Soil Dynamics
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Lecture 46
Analysis of Machine Foundations (For Impact Type Machines- Part II)

Welcome friends, today in the course soil dynamics, we will continue our discussion on analysis of machine foundations for impact type machines. So, today what we will do? We will solve one or two numerical problems to understand how to do the analysis of such kind of machine foundations. So, let us see the first numerical problem.

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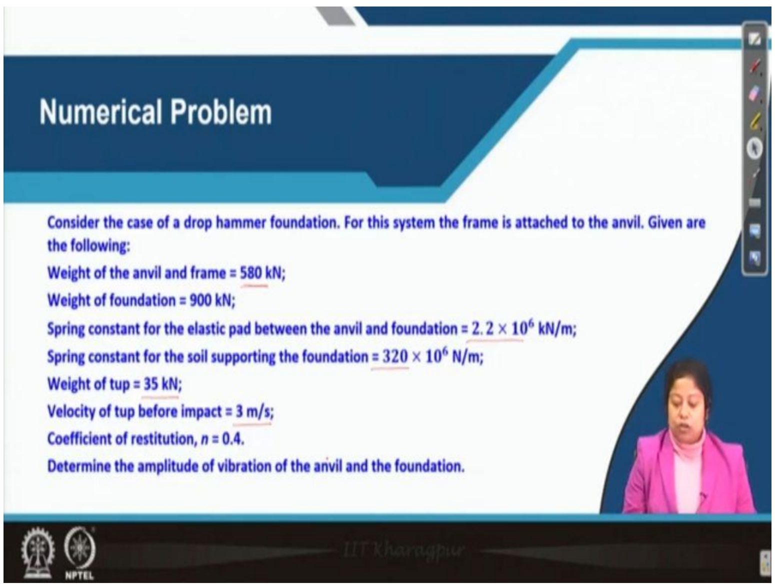

Here is the problem statement; so, let me read the problem first. Consider the case of a drop hammer foundation. For this system the frame is attached to the anvil, given are the following. Weight of the anvil and frame together is given which is 580 kilo Newton, weight of foundation that is 900 kilo Newton. Spring constant for the elastic pad between the anvil and foundation is provided; and it is 2.2 into 10 to the power 6 kilo Newton per meter.

Then, spring constant for the soil supporting the foundation that means soil below the foundation is provided which is 320 into 10 to the power 6 Newton per meter. Weight of tap or hammer is mentioned that is 35 kilo Newton. Velocity of the tup before impact is 3 meter per second; coefficient of restitution, n is given and that is 0.4. We are asked to determine the amplitude of vibration of the anvil and the foundation. That means, if I will take anvil as m2 and foundation as m1; then we need to find out z1 and z2 for the foundation and the anvil respectively. So, let us start to solve these numerical problems.

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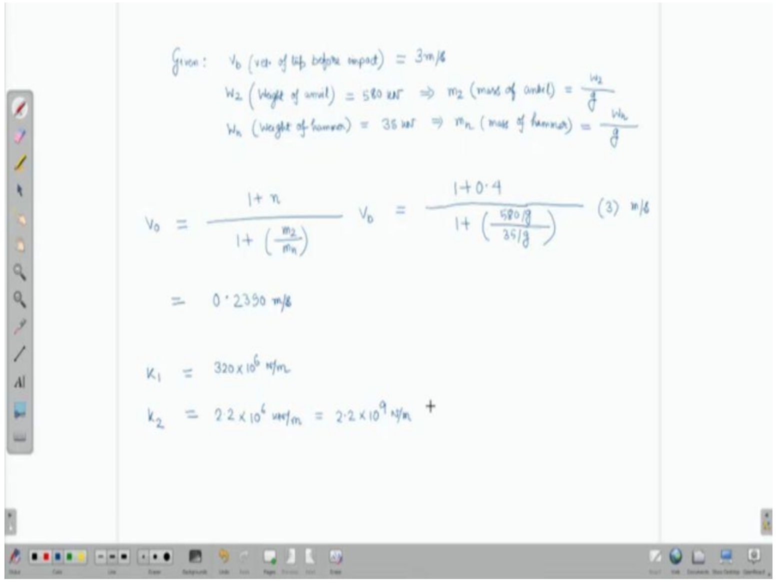

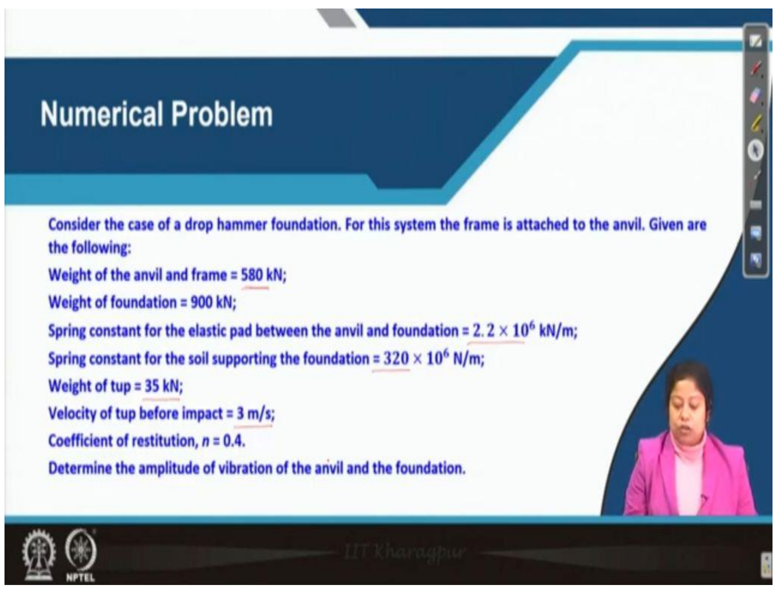

In this problem, given parameters are, let me write first. What are the parameters given vb; vb is the velocity of that tup before impact. So, this is given and it is equal to 3 meter per second. Also, weight of the hammer is given; sorry, not weight of the hammer, weight of the anvil is given. So, I can write it as weight of anvil which is 580 kilo Newton. So, from this we can calculate m2 which is mass of the anvil; and that is equal to w2 divided by h. Also weight of the hammer is given which is equal to 35 kilo Newton. So, from this we can calculate mass of the hammer which is equal to the wh divided by g, where wh is weight of hammer.

Now, we need to calculate v0; and v0 is equal to 1 plus m divided by 1 plus m2 divided by mh times vb. So, n is coefficient of restitution which is given 1.04, not 1 point; n is 0.4. So, 1 plus 0.4 divided by 1 plus 580 by g, divided by 35 by g. Since, m2 and mh gives a number; so, and w2 and wh both are given in same unit kilo Newton. So, I am not converting it in SI unit; because the ratio of m2 by mh should be equal to w2 by wh.

Now, vb is equal to 3 meter per second. So, v0 we are getting which is equal to how much? v0 is equal to in this case 0.2390 in meter per second. What else are given to us? k1 and k2 are given to us. What is k1? k1 is the stiffness of the foundation soil, which is 320 into 10 to the power 6 Newton per meter.

You can see here it is 320 into 10 to the power 6 Newton per meter, k2 is the stiffness of the elastic pad which is 2.2 into 10 to the power 6 kilo Newton per meter; so, we can write it as 2.2 into 10 to the power 9 in Newton per meter. So, now we have all the parameters.

