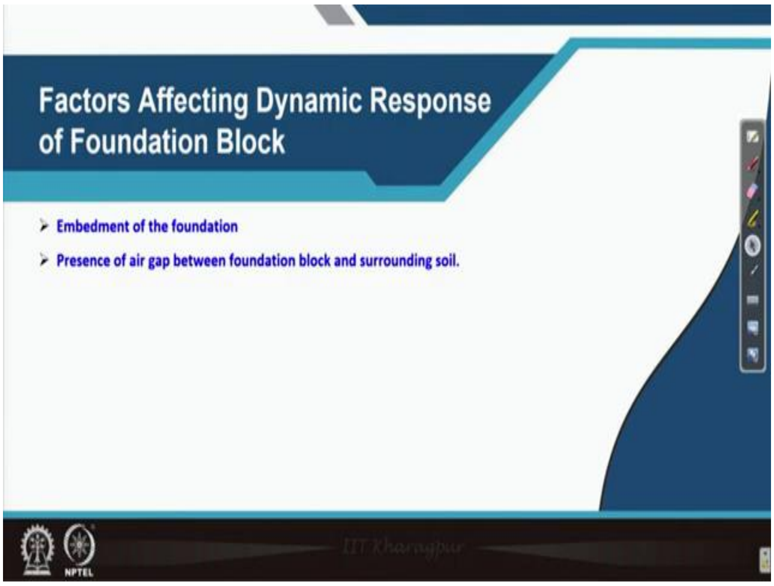
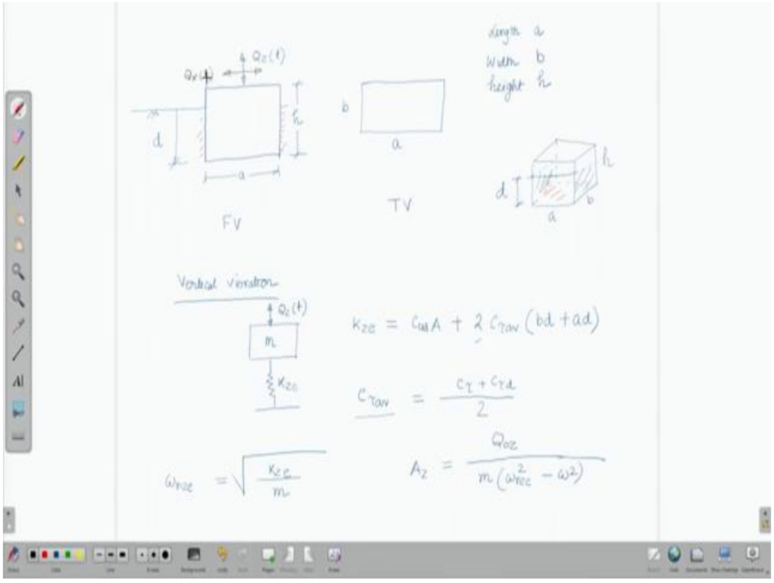
Soil Dynamics
Professor. Paramita Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture No. 43
Analysis of Machine Foundations
(For Reciprocating Machines - Part 3)

Hello everyone, welcome to the course Soil Dynamics. So, in this course under the heading analysis of machine foundation, we have discussed several things like how to do the analysis of machine foundation by using elastic half space theory. For these, we have spent one week. Now, in the current week, we are studying how to do the analysis of machine foundation for reciprocating machines by using linear elastic weightless spring method.

Today, what we will do, we will first discuss the factors which affect the final value of the natural frequency of the machine foundation system, machine foundation soil system and then we will study that we will do some numerical problem so that our understanding on analysis of machine foundation for reciprocating machine should be clear.

(Refer Slide Time: 2:07)





So, let us see what are the factors that affects the response dynamic response of foundation block. So, here you can see there are two factors mentioned, one is the embedment of the foundation block and the second one is the presence of air gap between foundation blocks and the surrounding soil. So, let us talk about the first one that means that embedment of the foundation.

So, what we have discussed so far under the heading of analysis of machine foundation is that we have considered that the foundation block is resting on the soil, that means on the ground surface. Now, what will be happening if the foundation is embedded into the soil up to some depth. That is the practical case actually. So, let us see that. So, here first I would like to draw the problem which I am discussing right now.

So, let us take a foundation block, this is our foundation block, it is the front view. So, the size of this foundation block we can take length a, it is height let us take small h and it is width perpendicular to along the direction perpendicular to the board that means in this direction if you consider, so b is how much that is b actually. So, I am writing here what I have seen the length of the block is a, width, this is b and height of the block is small h.

Now, let us consider that this block is embedded and the depth of embedment if you see that means the portion of the foundation block below the ground surface that is your small d. So, depth of embedment is small d. I can show that plan view of the same. So, in plan view, it looks like this a and this is b. So, I can write plan view means top view. So, I can write it as TV here.

Now, what will be happened in this case? If you see these portion, I can use different color these portion of the foundation block is inside the soil. So, what will be happening when it is subjected to vibration, the surrounding soil will also offer some sort of resistance which are provide some strength to the foundation. So, let us see when this foundation block is subjected to vertical vibration.

