, even in case where the transverse beam is eccentrically placed with respect to the centre line of the column.
- Both the columns and beams can be replaced by weightless elements with the masses lumped at a few points by equating the kinetic energies of the actual and idealized systems.
- The effect of elasticity of subsoil is neglected, it being relatively much flexible.
- When considering horizontal displacement the upper slab is regarded as a rigid plate in its own plane.

So, we will study two-dimensional analysis or two dimensional method. So, before studying that, we need to know what are the assumptions based on which this analysis will be carried out. So, here you can see the different assumptions, I am reading it for you. The difference between the deformations of individual frame column is insignificant; that means we are not considering any kind of differential settlement in this case. The deformation of the longitudinal and transverse beams is almost identical. The torsional resistance of the longitudinal beams is insignificant in relation to the deformation of the transverse beams.

The vertical vibrations of the frames can be determined for each frame individually. The weight transmitted from the longitudinal beam can be considered as a load supported by the column head, even in case where the transverse beam is eccentrically placed with respect to the centerline of the column.

Both the column and the beams can be replaced by weightless elements with the masses lumped at a few points, by equating the kinetic energies of the actual and idealized system. The effect of elasticity of subsoil is neglected; it being relatively much flexible. Another assumption is that when considering horizontal displacement, the upper slab is regarded as a rigid plate in its own plane.

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Resonance Method

- Natural frequencies of the system in relation to the operating speed of the machine is found out by considering the frame foundation as a single degree of freedom system.

Fig. 47.4 A typical transverse frame with idealized spring-mass model

W_1 = self weight of the machine and bearing,
 W_2 = load transferred to the columns by longitudinal beams,
 q = intensity of uniformly distributed load due to self weight of cross beam,
 F_z = amplitude of the unbalanced vertical force due to machine operation,
 ω = operating frequency of the machine.

So, now we will study how to do the two-dimensional analysis. Already I told there are three methods; first one resonance method, second one amplitude method, and the third one is the combined method proposed by major.

So, first we will learn resonance method. In this method, natural frequencies of the system in relation to the operating speed of the machine is found out by considering the frame foundation as a single degree of freedom system. I am repeating in this method, natural frequencies of the system in relation to the operating speed of the machine is found out by considering the frame foundation as a single degree of freedom system.

Here you can see a typical transverse frame in Figure a; and that is mathematically represented by mass-spring model in figure b. So, what are the different components we can see in figure a transverse beam? So, on transverse beam you can see different loads are acting; so, we need to know what are these different loads.

So, you can see here W_1 is the self weight of the machine and the bearing; whereas, W_2 this is the load which transfer to the columns by the longitudinal beam. So, longitudinal beam means beam in this direction. So, I am just erasing what I have drawn just now.

So, W_2 as I say is the weight of the transverse beam; sorry weight of the longitudinal beam which is transferred to the column this one and this one. Now, q is the intensity of uniformly distributed load due to self weight of this cross beam. Next is F_z , it is the amplitude; you can see

here $Fz \sin \omega t$ is the unbalanced force. So, Fz is the amplitude of the unbalanced vertical force due to machine operation; and ω is the operating speed of the machine.

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- Mass m as shown in Fig. 47.4b is: $m = \frac{W_1 + 2W_2 + qL}{g}$... (7)
- Stiffness constant k_z is equal to: $k_z = \frac{W}{\delta_{st}} = \frac{mg}{\delta_{st}}$... (8)
- Total static displacement can be calculated as: $\delta_{st} = \delta_1 + \delta_2 + \delta_3 + \delta_4$... (9)
- Here,
 - Vertical deflection of beam due to load W_1 is equal to: $\delta_1 = \frac{W_1 L^3}{96 E I_b} \cdot \frac{2K+1}{K+2}$
 - Vertical deflection of beam due to distributed load q is equal to: $\delta_2 = \frac{q L^4}{384 E I_b} \cdot \frac{5K+2}{K+2}$
 - Vertical deflection of beam due to shear: $\delta_3 = \frac{3}{5} \cdot \frac{L}{E A_b} \left(W_1 + \frac{qL}{2} \right)$
 - Axial compression in column: $\delta_4 = \frac{H}{E A_c} \left(W_2 + \frac{W_1 + qL}{2} \right)$
- where, I_b is the moment of inertia of beam about the axis of bending and I_c is the moment of inertia of column
- K is the relative stiffness factor $= \frac{I_b}{I_c} \cdot \frac{H}{L}$
- L and H are effective span and effective height of frame
- E is the Young's modulus of concrete

The diagram shows a mass M connected to a fixed support by a spring with stiffness K_z . The support is represented by a horizontal line with diagonal hatching underneath it.