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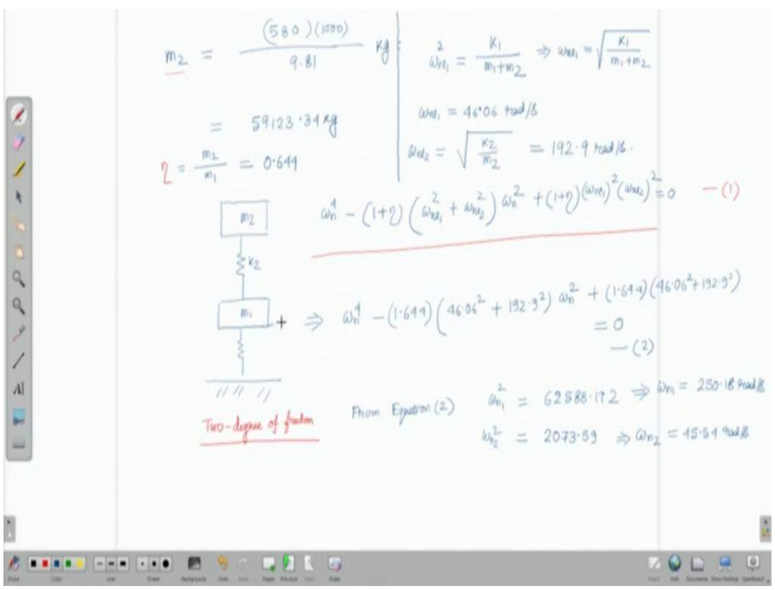

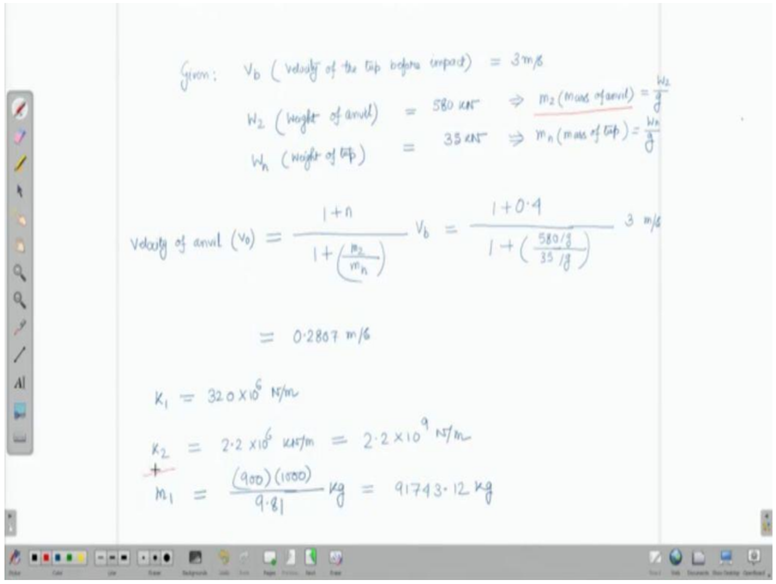

So, mass of the anvil is how much including the mass of the frame? It is 580 times 1000; this is in Newton divided by 9.81 in kg. So, how much it is coming? 580 times 1000 divided by 9.81; and I am getting it as 59123.34 in kg. I can, that means I can see 59123.34 kg.

Now, from these what we need to do? We need to construct the equation to get the natural frequency of the system. So, this system can be represented by a two degree of freedom system like this; already know what is m1, m2 and k1, k2, so I am not repeating. So, how we get the natural frequency for this system? Last class we have discussed, we need to solve this equation omega n to the power 4, minus 1 plus eta times omega nl1 square, plus omega nl2 square times omega n square, plus 1 plus eta times omega nl1 square, times omega nl2 square; and that is equal to 0.

So, now, we need to solve this equation; I can give it a number equation-1. We need to solve to get omega n. Since, this is two degree of freedom system, so we will get two positive roots for omega n, two-degree of freedom. So, what is the value of eta here? Eta means, if I will write it here, eta means it is the ratio of m2 upon m1, which is coming 0.644.

So, now I can write rewrite the equation-1 using the writing the coefficients of omega n to the power 4 and omega n square. So, there it is coming, omega n to the power 4, minus 1.644 times. I need to also calculate omega nl1 square and omega nl2 square. What is omega nl1 and omega nl2 here? Omega nl1 is equal to, or omega nl1 square is equal to k1 divided by m1 plus m2.

So, omega I can write omega nl1 is equal to square root of k1 divided by m1 plus m2; I can make the border here. So, omega nl1 is coming then equal is equal to 46.06 radian per second; likewise we can calculate omega nl2, which is equal to square root of k2 divided by m2. And we already know the value of k2 here, and also we know the value of m2 here. So, from this what we can get? We get the value for omega nl2. And it is coming, let me check, let me calculate; square root 2.2 into 10 to the power 9, divided by 59123.34. So, it is coming 192.9; actually coming 899 something, so 192.9 in radian per second.

So, these are the two values for omega nl1 and omega mnl2. So, I can now write it here 46.06 square plus 192.9 square, plus 1.644 times 46.06 square plus 192.9 square; and that is equal to 0. So, now we need to solve this equation; I can write it as equation-2. So, from equation-2, we get omega n1 square and omega n2 square; so, let me calculate. I can calculate directly the value of omega n1 square and omega n2 squared. So, omega n1 square is coming 62588.112; so, that means omega n1 is 250.18 in radian per second. Likewise, omega n2 square is coming 2073.587; or you can write even point 59, from which omega n2 is coming 45.54 in radians per second.

So, now we know the natural frequencies of two different modes of vibrations for these machines foundation soil system. Now, from these we need to calculate the values for amplitudes of the anvil and the amplitudes of vibration for the foundation.

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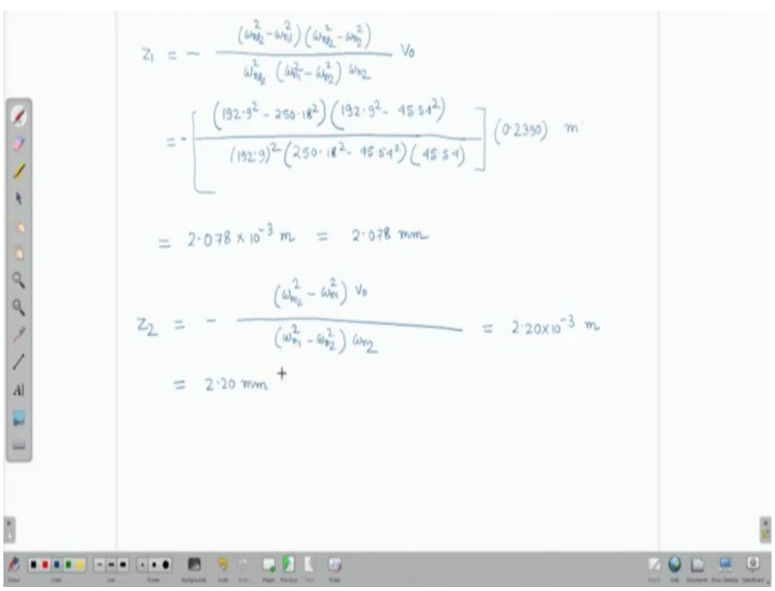