So, for vertical vibration what is happening that means here one dynamic force in vertical direction that means in z direction is considered. So, when this foundation block was resting on the ground surface, that time the entire foundation soil system was represented by a mass and a spring. So, the mass represented the total weight of the foundation block and the machine together, whereas spring represents the steepness of the soil. In this case, we have considered the spring is massless.

So, the same thing we will do here that means, we will represent this foundation and machine together by a mass m and the soil by it is stiffness that means this spring kz, it so the foundation is subjected to vertical vibration. Now, this is when the foundation block is placed above the ground on the ground surface. Now, when it is embedded that times instead of kz which represents the stiffness of the spring instead of that we are taking kze.

Now, what is kze it is the soil stiffness but this time as I already said that the surrounding soil of the foundation now will provide some sort of resistance, whereas earlier case only the soil below the foundation block provides resistance. So, this time and because of that additional resistance we will see the value of kze will change. So, how we will calculate, now kze? In this case, this kze is equal to C ud, C ud means c u value which is the coefficient of elastic uniform compression d stands for at a depth d; at a depth d means below at the base of the foundation.

So, C ud times the base area plus 2 times C tau average, C tau average is the average coefficient of elastic uniform shear. So, that is why it is appearing. If you see the foundation that means the front view what we can see when the foundation is subjected to vertical vibration that time within the soil, what will be happening? The soil which is adjacent to the sides of the foundation block will provide shear resistance.

So, because of that we here you we are getting that term saw a C tau av and that will be multiplied by the area, area mean which area, if this is our foundation block front view, then you can consider this phase. So, this is b, this is a and this is h now up to this portion we will see that shear resistance force, so what is the area for these I can show you the other side as well.

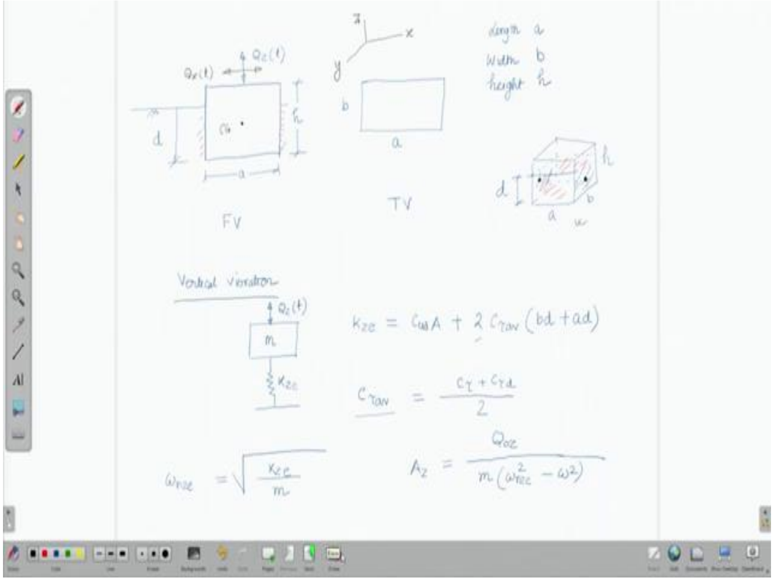
So, here also the same thing, it should be dotted line not the form line. So, this is our ground surface. So, this is the two phases which along which the shear resistance will appear. So, what is the magnitude of that shear resistance? 2 times C tau av into b times small d because small d is the data embedment, plus a times d. Why a times d? Because this front face is also attached to the surrounding soil and that will also apply resistance. That is the reason here we will see another term ad and there are two phases. So, that is taken care when I am multiplying it by 2 itself. So, in this way we can calculate kze.

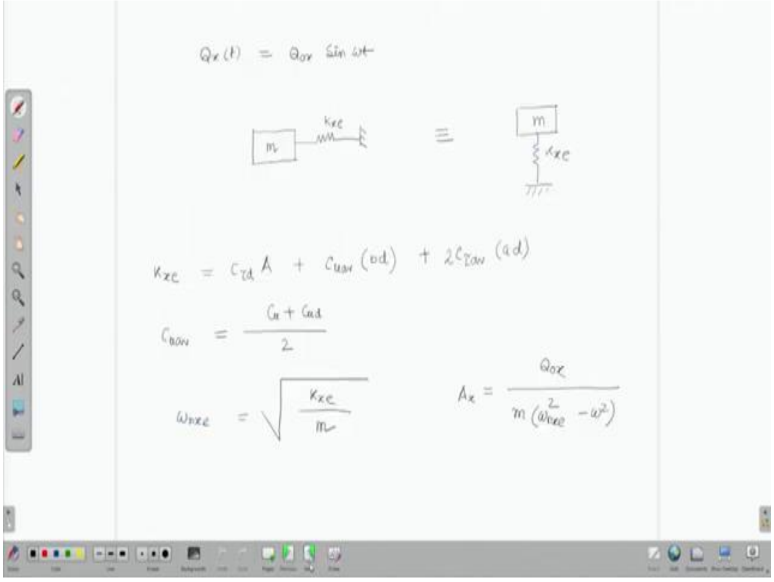
Now you can see tau av, how we will calculate, what is tau av? Tau av is the average value of the coefficient of elastic uniform shear. So, if we know the C tau value at the ground surface and if we know the C tau value at a depth d, then we can take the average of these two values as C tau average. So, we these now you can calculate omega nz it which is square root of kze divided by m and also you can calculate the amplitude of vertical displacement.

