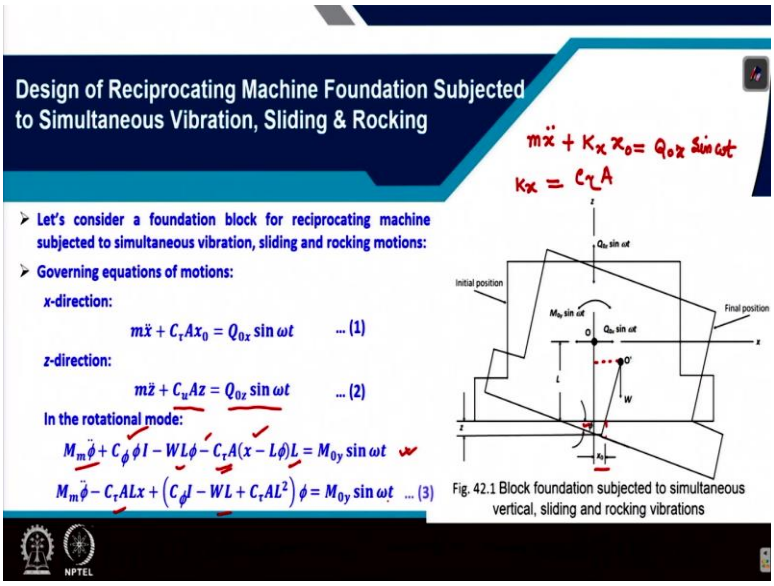
**Soil Dynamics**
**Professor. Paramita Bhattacharya**
**Department of Civil Engineering**
**Indian Institute of Technology, Kharagpur**
**Lecture No. 42**
**Analysis of Machine Foundations**
**(For Reciprocating Machines – Part 2)**

Welcome to the class Soil Dynamics. Today, we will discuss we will continue our discussion on analysis of machine foundations for reciprocating machines.

(Refer Slide Time: 0:40)




So, till now, we have studied how to find out the natural frequency and the amplitude of vibration when the machine foundation block for the reciprocating machine is subjected to vertical vibration, pure sliding and pure rocking motion. Now, today, if you see the finger 42.1 what we can see that foundation block for the reciprocating machine is subjected to vertical vibration where Q 0z times sine omega t is the vertical force. It is subjected to sliding and rocking simultaneously. So, the amplitude of sliding vibration is Q 0x and its direction is in half cycle, it is in positive x direction and in next half cycle, in the negative x direction.

Whereas, the rocking vibration is represented by the moment about y-y axis, which is M 0y times sine omega t. In this case, omega is the operating frequency for the machine. So, under that stated condition, this machine foundation is subjected to simultaneous vibrations sliding and rocking motions. So, the governing equations of motion for the 3 cases are when we are considering the x direction that time it is m that is mass of the machine and foundation together. So, m times x two dot plus C tau times A times x0 plus sorry is equal to Q 0x times sine omega t.

Now, in this equation 1, what is M? I have already I think told that M is the total mass of the machine and the foundation together. What is C tau here? Here, C tau is the coefficient of elastic uniform shear and how it comes in equation 1? We already know that for single degree of freedom system when any object is subjected to vibration, then what we can write? We can write the equation of motion in this format; mass times acceleration plus coefficient of stiffness for horizontal vibration times x, x is the displacement in horizontal direction that, so, this sum of these two is equal to the external force in this case it is Q 0x times sine omega t.

Now, in the problem which is stated by figure 42.1 if you see here, in place of x, we need to write x0 which is the displacement for the base of the foundation. So, at the base if you see that displacement is x0. Now, in place of Kx what we can write? We can express Kx is equal to C tau times A, where C tau is the coefficient of elastic uniform shear and A is the base contact area for the foundation block. Now, similarly, we can find out the equation of motion for z direction, which is represented by equation 2 here.

So, in this case, what we have seen is that mass times acceleration in z direction which is called also as inertia force in z direction plus C tau times A times z, where z is the displacement in z direction and Cu is the coefficient of elastic uniform compression. So, this is nothing but the stiffness force, where stiffness represents the soil. So, these two forces are equal to the external force applied to the foundation. The third equation of motion is for the rotational mode and you can see the third equation of motion here.