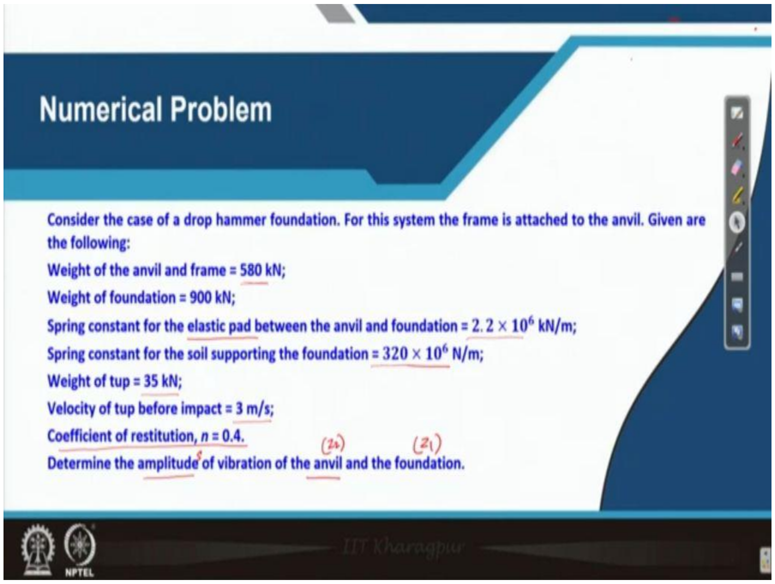

So, now we need to calculate z1 and z2. What is z1? z1 is the amplitude of vibration that is displacement of the foundation. And that we can calculate by using this equation which I am writing here. Omega nl2 square minus omega n1 square, times omega nl2 square minus omega n2 square, divided by omega nl2 square times omega n1 square, minus omega n2 square times omega n2. And this whole thing is multiplied with v0. So, now here if we write the values of omega nl2, omega nl; sorry, omega n1 and omega n2. Also if we use the value of v0, then we will get z1; so, let us do that.

Omega nl2 is 192.9 square minus 250.18 square. This is a 250.18, it is the value of omega n1; the second term is 192.9 square minus 45.54 square, where 45.54 is the value of omega n2. So, these things will be divided by 192.9 square; this will be multiplied with 250.18 square minus 45.54 square. And these two things will be multiplied with 45.54 which is omega n2. And this entire thing will be multiplied with v0 which is 0.2390 in meter per second. So, z1 what we will get that will be in meter. So, how much you are getting? Here we are getting the value of z1 is equal to 2.0.

We are getting the value of z1, which is 2.078, into 10 to the power minus 3 in meter. So, we can convert it in millimeter and that is then 2.078 millimeter. Likewise, we can calculate z2 by using this equation which I am writing here. The equation is minus of omega nl2 square minus omega n1 square times v0, divided by omega n1 square minus omega n2 square times omega n2. And that is equal to if we will write the value of omega nl2, omega nl1 and v0 in this equation, then what we will get? We are getting 2.20 into 10 to the power minus 3 meter. So, we can convert it in millimeter, then z2 will be equal to; then z2 is equal to 2.20 millimeter.

Here actually we are calculating two amplitudes; so, amplitudes of vibration of the anvil which is z2, and foundation which is z1. I hope this problem and how to do the analysis is clear to all of you.

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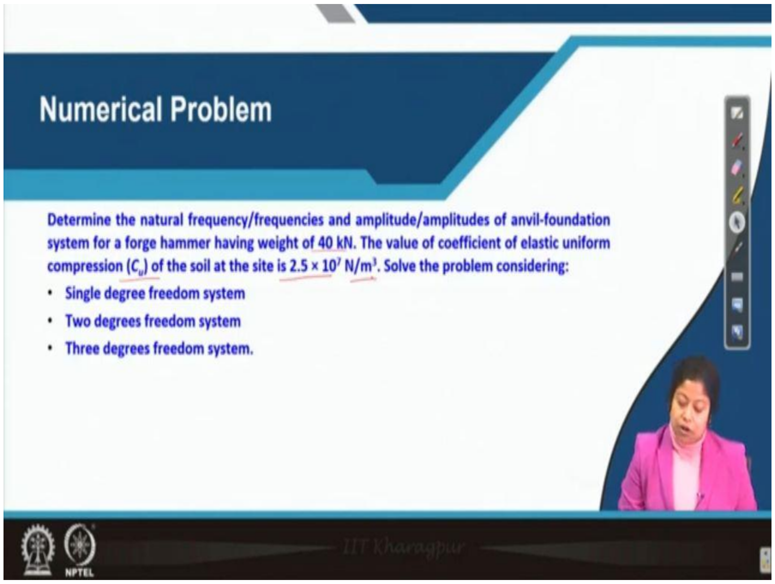

Now, we will see the second problem of this machine foundation, hammer type machine Foundation. So, this problem is little bit bigger problem. So, what I will do? I will solve partly for you and will demonstrate the remaining part for you. And I will expect that you will complete the, you will be able to solve the problem properly and will get the final answer.

So, first let me read the problem statement. Determine the natural frequency or frequencies and amplitude or amplitudes of anvil foundation system for a forge hammer having weight 40 kilo Newton. The value of coefficient of elastic uniform compression which is Cu of the soil at the site is given, that is 2.5 into 10 to the power 7 in Newton per meter cube.

And you are asked to solve the problem considering single degree of freedom system, two degrees of freedom system and three degrees of freedom system. So, we already know in under which condition the problem will be treated as a single degree of freedom system, when it will be treated as two degrees of freedom system, and when it will be treated as my three degrees of freedom system.

So, if we are providing elastic padding between anvil and the foundation block, then the problem will become a two degree of freedom system. Likewise, if we will provide another elastic padding below the foundation and then there will be a (()) (24:13); then it will be treated as three degree of freedom system. So, let us solve this numerical problem one by one.

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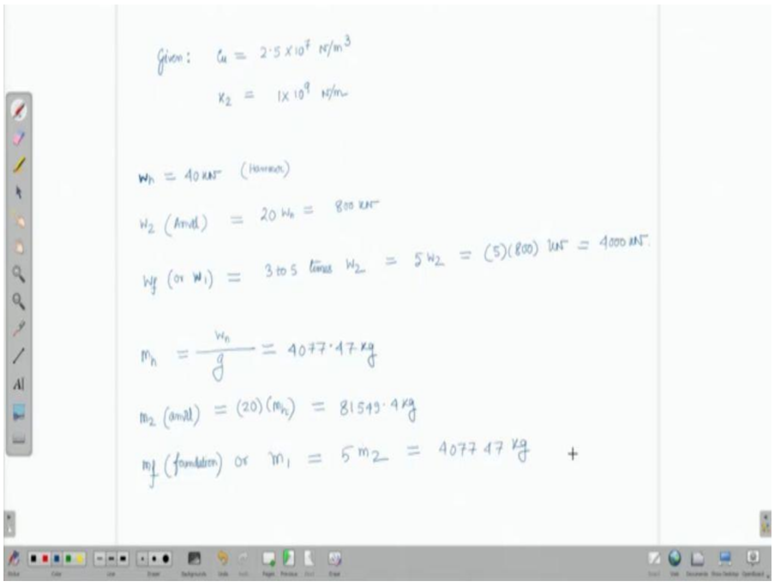