So, that also you know how to calculate and just writing here Q 0z is the amplitude of vertical force divided by m times omega nz or you can write it omega n ze also when e times for embedment. So, omega n ze square minus omega squared times m this will give you the amplitude of vertical vibration vertical displacement.

Now, the second case is sliding so I am just considering the same figure now what we are adding here is sliding force. So, if sliding force occur at that top, then it will look like this; if it is at the cg, then it may be like this. So, I am considering at the top and sliding force can be represented by Qx t.

(Refer Slide Time: 14:44)





So, what is here Qx t, that we can represent by Q 0x times sin of omega t, where omega is the operating frequency of the machine. If I go back to previous figure, what we can do, we can represent this case, I am considering previous sliding now. So, that time what we can, how we can represent this foundation soil system, by a mass and touched with spring. So, let us draw a mass. Here these mass m represents the mass of the foundation and the machine together and this is attached to a spring. When I am talking, sliding, I can show spring like this way also or for simplicity generally we draw it this way.

Now, why I have used the subscript e also because right now we are considering the effect of the embedment now if I will go back to the figure, what will be happened when this foundation block is subjected to the horizontal sliding, the block will move in the horizontal direction. So, if I am taking these three direction x, y and z this way, then right now I am considering the block is subjected to sliding vibration in it is x direction.

So, this time also because of the surrounding soil, we will get additional resistance force from the soil. So, that should be taken care when we are calculating the soil stiffness. So, that is the reason when we will calculate kxe that time how do we do so, if the foundation is resting on the ground surface, then it is C tau d times cross sectional area or the base area. Now, it is embedded. So, I need to consider the effect of the embedment for which you can see when these foundation is subjected to horizontal sliding, that time the front phase that means the area ad in this figure will be subjected, will experience shear resistance from the soil. Likewise, the back surface will also experience shear resistance from the surrounding soil.

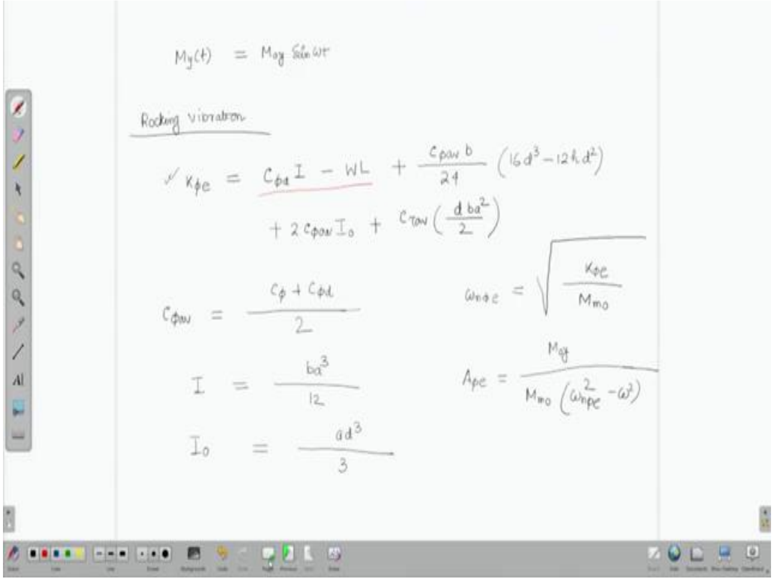
Now, when we are thinking the, this surface that means the surface which is perpendicular to the x direction and its area is b times d, what will be their resistance will be because of the Cu factor. So, accordingly now we can here calculate, we, first am considering the forces. So, here you can see it will be Cu times Cu average times b times small d. So, I am once again going back so as I said b times small d that means I am considering this phase and this phase only. So, there are two phases. So, it is multiplied 2.

But in this case, if you see when it is going in one direction, only one piece is subjected to this resistance force. So, and the other side they are maybe there is a kind of gap. So, we will consider alternately either these phase or these phase. So, let us do this. So, we will not multiply here 2, it is just bd plus here you can see what are the other phases where shear resistance or shear force up will be developed, that is ad front and ad back.

So, then I can write here as C tau average, there are two phases, so I am multiplying it with 2 times a times d. What is Cu average here? Cu value at the ground surface plus Cu value at a depth d whole divided by 2, that means we are averaging these two Cu value So, from these now we can calculate omega n xe which is k xe divided by m. Likewise, we can find out ax which is the amplitude of horizontal displacement using this equation. So, let me correct it m time times omega n xe squared minus omega squared.

Now, let us see the third case. So, third case when, what is the third case? Let us take this is the CG. Now, when it is subjected to the horizontal force, what will be happened at the CG? If you see moment about y axis we will be imposed to this foundation block. So, we can consider that moment.