So, in this equation, what we can note is that the first component is for the inertia whereas, the second component is due to the reaction of the soil. However, in this case, during rotation, what is happen? Non uniform stress distribution we can see at the base of the foundation or at the interface of the base and the soil. So, that resistant force basically causing moment. So, that moment you can represent it by C phi times phi times I. So, what is phi here? If you see the figure, phi is the angle of twist, here you can see this is the phi and what is the W here in the next term?

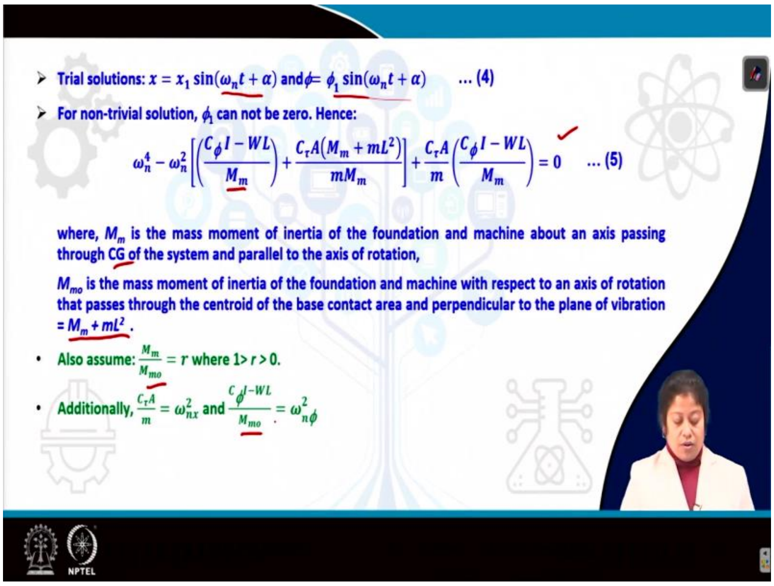
So, in the third term, W causing the trying to rotate the foundation block that you can see here. So, W which is acting always downward direction, which is the total mass of the machine and the foundation together causes rotation and therefore, that causing moment for W is equal to W times L times 5. So, what is A here than? A is the height of the CG of the foundation from the base of the foundation itself. And if you refer the figure, what we can see? Because of the rotation, originally CG was at O, but because of the rotation of the foundation CG is shifted from O to O dashed.

So, because of that, now, what is the horizontal distance between O and O dashed these distances, A times phi. So, the moment causing by the W at O is equal to W times L times phi. Now, see the fourth component, which is C tau times A times x minus L phi times L. So, here C tau times A, it is what is C tau? C tau is coefficient of elastic uniform shear. So, basically because of the sliding, what is happening? We can see the uniform shear stress distribution at the base of this foundation.

But, because of the rotation there is or because of the rocking, there is some displacement already which is L times phi. So, then how much is the actual displacement in x direction because of this sliding that is x minus L 5. So, this resistance force is how much then C tau times A times x minus L phi and that causes moment. In this case, it is opposing. So, we are using negative sign here and how much is the moment that about the CG that is, O that C tau times A times x minus L phi these things give the force times the distance which is L here.

So, in this way we can get the left-hand side and right-hand side is the external moment, which is imposed to the foundation from the machine and that is expressed by M 0y times sine omega t, whereas, M 0y is the amplitude. So, now, if we will take the coefficient of phi in one place and coefficient of x in other place, then we can represent this equation of motion for rotational mode by equation 3.

(Refer Slide Time: 12:49)