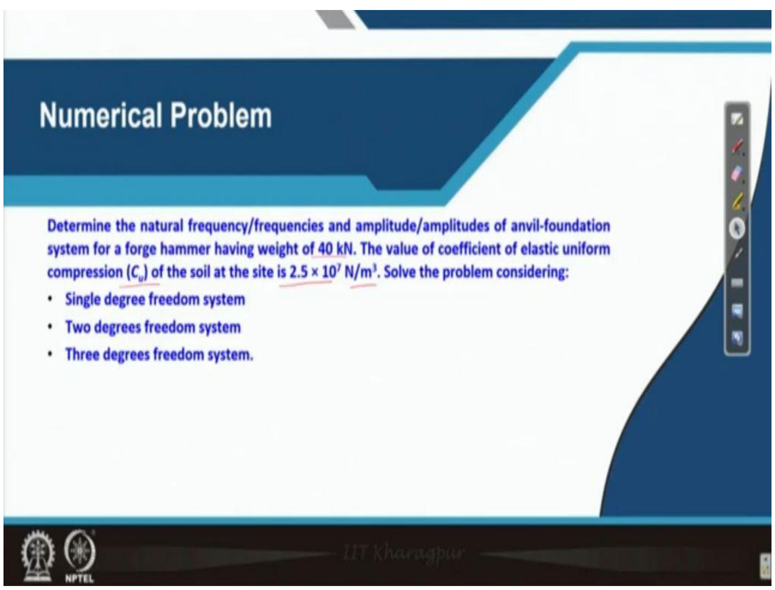

So, what is given us? Elastic coefficient of elastic uniform compression that is Cu; and the value of Cu is 2.5 into 10 to the power 7 in Newton per meter cube. Also, k2 which is the spring stiffness for the elastic padding is given that is 1 into 10 to the power 9 in Newton per meter.

Now, what is said about mh? It is said that the weight of the hammer which is the wh is equal to 40 kilo Newton;

So, I can write here the wh is 40 kilo Newton. Now, if you recall the design guidelines for the hammer type machine foundation 40 said there, it is said that the weight of the anvil will be 20 times of weight of the hammer. So, the weight of the anvil which is w2 here, I can write anvil just; so, this is for hammer.

So, we talked the anvil is 20 times the wh; so, it is coming then 800 kilo Newton. It is also said that the weight of the foundation in such case will be 3 to 5 times the weight of the anvil. So, weight of the foundation or I can say w1 also; that is equal to 3 to 5 times w2. So, let me consider the wf is 5 times w2; so, 5 times 800 in kilo Newton, it is coming 4000 kilo Newton. Now, we can get the masses of hammer anvil and foundation one by one. So, mh which is mass of hammer is equal to how much, let me find it out. So, mass of the hammer is coming 4077.47 kg.

Then, mass of the anvil is coming 20 times off m2; so, it is coming, yes. So, m2 is coming 81549.4 kg that means 81,549.4 kg. Now, mass of foundation this is for foundation, or I can write mf also or m1; it is how much? It is 5 times of, achha, please correct one line, it is 20 times mh; so, this is mh. Now, mf is how much? It is coming; mf is coming 5 times of m2; that means it is 4077.47 kg. So, it is actually 4 lakhs 7747 kg. So, now we know the weight of the foundation block. So, from this what we can do?

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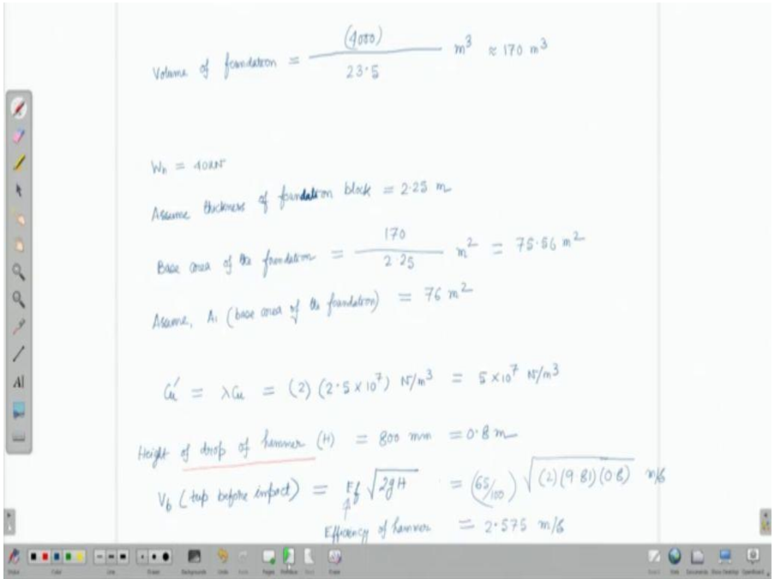

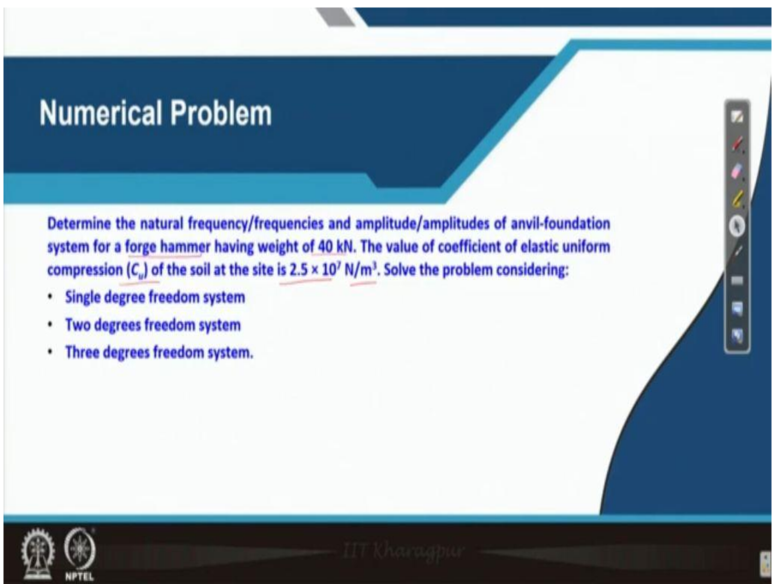

We can find out the volume of the foundation block. So, volume of foundation block we know the weight which is, if you see the weight is 4000 times 1000 for; we are converting it in Newton per meter Newton, or I can keep it kilo Newton also. In that case I need to write the unit weight of concrete in kilo Newton per meter cube; and that is if that is 23.5. Then, I will give the volume of foundation; and it is approximately 170 meter cube. This is the volume of the foundation.

Now, if you have seen that the weight of the hammer is 40 kilo Newton; and guideline says if the weight of the hammer is this much, then we need to consider the thickness of the foundation block equals to 2.25 meter.

So, we need to assume thickness of foundation block 2.25 meter. So, from this we can get the base area of the foundation; how much it is? 170 divided by 2.25 in meters square. So, we are getting 75.55 Or 56 meter square. So, I can assume A1 which is base area of the foundation is equal to 76 meter square.

Now, another important thing I need to calculate which is modified value of the coefficient of elastic uniform completion, which is Cu dashed; and Cu dashed is equal to lambda times Cu. So, in this case, we can consider lambda is equal to 2; we can take any value in between 1 and 2. So, I am assuming it 2 here times Cu, which is given 2.5 into 10 to the power 7 in Newton per meter cube.