(Refer Slide Time: 21:59)



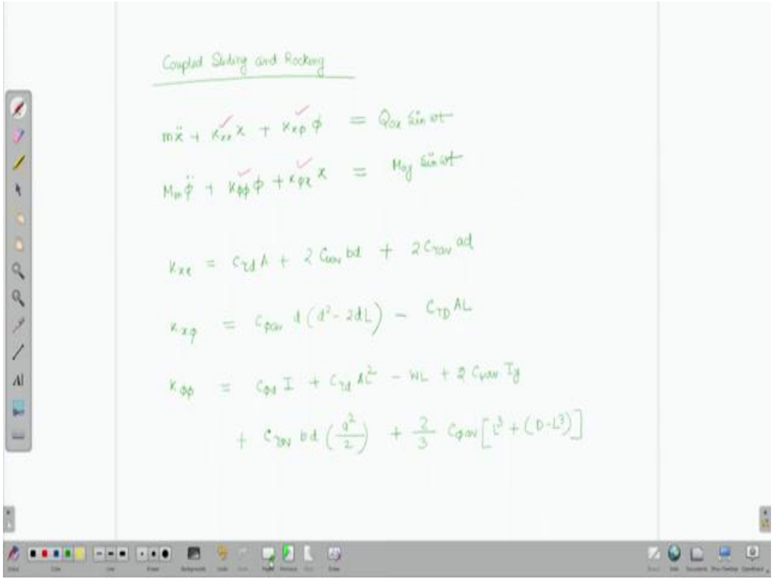
So, here we can write My t is equal to M 0y times sin omega t, we can calculate these M 0y for this case. So, now, when rocking is working that time what will be the value of spring stiffness that is k phi e which is equal to C phi at d times i that means it is the same case when there is when the foundation is resting on the ground surface then now minus WL; the W is the total weight of the foundation and L is the height of the CG from the base of the foundation. So, this condition we have already seen earlier.

Now the additional resistance will be considered and moment of resistance in this case we will consider. So, how we will write it, it is C phi av times b which is the width of the block divided by 24 times 16 d cube minus 12 times h times t square, this is one factor, then plus 2 times C phi av that means, C phi average times I0 plus C tau av times d times b a square divided by 2 here C phi average can be calculated by knowing C phi at the ground surface and C phi value at the sorry at a depth, so C phi value at a depth small d. Now, summing these two, we will take the average.

So, what is C phi here, here C phi is the coefficient of elastic non-uniform compression and C phi average is the average value of coefficient of elastic non-uniform compression. So, now you can see there are two terms one is I. So, I if I know that geometry of the foundation, I can write the expression for I and also for I0. So, if we are able to calculate k phi e, then from these we can calculate the natural frequency omega n phi e which is equal to square root of k phi e divided by capital M mo this is about y axis and what is the amplitude of rotation, these is M 0y divided by M mo times omega n phi e squared minus omega squared.

So, these terms are these two are known to us, the only thing which we have changed here is k phi e, earlier we have considered only first two terms. Now because of the surrounding soil and resistance force from that, we need to consider the resistance moment and then we will we have calculated k phi e.

(Refer Slide Time: 27:08)



Now, that coupled case. We all know that rocking and sliding can be coupled. So, what is the equations of motion for the couple case, first let me write that. So, right now we are discussing couple case, better I should use green color rather than this, coupled sliding and rocking. So, these time what is the equation of motion? When we are considering the sliding, that time we can write it like this, this should be equal to Q 0y x, we are considering the horizontal force in x direction, so it is Q 0x times sin omega t and for the rocking we can write here we will write k phi phi times phi last k phi x times x.

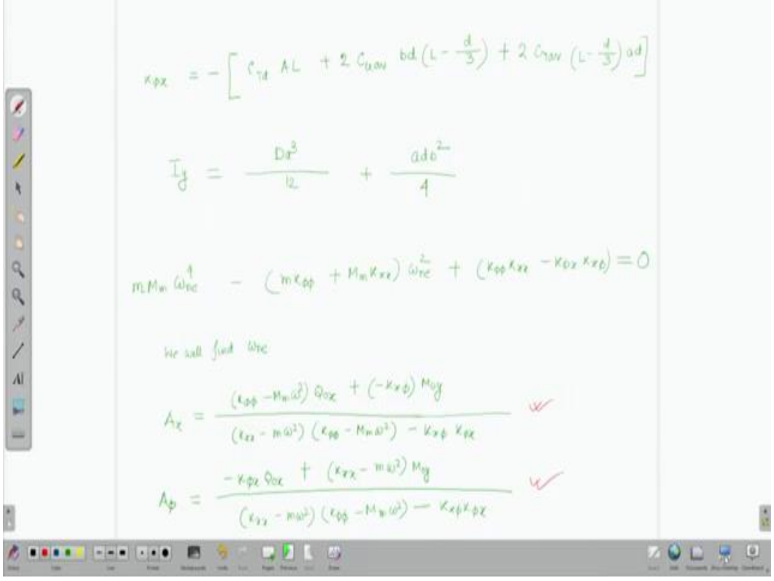
So, if you see here, here we are getting a few new terms what are those K xx, K xy, K phi phi and K phi x, these four stiffness coefficients are now new for us. So, I am just writing here how we are getting K xx, because, we have already discussed in detail how to, how were we are getting the resistance proves and from that resistance moment and from that, how we are getting the rest of the things K xx etcetera. So, I am not discussing it elaborately. I am writing it for you. So, K xx here is C tau at depth d times base area plus 2 times of Cu average times bd plus 2 times of C tau average times ad.