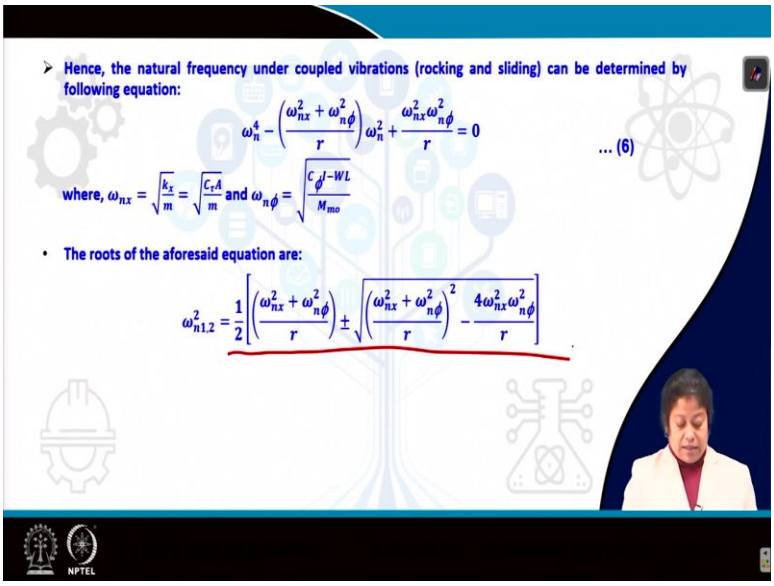


Now, we need to do what is the solution for these three equations? So, for that, we consider trail solutions you can see, we have chosen trial solution x is equal to x1 times sine omega n t plus alpha, alpha is the phase angle and phi is equal to phi 1 times sine of omega t plus alpha. Now, for the non-trivial solution phi 1 cannot be 0, so, we can get this equation which can be numbered by 4. So, now, if we solve the equation 5, then we can get omega n which is the natural frequency of the system. Now, when we have written equation 5, we have used a few new terms.

So, what are the meanings of those terms here? So, you can see Mm is one new term, what is Mm here? Mm is the mass moment of inertia of the foundation and machine about the y axis passing through CG of the system and parallel to the axis of rotation. So, it Mm is the mass moment of inertia, but it is at the position of CG. So, how we can get M mo, which is the mass moment of inertia of the foundation and the machine with respect to an axis of rotation that passes through the centroid of the base area. For that, we need to use the plane axis theory and we can get M mo, Mm plus small m times L square, whereas, small m is the mass of the foundation block and capital L is the distance between the CG and the foundation base.

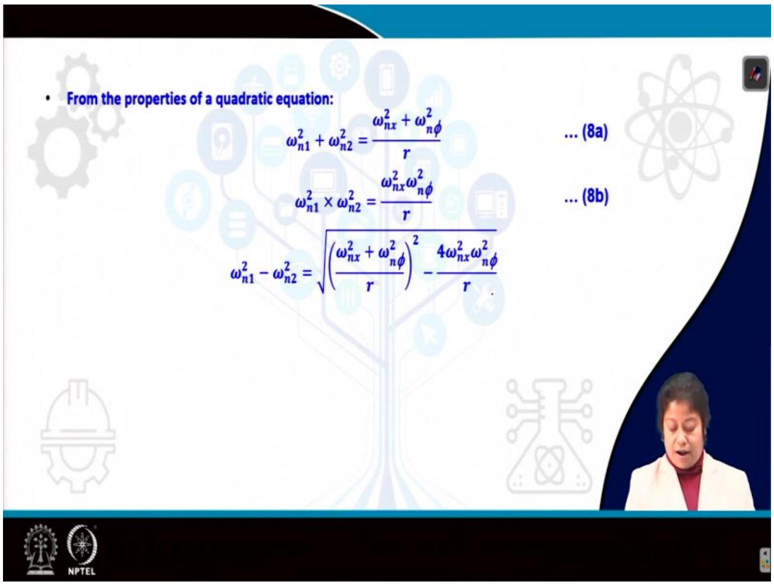
Now, if we consider that the Mm divided by M mo by ratio r, where r lies between 0 and 1 and if we write in place of C tau times A divided by A as omega nx square and in place of C phi I minus WL divided by M mo is equal to omega n phi square then what we can get?

(Refer Slide Time: 16:28)




Then, we can get this equation where as I already explained what is r, what is omega nx and what is omega n phi? So, in equation 6 if you see, we know what is the value for omega nx, we know what is the value for omega n phi, we know what is r? So, there is only one unknown which is omega n, which represents the natural frequency. So, how many roots we can get from this equation 6? We can get 4 roots actually. So, we will get 2 negative and 2 positive roots. So, we are only interested for the positive roots. So, here you can see, I have just written it as omega n1 or omega n2 square and that can be represented by this form.