So, we are getting 5 into 10 to the power 7 in Newton per meter cube. So, Cu dashed we calculated. Now, what is next? Now, if we, let us take we need to also know what is the velocity of the anvil. So, for that what is given to us? If you see only the specification or type of the hammer is mentioned here; so, we can assume the height of drop of the hammer. So, we need to assume height of drop of hammer which is capital H if; and from that we can find out the velocity of the tup before impact. This is for tup before impact and how we can calculate it. This is Ef times square root of 2g capital H.

So, let us assume this height of drop of the hammer is how much? Let us take it 800 millimeter that means 0.8 meter. And we can consider the efficiency of the hammer; this is efficiency of hammer which can vary from 50 to 80 anything, so, let us take 65% efficient this hammer. Then, Ef is 65 divided by 100 times square root of 2 times g, which is 9.81. You can write here 9.81 also times h, which is 0.8; and this is in meter per second. So, vb is coming in this case, then 2.575 in meter per second. Now, we know the vb which is that velocity of the tup v before the impact.

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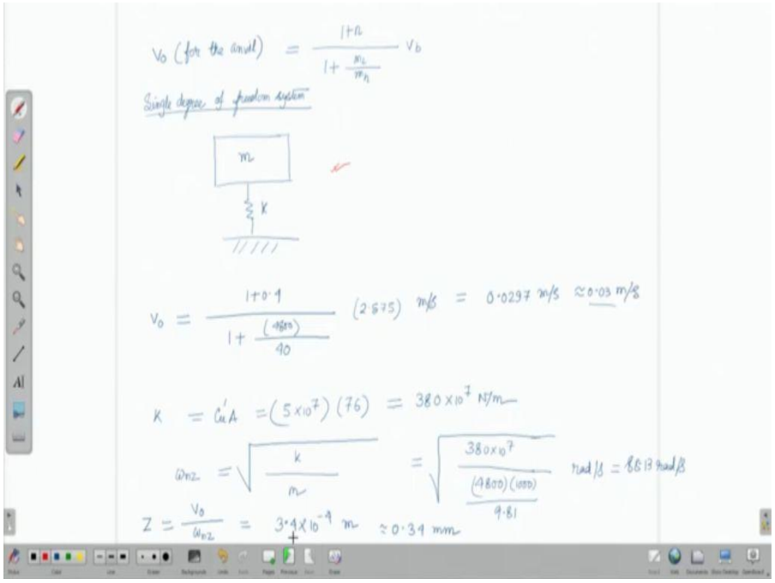

Then, from that we can calculate v0 for the anvil. Already in the first problem, we have seen how to calculate it; 1 plus n divided by 1 plus m2 divided by mh times vb. So, let us assume n is equal to 0.4, because in this problem nothing is mentioned. So, 1 plus 0.4 divided by 1 plus.

So, now, actually if you see what is m2. Now, way we will consider the problem as a single degree of freedom system; that means there is no m2, m1 separately; we have total mass that is connected to the soil like this. So, this is for single degree of freedom system. So, for this type of problem, if I will write it as K, then m2 means m itself which is the total mass of the anvil and the foundation.

So, I am just erasing this line. Now, for single degree of freedom system what will be v0 that I am calculating. So, 1 plus 0.4 divided by 1 plus; in this case in case of m2, it will be m; which is in your case it is you can get it as 48,000. Sorry, 4800 divided by 40 times 2.575 which is vb in meter per second. So, it finally you are getting 0.0297; or you can take it even 0.03 meter per second.

So, in this way, you can calculate v0 which is the velocity of the anvil for the single degree, considering single degree of freedom system. Now, after calculating v0 what we can do? We can calculate K1 or K in this case; for single degree of freedom system in place of K1, I can write K which is Cu dashed times base area.

So, Cu dashed we have already calculated, this base area we have assumed 76 square meter. So, K is 380 into 10 to the power 7 in Newton per meter. So, from this for a single degree of freedom system we can calculate natural frequency for vertical vibration, which is omega nz. So, we can use this equation 380 into 10 to the power 7; m means 4800 it is in kilo Newton, now converting it in Newton, divided by 9.81.

So, finally, I am getting it as 88.13 radian per second. And from this I can calculate z, not z1; because in this, it is a single degree of freedom system. So, z is the amplitude of displacement and that is v0 divided by omega nz; omega nz 88.13 and v0 0.03 meter per second.

So, finally it gives 3.41 into 10 to the power minus 4 in meter, or 0.34 millimeter. So, in this way we can solve the problem considering it as single degree of freedom system as shown in this diagram. For multiple degree of freedom system specifically for this two degree of freedom system, how we can analyze the problem? Let us see.

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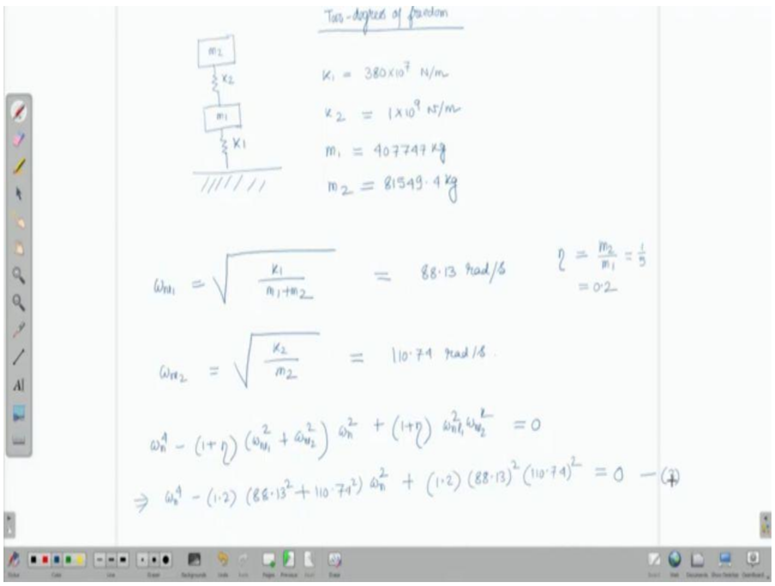