So, I am just saying what is Cu ab and C tau av, Cu av is the average values of the average value of the coefficient of elastic uniform completion, whereas C tau av is the average value of the coefficient of elastic uniform shear. So, how we calculate these two coefficients that we have already discussed. Now, for K x phi, we will use that expression C phi average which is that average value for the coefficient of elastic non-uniform compression times d times d squared minus 2 times d L, here L is the distance so I can go back to the slide so a is the height of the CG from the base of the foundation block.

So, here it is coming this then minus C tau d times AL. So, basically, if negative sign is associated, it means that these force when we are thinking in terms of force or moment, that is causing the disturbance and if it is offering resistance that means if it is resistance moment, then that should be associated with the positive sign here.

Next is K phi phi that is C phi at depth d times I plus C tau d sorry plus C tau d times AL squared minus WL plus 2 times C psi av times Iy. So, here the question what is C psi av. So, c shy av is the average value of the coefficient of elastic non-uniform shear plus C tau av times bd times is square by 2 plus two thirds of C phi average times L cube plus d minus L cube. So, now we know the three stiffness coefficient.

(Refer Slide Time: 33:38)



The fourth one which is K phi x is minus of C tau d times AL plus 2 times c u average times b d times, I should write it as C tau small d AL plus C U av times bd, bd is the surface area inside of the foundation block. So, that times L minus d by 3 because of the pressure distribution and then from that we are calculating moment plus 2 times C tau av times L minus d by 3 times a times d. So, in this way we can calculate the four different stiffness constant wanting what is Iy in the previous slide.

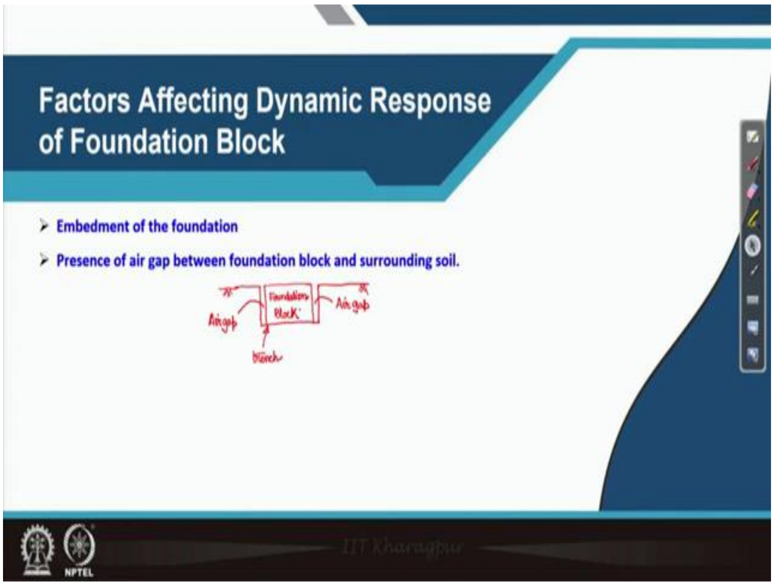
So, here Iy is, if you see Iy is equal to d AQ divided by 12 plus a db square by 4. So, from these now what we need to know, we need to know how we can calculate the natural frequency for the system. So, for that, we need to write to the next equation using which we can calculate the natural frequency.

So, we can write this equation omega ne, this time it is to the power 4 not 2, minus m times k phi phi plus capital Mm K xx times omega n e squared plus K phi phi times K xx minus K phi x times K x phi that is equal to 0. So, from this equation what we will get, we will find out we will find out omega ne that means natural frequency for the machine foundation so I will system when foundation is embedded to the soil up to date small d.

Now, after knowing that omega ne, we can calculate the amplitude of horizontal displacement which is ax and also we can calculate amplitude of rotation. So, for amplitude of horizontal displacement, we need to use this equation which I am writing right now. So, the first component for the horizontal force plus minus K x phi times m 0y that means the second component for the rocking this entire thing should be divided by K xx minus small m omega squared times K phi phi minus Mm omega squared minus k x phi times k phi x. this using this equation we can calculate Ax which is the amplitude of horizontal displacement.

Now, if I that can be calculated by using that expression which I am writing here so Q 0x plus K xx minus small m omega squared times M 0y whole thing should be divided by K xx minus m omega squared times K phi phi minus Mm omega squared minus K x phi times k phi x. So, using these two equations we can find out the amplitudes of horizontal displacement and the rotation respectively.

(Refer Slide Time: 39:01)