Now, what is mass here? Mass means I am just we have seen that the system can be represented by a mass-spring system. So, mass-spring system means we have a mass here which is M and that is connected to a spring K_z in this case.

Now, what is in the M then? M is W_1 plus 2 times of W_2 plus qL divided by g . From this we can calculate stiffness constant K_z . So, K_z is equal to capital W divided by δ_{st} ; what is capital W here? Capital W means m times g . So, I can also write here as m times g divided by

delta is st. Now, what is delta is st here? Total static displacement is delta is st; and that can be calculated by some of delta1, delta2, delta3 and delta4.

Now, the question what are delta1, delta2, delta3 and delta4? So, let us see here. Vertical deflection of beam due to load W1 is equal to delta1; and that can be calculated by this equation. Next is delta2, which is the vertical deflection of the beam due to the distributed load of intensity small q; and delta2 can be calculated using this equation. Delta3 is the vertical deflection of the beam due to shear. So, here you can see the expression that we can use to calculate delta3. Next is to find out the axial compression in column which is written here as delta4.

So, delta4 is equal to h divided by E times Ac, whole thing is multiplied with the W2 plus W1 plus q l divided by 2; where you can see the meaning of different parameters. So, here we have used Ib which is Ib, that is written the moment of inertia of beam about the axis of bending. In this equation, what new term we have seen? We have used H; let us write it capital H, it is not small h. So, capital H is the effective height of the frame whereas, capital L here this is the effective span of the frame.

And you can see we have used E, what is E? E is the Young's modulus of the concrete, which is used for the construction of the frame. And Ac is the cross sectional area of the column. So, let us give the number of these three equations.

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• Here, $L = L_0 - 2ab$ and $H = H_0 - 2aa$

• α can be calculated from the following Fig. 47.6

Fig. 47.5 A typical frame with haunches

Fig. 47.6 α vs b/L_0

L_0 = centre to centre distance between columns
 H_0 = Height of the column from the top of the base slab to the centre of the frame beam

In figure 47.5, what you can see? Here you can see typical frame with hunches. Now, if there is no hunch, then we can just take L and H, which represents which represent the effective length and effective height of the frame. However, if haunches are present, that time how do we calculate L that is shown here. L is equal to L0 minus 2 times of alpha b.

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The slide contains the following content:

- The natural frequency of a transverse frame in vertical vibrations: $\omega_{nz} = \sqrt{\frac{k_z g}{W}}$... (10)
- Average vertical natural frequency of the turbogenerator foundation is: $\omega_{nzav} = \frac{\omega_{nz1} + \omega_{nz2} + \dots + \omega_{nzn}}{n}$... (11)
- Average vertical amplitude of the turbogenerator foundation is: $A_{za} = \frac{\sum F_z}{(\sum k_z) \sqrt{(1 - r_z^2)^2 + (2Dr_z)^2}}$... (12)

where, frequency ratio $r_z = \frac{\omega}{\omega_{nzav}} = \frac{\omega}{\omega_{nzav}}$

$\sum F_z$ is total vertical imbalance force,
 $\sum k_z$ is sum of the stiffness of the individual frames
 D is damping ratio

Special case: under tuned machine with $\omega < \omega_{nzav}$

Handwritten notes on the slide: $\omega = \omega_{nzav} \Rightarrow r_z = 1$ and $A_{za} = \frac{\sum F_z}{(\sum k_z) (2D)}$

So, if we are able to calculate the mass or total weight W, either total mass m or total weight W; from that we can calculate the natural frequency of a transverse frame subjected to vertical vibration using this equation. Already, we know how to calculate W.

So, for each frame if we calculate omega nz, then the average value of all the natural frequencies can be considered as the average vertical natural frequency for the turbogenerator foundation. So, you see here omega nz average can be calculated by averaging all the natural frequency for all the frames. Likewise, we can calculate also the average vertical amplitude of the foundation using equation-12.

So, what is rz here? rz is the frequency ratio; so, you can see here this is rz not just r, rz is equal to omega divided by omega nz average; I can write it as average. So, I am just rewriting omega divided by omega nz average. What is summation of Fz? It represents the total vertical imbalance force; whereas, Kz it summation of Kz represents the sum of the stiffness of individual frame here; and D is the damping ratio.