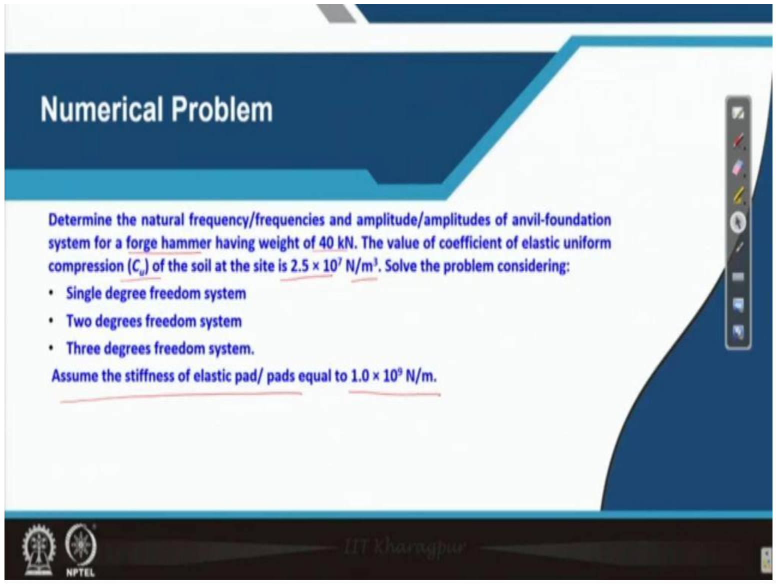

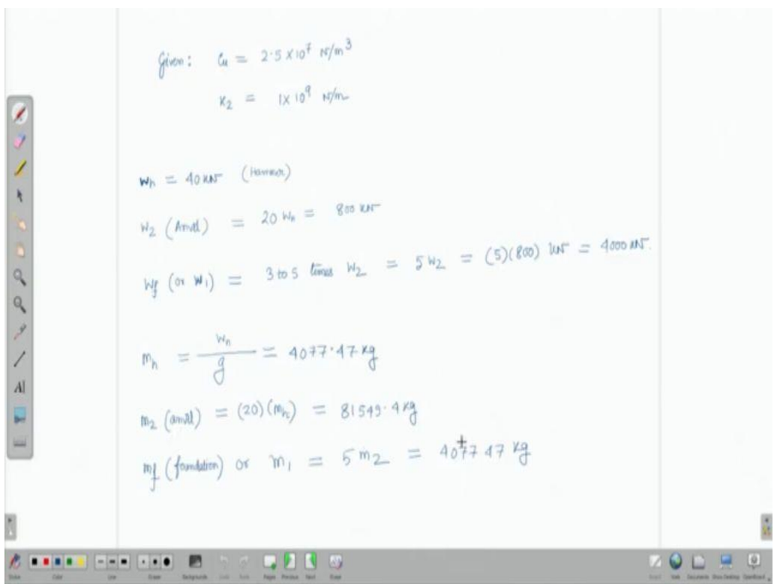

So, there are two masses m2 represent anvil, K2 is the stiffness constant for the elastic padding. Now, that is connected to m1 which is the mass of the foundation connected to the spring K1; K1 represents the soil stiffness. So, in this way, we can first construct a problem as two degree of, two degrees of freedom.

Now, what we need to do? We need to solve this problem. Already, we know what is the value of K1; so, K1 here is 380 into 10 to the power 7 in Newton per meter. Likewise, K2 it is already given if you see, let us see I think it is given. Assume the stiffness of elastic padding 1 into 10 to the power 9 Newton per meter; so, 1 into 10 to the power 9 Newton per meter. So, K2 is also given.

Now, we know the value of w1 and the w2; from that we can calculate m1 and m2. So, here you can see we have already calculated m1 and m2. So, I will directly use these values of m1 and m2. Just I am writing here, m2 for the anvil which is; anvil it is 800, so 549.4 kg. Now, first thing to calculate omega nl1 just like the numerical problem one, which is K1 divided by m1 plus m2. So, it will be equal to actually 88.13, the same value which we get for single degree of freedom system omega nz; that is here omega nl1. Omega nl2 is equal to square root of K2 divided by m2.

So, now if I will use this parameter, I will get it is 110.74, 110.74 in radian per second. So, omega nl1, nl2 now we have calculated; so, next step is to construct the equation. We already know the general form which I am writing here first. How we are getting that is already discussed when we discussed the theory of vibration and also the last class; so, I m not repeating those things here.

So, here what is the value of eta? Eta is 0.2. What is eta? It is m2 divided by m1. So, m1 we have taken 5 times of m2, so this is 1 by 5 or 0.2. As a result, we can write this equation once again as this form, 88.13 square; sorry, plus 110.74 square, multiplying it to omega n square, plus 1.2 times 88.13 square times 110.74 square is equal to 0. So, if we have this equation, then from let us write it as equation 3. So, from equation-3, we can get four roots of omega n; there are two roots positive roots, so that routes are useful for us.

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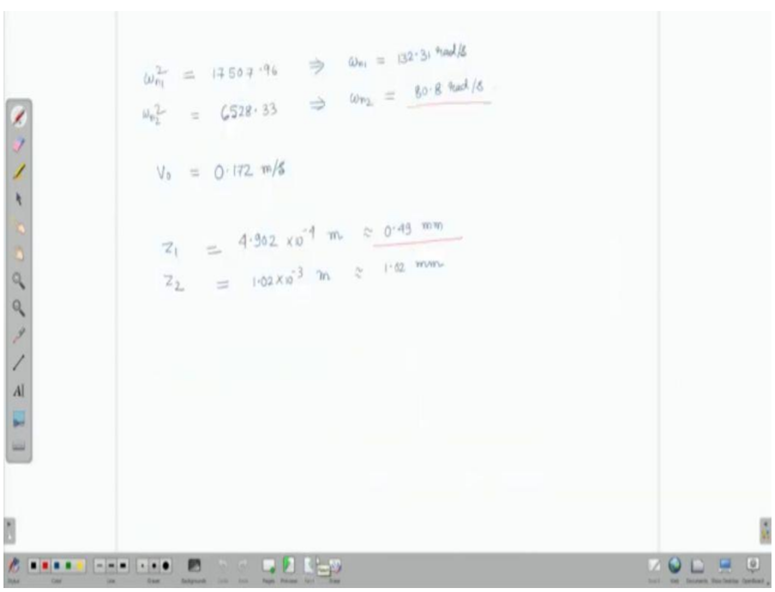

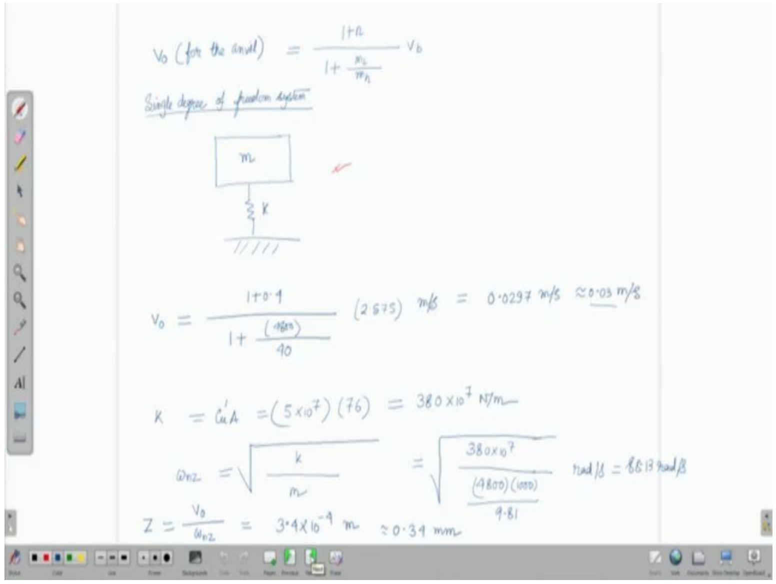

So, the positive two roots are omega n1 square and omega n2 square; omega n1 square is coming 17507.96, from this we can get omega n1. It is coming omega n1 is 132.31 in radian per second. Likewise, omega n2 is coming 66528.33, from which I can write omega n2; this is 80.8 radian per second. Already we know how to calculate v0; so, we will use this equation. But, in this case now m2 is the mass of the anvil, and mh is the mass of the hammer. So, mass of the anvil means it is 800, the mass I cannot say; weight of the anvil is 800 kilo Newton, and hammer is 440, sorry hammer weight is 40 kilo Newton.