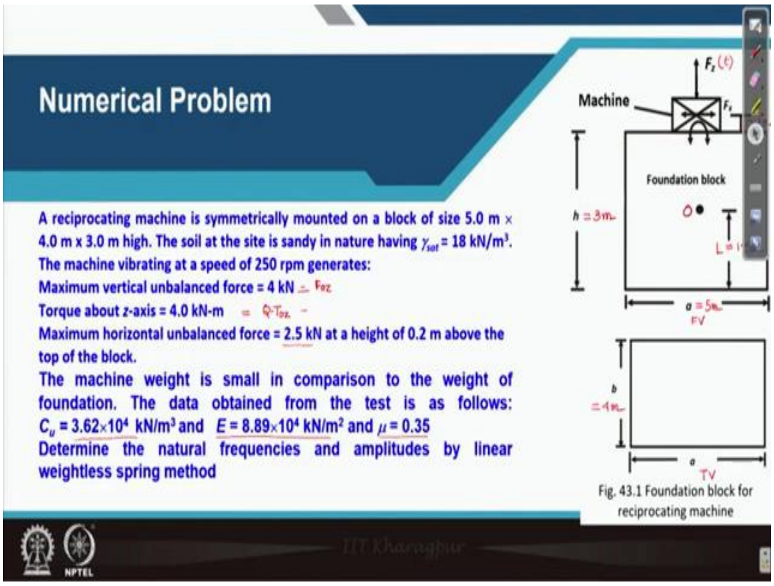


So, in this way we can find out the effect of the embedment of the foundation. The next factor is the presence of air gap between foundation block and the surrounding soil. So, what is happened generally, generally in the soil we first excavate our trench as you can see here. So, this is the trench. Now the foundation block is placed within these trench keeping some gap between the once of the foundation block and the surrounding soil.

So, here you can see the air gap, same thing here also this is your air gap and this is your foundation block. Now, in presence of air gap what is happening the natural frequency of the foundation soil system decreases, however, the amplitude of vibration increases in presence of air gap.

So, in this way we have learned of different factors affecting that dynamic response or foundation block we have also studied how to calculate the natural frequency and amplitude of vibration for foundation block resting on the ground surface and foundation block embedded in the soil. Now, with this knowledge we will now solve one numerical problem let us see the problem statement here.

(Refer Slide Time: 41:21)



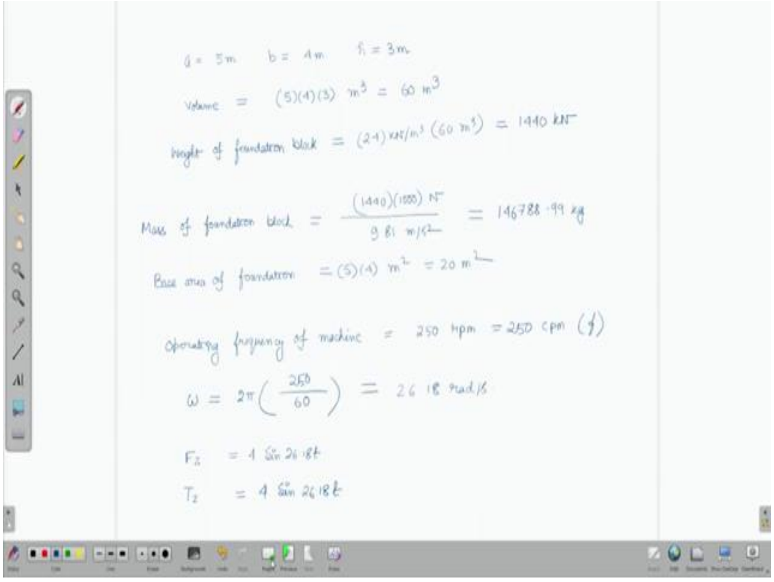
So, in the problem statement what is mentioned that first let me read, a reciprocating machine is symmetrically mounted on a block of size 5 meter by 4 meter by 3 meter. So, here if you see these what you can see a is the length of the block which is set 5 meter. So, a value is I am writing here 5 meter. What is the b, b is 4 meter. So, this is a front view, this is top view. So, b is 4 meter and h which is representing the height of the foundation block is 3 meter.

And you can find out from these information, what is the height of the CG from the base of the foundation which is eventually 1.5 meter in this case this is only the location of the CG from the base of the foundation. Now, it is also said that the maximum vertical unbalanced force is 4 kilo newton that means, if I will consider here as per this diagram capital Fz is that dynamic force in vertical direction, then it is a function of time of course, and it is amplitude which I can write as F 0z it is 4 kilo Newton. Torque about z axis that is also 4 kilo Newton. So, I can 4 kilo newton meter. So, I can write it as Q, not Q it is T0, T0 about z axis is 4 kilo and 4 kilo newton meter.

Now, the maximum horizontal unbalanced force that means, the amplitude of Fx is given that is 2.5 kilo newton and it is acting at the height 0.2 meter above the top of the block that means from this level to the line of action of Fx these distances 0.2 meter. The machine weight is small in comparison to the weight of the foundation. If so, then we can consider that the combined CG is located almost at this point O.

What is the soil properties are given, Cu which is the coefficient of elastic uniform compression is equal to 3.62 into 10 to the power 4 kilo newton per meter cube, e value is also given which is the dynamic elastic modulus or Young’s modulus and poisons ratio also provided. So, we are asked to find out the natural frequencies and amplitudes by linear weightless spring method. I hope the problem statement is clear to all of us. So, now, we will solve the numerical problem.

(Refer Slide Time: 45:53)



So, let us go to a new page. So, what we need to do basically for these probably we need to check several things, first is the vertical for checking for the foundation blocks objected to vertical vibration, second checking is for horizontal sliding, third checking for the torsional vibration about z axis. Then we will check coupled, we will check the coupled rocking and sliding vibrations. So, for today, we will do two tasks, we will check we will find out the natural frequency and amplitude for the vertical vibration and torsional vibration.