So, if we know the damping ratio, if we are able to calculate the average vertical natural frequency, then we can also find out average vertical amplitude for this foundation system. Now, the special case is under tuned machine when you can see omega is less than omega nz average.

So, that time what we can do? We can consider omega is equal to omega nz average, because this is the maximum value possible and for under tuned machine. So, what will be A za at that time? So, basically rz in that case will be equal to 1. So, the first term here 1 minus rz square whole square, this will be 0; only the second term will be left. So, we will get summation of Fz divided by summation of Kz times 2D, rz it is already 1. So, this is the value of average vertical amplitude of the machine foundation when the machine is under tuned.

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- Lateral stiffness of an individual transverse frame $k_x = \frac{12EI_c}{H^3} \frac{6K+1}{3K+2}$... (13)
- The natural frequency the frame foundation in horizontal vibrations: $\omega_{nx} = \sqrt{\frac{\sum k_x}{W_T}}$... (14)
- Average horizontal amplitude of the turbogenerator foundation is:

$$A_{xa} = \frac{\sum F_x}{(\sum k_x) \sqrt{(1 - r_x^2)^2 + (2Dr_x)^2}}$$
 ... (15)

where, frequency ratio $\xi = \frac{\omega}{\omega_{nxa}}$

$\sum F_x$ is total vertical imbalance force,
 $\sum k_x$ is sum of the stiffness of the individual frames
 D is damping ratio

$\omega_{nxa} = \frac{\omega_{nxa1} + \omega_{nxa2} + \dots}{n}$

Now, when we are interested to do the horizontal details for the horizontal vibration, what we need to do? First, we need to calculate Kz. Sorry, Kx which is the stiffness or I can say it is as lateral stiffness of an individual transverse frame; so, that can be calculated using this expression 13. Then, from Kx we can calculate the natural frequency of the frame foundation in horizontal vibration.

So, here next step is to know the average horizontal, average horizontal amplitude of the foundation; and that can be calculated using this equation. So, here also not r, this is rx; rx is frequency ratio, which is equal to omega divided by omega nx average.

So, what is $\omega_n x_a$ here? Here $\omega_n x_a$ means some we are averaging $\omega_n x$ value for all the frames, divided by n ; n is total number of frames. And so, from this we can calculate r_x , then summation of F_x is total vertical imbalance force; and summation of K_x is the sum of the stiffness for individual frame.

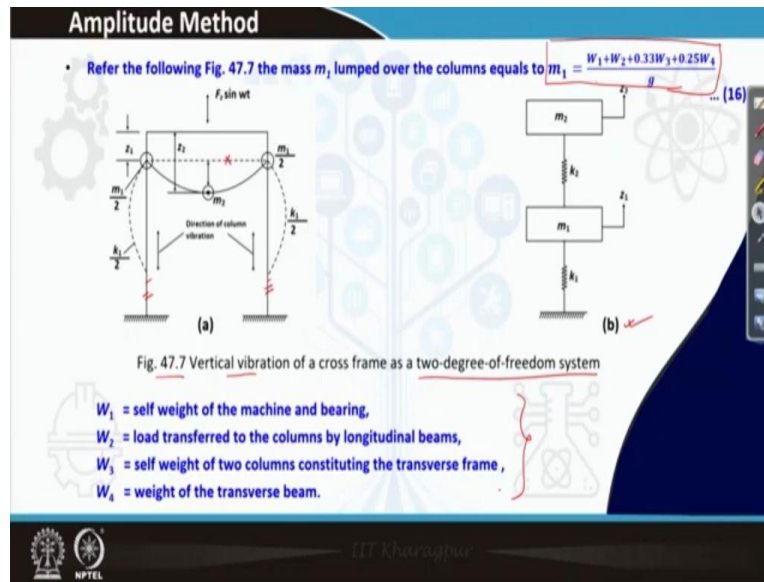
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**Two Dimensional Analysis-
Amplitude Method**

- The vibration analysis in amplitude method is carried out for each transverse frame independently.
- Here, the amplitudes due to forced vibrations are within permissible limits.

So, next method is amplitude method, which also in two dimensional analysis. As I already said, there are three methods which we can use. First one is resonance method, second one is amplitude method, and third one is the combined method. So, already we have studied resonance method, the second one is amplitude method. So, the vibration analysis in amplitude method is carried out for each transverse plane independently. Here the amplitudes due to the forced vibrations are within permissible limit.