So, the ratio is m2 by mh is coming 800 divided by 40, which is 20. So, from that we can calculate v0 which is coming 0.172 in meter per second. Here I have taken n is equal to 0.4. So, already I have shown how to calculate z1 and z2; so, I am not writing once again the equation. So, I am writing the final value for z1 here, which is coming 4.902 into 10 to the power minus 4 meter; or in millimeter, it is 0.49 millimeter. Likewise, z2 is 1.02 into 10 to the power minus 3 meter; in millimeter, it is 1.02 millimeter. So, in this case the displacement of the foundation if you see it is coming 0.49 millimeter.

Now, if I will go back to our previous calculation there you can see the displacement of the foundation was 0.34 millimeter; but, the natural frequency was very high which is 88.13 in radian per second. Whereas, in present case when we consider, it has two degree of freedom system; that means we provided elastic padding in between anvil and the foundation, that time we are getting it this value 80.8. So, just I am going back, so 88 is now reduced to 80 radian per second.

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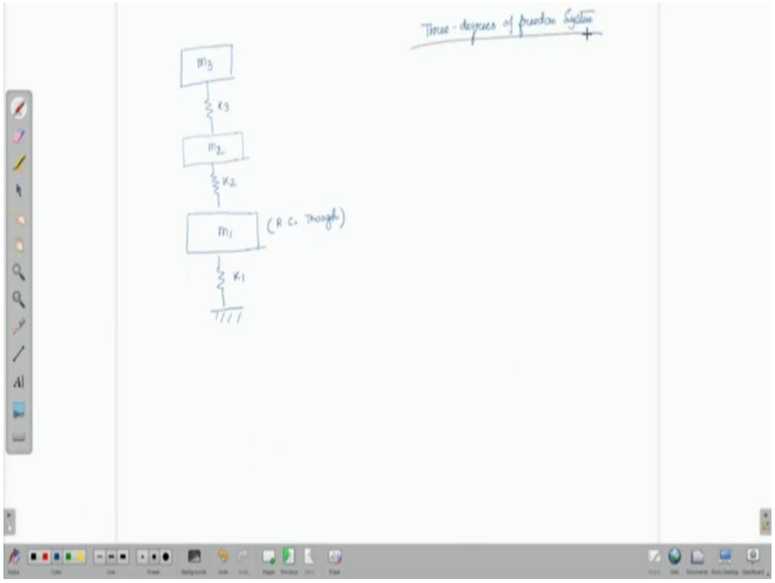

Now, we can solve the same problem by introducing another elastic pad or spring absorbers between foundation and that trough, RC trough. So, in that case, I can write it like this; I am quickly just writing this thing. So, here, m3 is the mass of the anvil, K3 is the spring stiffness which represents the stiffness of elastic Padding. Now, it is connected to the foundation, mass is m2 this case, and the stiffness spring the K2 is connected to m2; that K2 represents the stiffness of the elastic padding provided in between foundation and the RC trough. Now, m3, m2 then m1; so, this is your, this is the three degrees of freedom system. Now, I can go to the next page.

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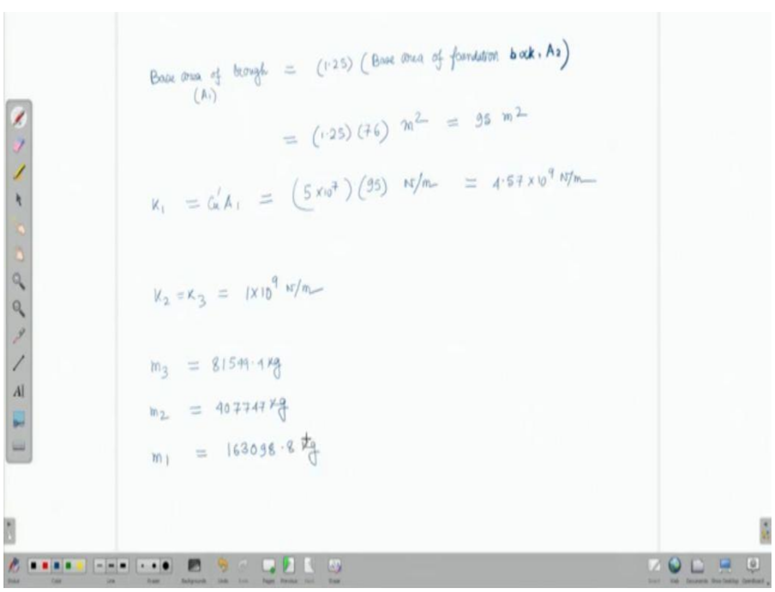

So, here first I need to find out, I need to consider also something. So, let us consider the base area of the trough because this is only unknown for us. We can generally take it as 1.25 times of base area of foundation block which is A2. Now, from already we know what is the base area of the foundation block, we consider it as 76 square meter. So, then base area of brough is coming 9; it is approximately coming 95 meter square.

Now, then K1 value will be how much? Cu dashed times A1, so this is A1; so, A1 means 95 square meter. So, finally, what we are getting is 95 and it is now Newton meter; and we are getting 4.75 into 10 to the power 9 Newton meter.

Likewise, we are getting K2 and K1 which is already; sorry, not K2 and K1, K2 and K3 which is already given K2 and K3 are same. And that is 1 into 10 to the power 9 Newton meter; so, this is given. Already we have calculated what is m3, mass of the anvil; and that is 81549.4 kg, m2 which is mass of the foundation is equal to 407747 in kg; and finally m1, this is 1 lakh 63; this is 163098.8 kg. So, m1, m2, m3 these three are given.

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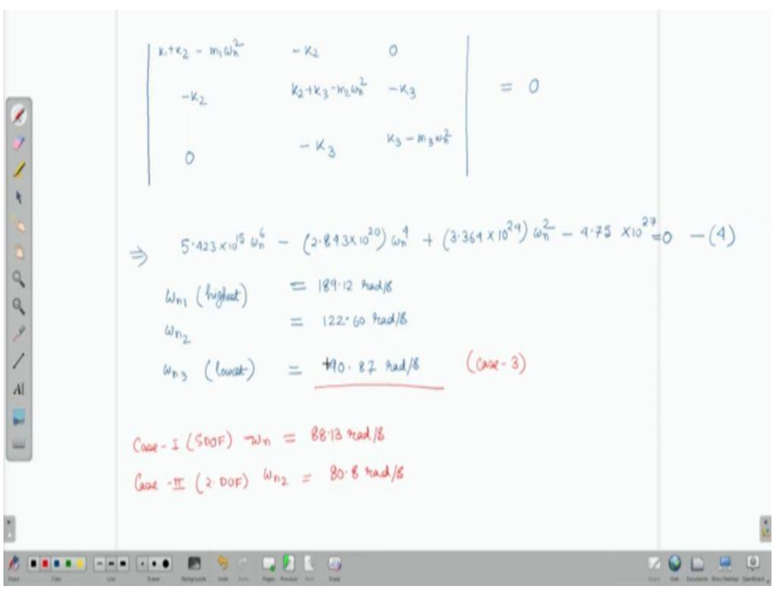

So, we know for non-trivial solution, what is the condition? So, I am writing the determinant of this which I am writing here, K1 plus K2 minus m1 times omega n square. This is the first row first column element, then minus K2 is the first row second column element. Third row, sorry first row third column is 0. Likewise, we can write the elements in other rows and columns; let me do that, it is 0 not. So, this determinant is equal to 0.