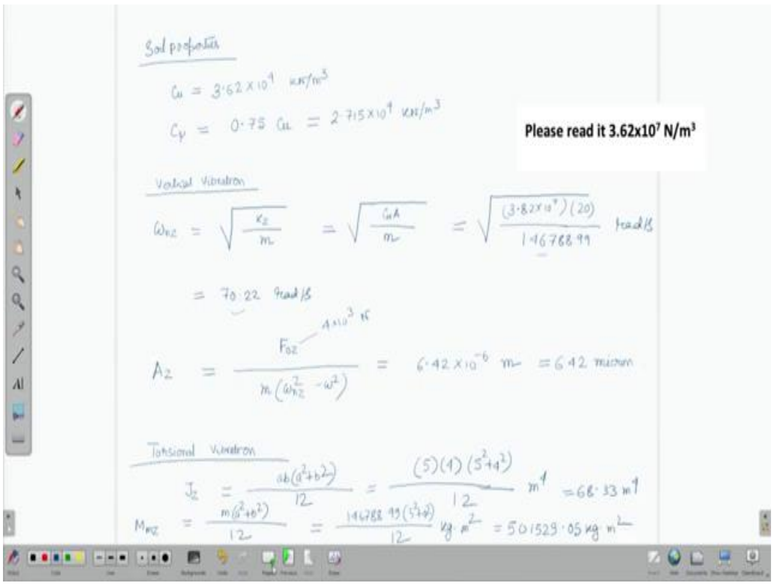
Next class we will continue the same problem for rocking and rocking sliding and coupled rocking and sliding cases. So, first what is given to us, we know the geometry of the foundation which is 5 meter by 4 meter by 3 meter. So, from these we can calculate the volume of the foundation that is 5 times 4 times 3 in meter cube that means. 60-meter cube. From this we can calculate the weight of the foundation block. So, weight of foundation block is equal to unit weight of the concrete which we are considering 24 kilo newton per meter cube times the volume of the foundation block. So, it is coming 1440 in kilo newton.

So, from weight now we need to know what is the mass of foundation block already it is mentioned that the machine weight is small in comparison to the weight of the foundation. So, we are neglecting the mass of the machine here. So, mass foundation block is 1440, it is converting a newton divided by g which is 9.81 meter per second squared. So, how much we are getting? Let me calculate, we are getting yes we are getting 146788.99 kg. So, weight is 146788.99 kg.

Now, what is the base area of foundation, that is 5 times 4 in meter square. So, 20-meter square is the base area of this foundation. Next thing which we need to know what is the operating frequency, operating frequency of machine it is already mentioned that 250 RPM or we can write it as 250 CPM - cycles per minute. So, this is f ,so from these we can find out omega circular frequency that is equal to 2 pi times 250 by 60 so we are getting 26.18 approximately in radian per second.

So, now I am just writing Fz, Fz it can be written as sin omega t, which is 26.18 times t. I can write Tz which is also 4 times sin omega t. So, these two because today we will solve the, we will do the analysis for Fz and Tz only.

(Refer Slide Time: 51:27)



Now, from the soil property what we can get, what is given to us soil properties given us Cu which is the coefficient of elastic uniform compression and the value is 3.62 into 10 to the power 4 kilo newton per meter cube. From this, we can calculate C psi which is the coefficient of elastic non-uniform shear 0.75 times c u which is coming 2.715 into 10 to the power 4 kilo newton per meter cube.

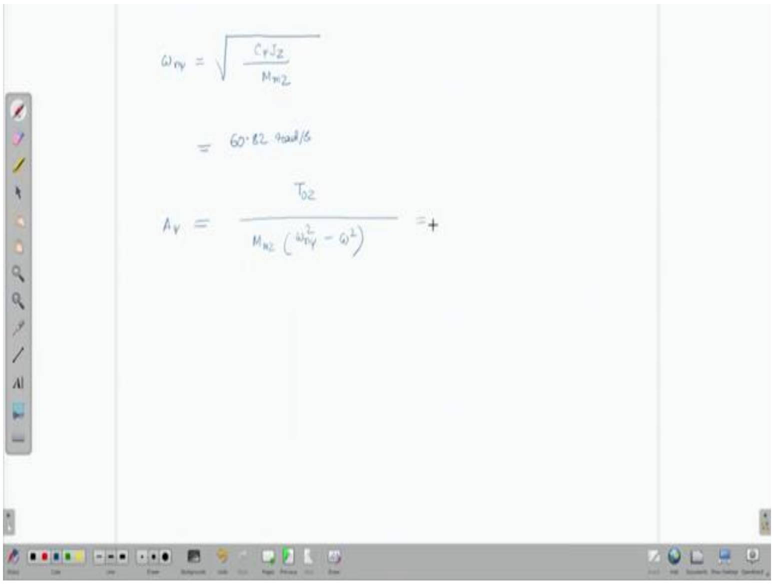
So, these two values are required for today’s work. So, with these two values let us find out the natural frequency when the foundation block is subjected to vertical vibration. So, right now, we are considering the case vertical vibration. So, omega nz is equal to kz divided by m in this case we are not considering the embedment because foundation is it is assumed that the foundation is resting on the ground surface.