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In figure 47.7, you can see a cross frame is subjected to vertical vibration; and this cross frame can be represented by a two-degree of freedom system as shown in figure b. So, in Figure a, what we can see? We can see the cross frame has different components; you can see the columns and the cross beam. So, what are the loads acting on this beam that we need to see and how to calculate the lumped mass over the column? So, in this figure, you can see total lumped mass acting over the column is equal to m_1 ; and that is divided into equally divided into two columns column one and column two.

So, m_1 is equal to here you can see W_1 plus W_2 plus $0.33 W_3$ plus 0.25 times W_4 , divided by g . And we can write it by equation-16; we can give a number 16. Now, what is W_1 , what is W_2 , what is W_3 and W_4 that is mentioned here. So, W_1 is self weight of the machine and the bearing; whereas, W_2 represents the load transferred to the column by longitudinal beams. W_3 is the self weight of the two columns constituting the transverse frame, and W_4 is the weight of the transverse beam.

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• Mass m_2 acting at the centre of cross-beam is: $m_2 = \frac{W_1 + 0.45W_4}{g}$... (17)

• Stiffness k_1 of both the columns of a transverse frame is equal to: $k_1 = \frac{2EA_c}{H}$... (18)

• Stiffness k_2 is: $k_2 = \frac{1}{\delta_{st}}$

where

$$\delta_{st} = \frac{L(2K + 1)}{96EI_b(K + 2)} + \frac{3L}{8GA_b}$$

... (19)

where,

- G is the shear modulus of the beam
- E is the Young's modulus of the material of column
- H is the effective height of the column
- L is the effective span of the column
- I_b is the moment of inertia of beam about the axis of bending and
- A_b are the cross-sectional area of the beam

Now, next is to calculate the mass m_2 . So, mass m_2 acting at the center of cross-beam is W_1 plus 0.45 times W_4 divided by g . Next, we need to calculate stiffness K_1 . K_1 is the stiffness of both the columns of a transverse frame; so, there are two columns. So, for both the columns, this stiffness K_1 can be calculated using equation 18. That can be calculated using the expression, 1 divided by δ_{st} , where δ_{st} is this expression shown in equation 19.

So, here what are the other symbols L ? Already we know what is L ; L is the effective span of the column; then k which is the stiffness already we have discussed. Then, next is E , E is Young's modulus of the material of column; G is shear modulus of the column material. A_b is the cross-sectional area of the beam.

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• Equations of motion:

$$m_1 \ddot{z}_1 + k_1 z_1 + k_2(z_1 - z_2) = 0 \quad \dots (20a) \text{ For } (m_1)$$

$$m_2 \ddot{z}_2 + k_2(z_2 - z_1) = 0 \quad \dots (20b) \text{ For } (m_2)$$

• Assume: $z_1 = A_1 \sin \omega_n t$, $z_2 = A_2 \sin \omega_n t$

• From Equations (20a) and (20b) the natural frequencies can be determined from the solutions of the following equation:

$$\omega_n^4 - (1 + \eta_m)(\omega_{n11}^2 + \omega_{n12}^2)\omega_n^2 + (1 + \eta_m)(\omega_{n11}^2)(\omega_{n12}^2) = 0 \quad \dots (21)$$

where, $\omega_{n11}^2 = \frac{k_1}{m_1 + m_2}$ and $\omega_{n12}^2 = \frac{k_2}{m_2}$ and $\eta_m = \frac{m_2}{m_1}$

• The natural frequencies will be found out using Equation (21).

• For forced vibration the equations of motion are:

$$m_1 \ddot{z}_1 + k_1 z_1 + k_2(z_1 - z_2) = 0 \quad \dots (22a)$$

$$m_2 \ddot{z}_2 + k_2(z_2 - z_1) = F_x \sin \omega t \quad \dots (22b)$$

Assume: $z_1 = A_{z1} \sin \omega t$, $z_2 = A_{z2} \sin \omega t$.

If we have several times solved different two-degrees of freedom system; so, we already know how to get the equations of motions for the two-degrees of freedom system. So, the problem which is already shown to you. For that also first we can draw the free body diagram for the two masses m_1 and m_2 respectively. And from that, we can get these two equations of motions which are equation 21 and 20, sorry 20a and 20b respectively; so, 20a for mass m_1 , 20b for mass m_2 . Now, after getting the equations of motion, we can assume trial solutions z_1 for the z_1 and z_2 as shown here.