Now, if we will expand it, what we will get? We will get 5.423 into 10 to the power 15 times omega n to the power 6, minus 2.843 into 10 to the power 20 times omega n to the power 4, plus 3.364 into 10 to the power 24 times omega n square, minus 4.75 into 10 to the power 27 is equal to 0.

So, give it a number equation-4. Now, if we will solve this equation 4, then we will get three positive roots of omega n. Let us take these three positive roots are omega n1, omega n2 and omega n3. And also consider omega n1 is the highest one and omega m3 is the lowest one among these three.

So, if we will solve equation-4, then what we will get? Let me write here. So, omega n1 is coming 189.12 in radian per second, omega n2 is equal to 122.60 in radian per second, omega n3 is equal to 40.87 radian per second. Now, here the interesting thing is that when I am getting the three routes that means when I am considering three degrees of freedom system.

And that time I am getting the three routes; and the smallest one if you see here is 40.87 radian per second. So, this is for the case-3. What are the smallest value of omega n for other two cases? For case-1, where we have considered single degree of freedom system if you recall, there we have determined only one root which is equal to 88.13 in radian per second. Whereas, in case-2 that is two degree of freedom system, there were two roots; and that smallest one which we determined is 80.8 radians per second.

So, from this what we can see when we are considering three degree of freedom system that means we have considered the two elastic padding material for vibration isolation. That time what we get? We get the smallest value of omega n which is 40.87 in radian per second.

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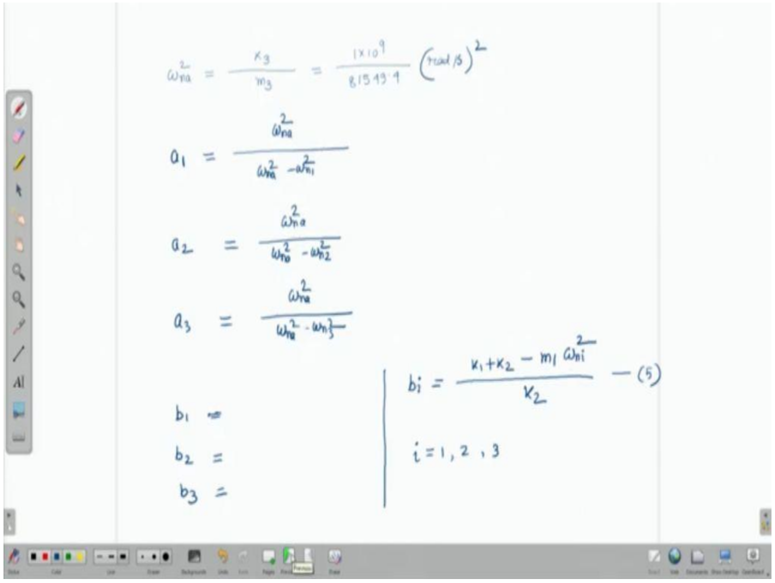

We will find out omega n square. If you recall, it is K3 divided by m3; K3 value is known for you. So, you will get no, it is radian per square per second square. So, then you can get A1, where you will use this equation; likewise, you will get A2 using this equation, and A3 using this equation. After knowing A1, A2, A3 you can also calculate b1, b2, b3.

So, I am writing in general it as bi; so, i may be 1, 2 or 3. So, what you are getting? How you will get that I am just writing minus n1 in i square, divided by K2; so, i is here 1, 2, or 3. So, for b1, omega in place of omega ni you need to write omega n1; for b2, you need to write omega n2 in this equation-5. And for calculation of b3, you need to write here omega m3 in equation-5. So, in this way you can get b1, b2 and b3.

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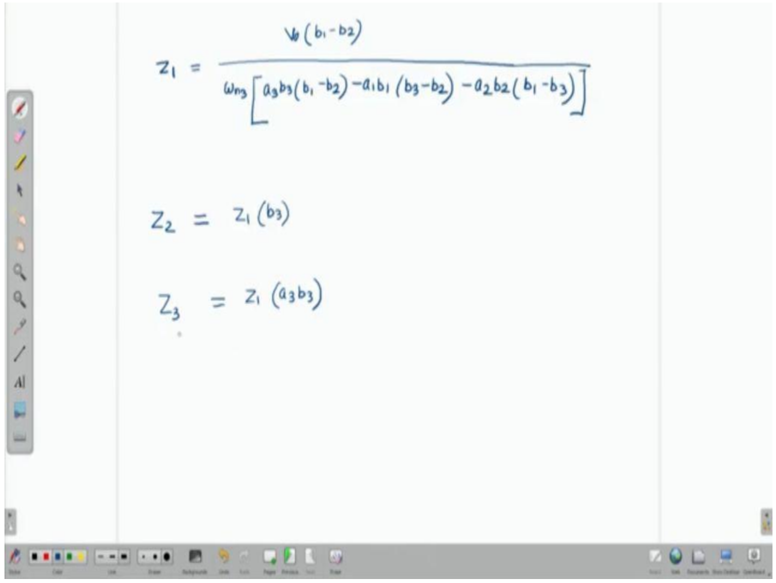

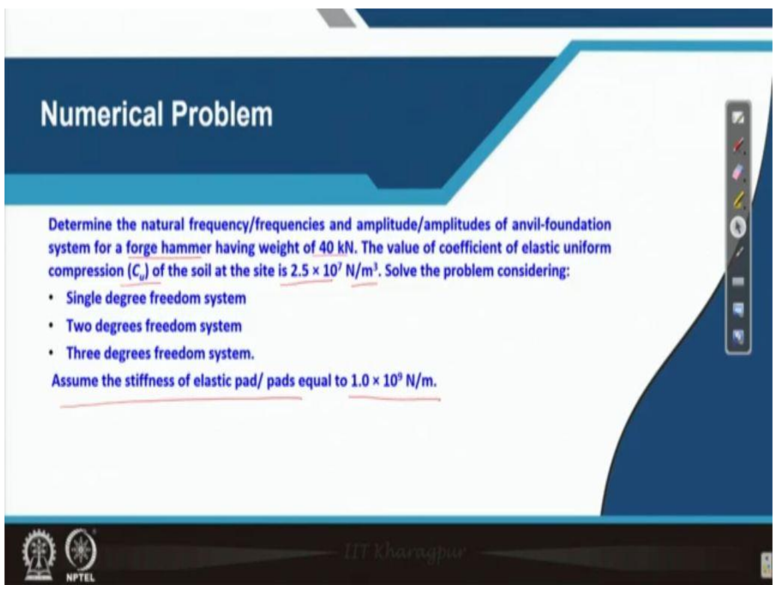

After knowing b1, b2 and b3, find the next step is to find out z1, z1 means it is your amplitude of displacement; so, for that you can use this equation. I am just writing it here. So, I hope you will be able to solve this equation; you will be able to solve this expression, not equation.

So, here next is z2. If you see, already I have shown the expression for z2 in the last class. So, that is actually z1 times we can multiply it by b3. Likewise, z3 is actually z1 times a3 times b3. So, in this way, you can give a try to calculate z1, z2, z3; and can check by yourself whether z3 is changing? Is it increasing or decreasing from the in comparison to the previous two cases?

So, today's lecture, we basically discussed two numerical problems, how to do the design of machine foundation for the hammer type machine. Here you can see the references. So, with this I am stopping today. Thank you.