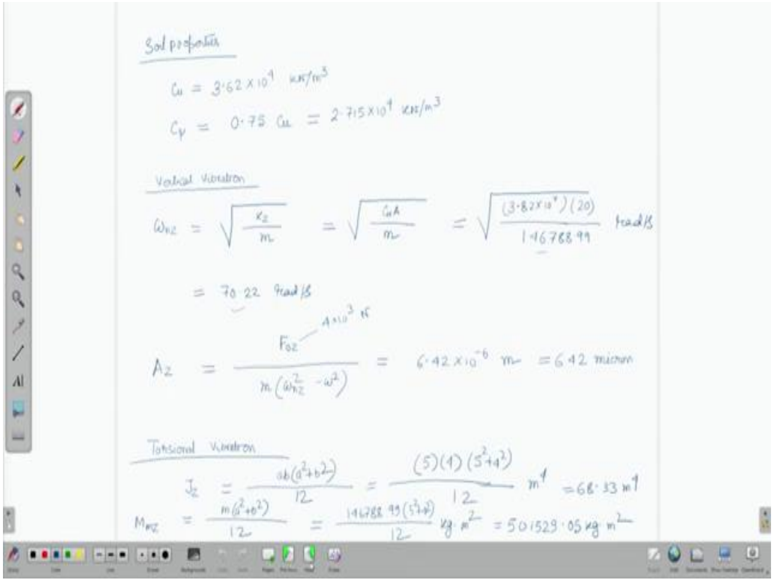
So, kz means Cu times A divided by m. So, Cu is already given 3.62 into 10 to the power 4 kilo newton per meter cube. So, converting it in newton per meter cube times in base area divided by mass, mass is how much let me check mass is 146788. So, then that circular natural frequency is coming 70.22 radian per second. From, this we can calculate Az which is the amplitude of vertical displacement.

So, this amplitude is coming you can take F 0z here divided by m omega nz squared minus omega square. So, I am directly writing the value F 0z is known which is 4 into 10 to the power 3 in newton, mass already I have mentioned you can see here omega nz is known and omega is 26.18. So, from that what we are getting is 6.42 into 10 to the power minus 6 meter or I can write it as 6.42 in micron.

Next is torsional vibration so for this case we need to know Jz which is calculated by using this expression it is already known, I hope to all of us from our knowledge in solid mechanics so divided by 12, it is in meter to the power 4. So, we are getting 68.33 in meter to the power 4 also we need to know M mz mass moment of inertia about z axis. So, it is coming 501,529.05 kg meter square.

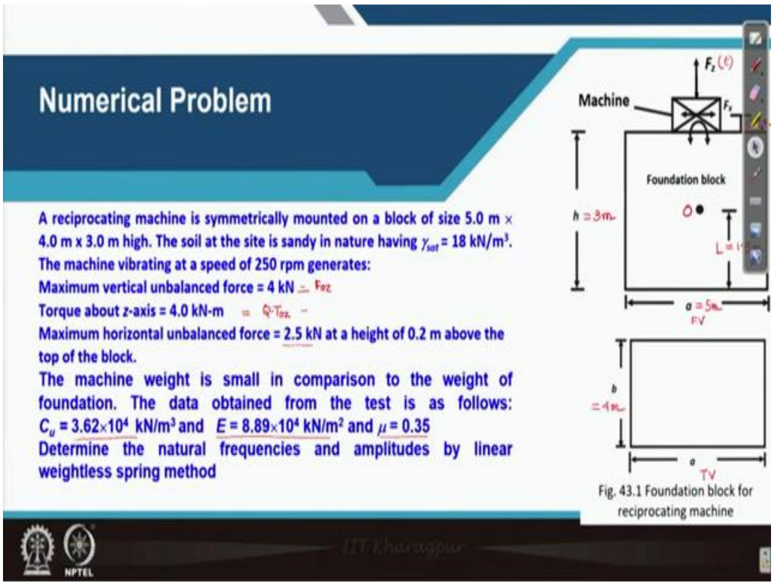
(Refer Slide Time: 56:45)





So, now, we can calculate omega n psi which is equal to square root of C psi times Jz divided by M mz. So, already we know all these value, so I am directly calculating using these values so, I am getting 60.82 a second and you can calculate also the amplitude of rotation, you know how to calculate it, amplitude of rotation if required you can calculate by using this equation in this case it is T 0z divided by M. So, using this equation if you wish, you can find out the amplitude of rotation. So, I am leaving this for you that last it.

(Refer Slide Time: 58:27)



Factors Affecting Dynamic Response of Foundation Block

- Embedment of the foundation
- Presence of air gap between foundation block and surrounding soil.

SUMMARY

In this lecture numerical problem on rigid foundation block for reciprocating machine under different (2 modes) modes of vibration is discussed.

REFERENCES

Text Book

1. Principles of Soil Dynamics by B M Das, PWS-KENT Publishing Company (available Indian Edition: by B M Das and Luo Zhe, Cengage Learning India Private Limited; Third edition)

So, we these I am stopping today's class. So, you here is the summary of today's class that we have discussed our numerical problem partly on rigid foundation block for reciprocating machine under. Right now, we have discussed on the two modes of vibration or different means, here two modes what are those two modes, vertical vibrations and torsional vibration and next class we will discuss how to do the coupled rocking and sliding vibration analysis. So, with this I am stopping here. The reference which you may follow, so thank you.