So, from these trial solution and the equations 20a and 20b, we can get the equation 21. If we solve equation 21 which is shown here, then from this equation finally we can get the natural frequencies. So, solving equation 21, it is 21 please correct it. So, solving equation 21, we can get the natural frequencies. So, in equation 21 what is ω_{n11} , then ω_{n12} , and η_m in that is those are explained here you can see. Now, if the, now if we consider the forced vibration which is the actual case in the current problem, then the equations of motion will change from 20a to 22a, and 20b to 22b.

So, in equation 22b, you can see the right hand side is now non-zero. In this case, if we assume the trial solutions as shown here, then solving this equation 22a and 22b, what we get? We get the values for A_{z1} and A_{z2} . So, what are these A_{z1} and A_{z2} ? These are the amplitudes of vertical vibrations.

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• Thus amplitudes of vibration are:

$$A_{x1} = \frac{\omega_{n11}^2 F_z}{m_1[\omega^4 - (1 + \eta_m)\omega^2 + (1 + \eta_m)(\omega_{n11}^2)(\omega_{n12}^2)]} \quad \dots (23)$$
$$A_{x2} = \frac{[(1 + \eta_m)\omega_{n11}^2 + \eta_m\omega_{n12}^2 - \omega^2]F_z}{m_2[\omega^4 - (1 + \eta_m)\omega^2 + (1 + \eta_m)(\omega_{n11}^2)(\omega_{n12}^2)]} \quad \dots (24)$$

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So, amplitudes of vibration A_{z1} can be expressed by this equation. Likewise, A_{z2} which is the amplitudes of vibration for the mass m_2 , that can be determined by using the second equation. So, equation 23 and 24 provide the amplitudes of vibration A_{z1} and A_{z2} .

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SUMMARY

In this lecture following topics related to design the foundation for turbogenerator have been studied:

- Essential features
- Design criteria
- Load calculation
- Design load
- Method of analysis- Resonance method

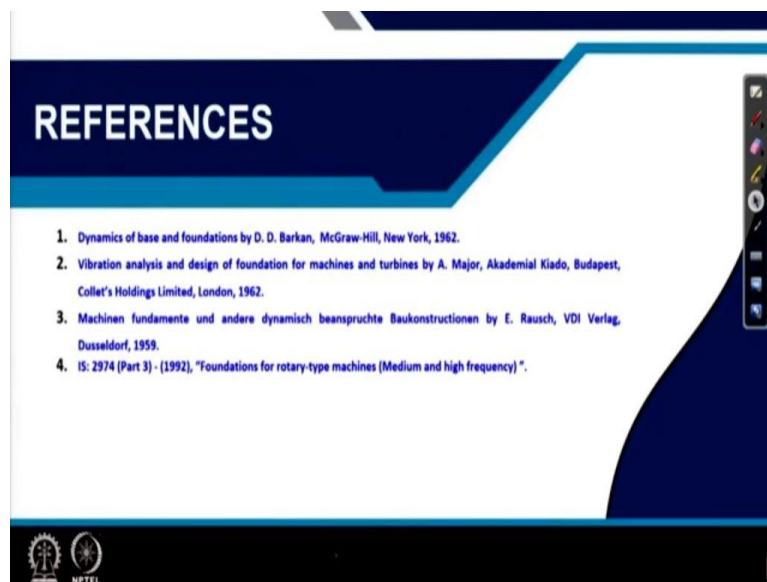
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So, come to the summary of today's class. In this lecture, we have discussed the frame foundation which is required for the turbogenerators; and what are the specific things of frame foundations we have discussed. We have discussed about its essential features, then we have discussed the design criteria. Then, we have discussed the load calculation; that means how we

will calculate different types of loads. And then how to calculate the design load that also we have seen.

After calculating design load, what we have done? We have studied how to do the analysis by two-dimensional method. In this for two-dimensional analysis, we have studied the resonance method for vertical and horizontal vibrations; whereas, for vertical vibration only, we studied the second method which is amplitude method. In next class we will continue the amplitude method, where we will study how to calculate the horizontal vibrations. That means, considering horizontal vibrations, how we will calculate the natural frequencies and the amplitudes of vibrations.

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So here you can see the references that I have used for today's class. Thank you.