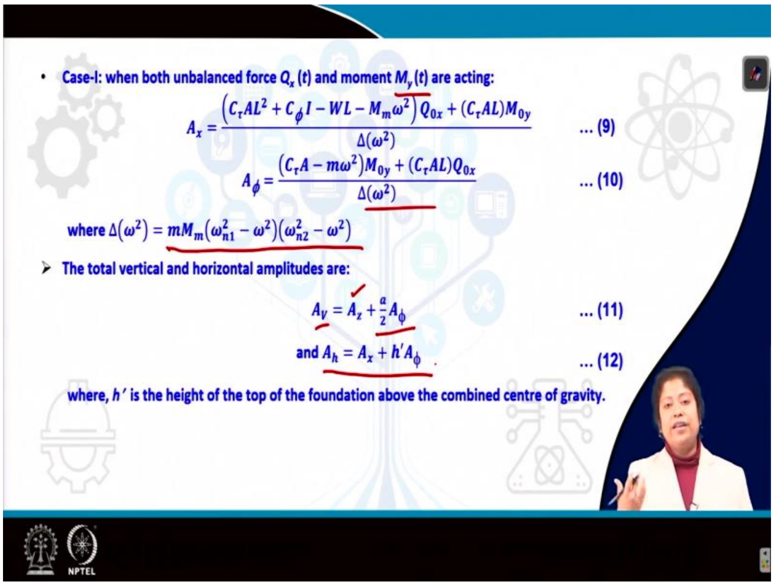
(Refer Slide Time: 17:53)




Now, using the properties of quadratic equation, what we can say? We can say that omega n1 square plus omega n2 square, which are the 2 roots of the equation, which is equation 6. So, the sum of these two is equal to omega nx squared plus omega n phi squared divided by r. What else we can say? We can say omega n1 squared times omega n2 squared is equal to omega nx squared times omega n phi squared divided by r. I hope these relationships are already known to all of us, I am just going through it, so that if required, you can use it.

And the third property is omega n1 square minus omega n2 square is equal to square root of omega nx squared plus omega n5 squared divided by r whole square minus 4 times omega nx squared minus omega n phi squared divided by r.

(Refer Slide Time: 19:28)




Now, in this case, there are 3 possibilities. In this case means, first we have assumed that foundation block for the reciprocating machine is subjected simultaneously to vibration, sliding and rocking. Here vibration means vertical vibration, sliding means horizontal vibration and rocking means moment about or rotation about y axis. Now, for all the rocking and sliding case it may be coupled, it may not be coupled, when we will consider coupled case when both unbalanced forces Qx t and the moment My t are acting together to the foundation block.

So, when both unbalanced force and moment are acting together to the foundation block that will be considered as case 1 here. So, that time the amplitude for the horizontal displacement can be represented by the expression shown here in equation 9. So, except delta omega square rest of the terms are now known to us, what is delta omega square? Delta omega square is this one.

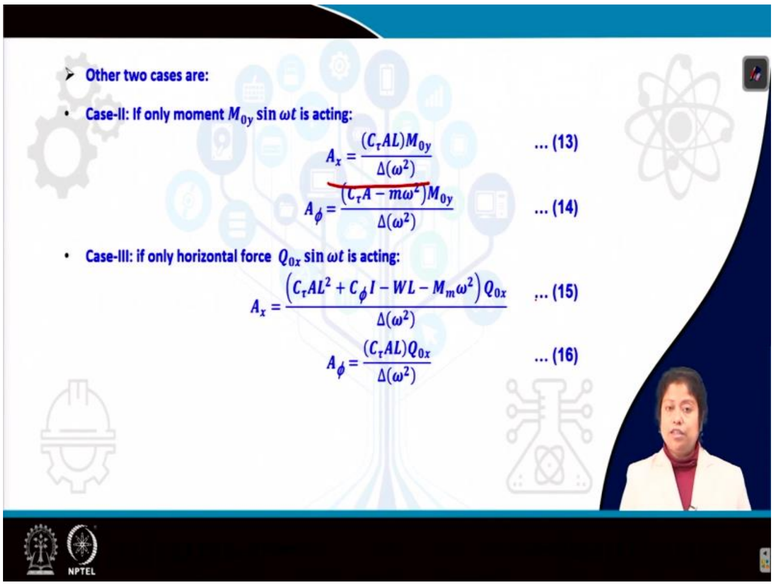
So, delta omega squared can be calculated if we know the value of omega n1 squared and omega n2 squared or in other words, if we know that natural frequencies for the solutions of the equation 6, then we can find out delta omega squared. And rest of the things are known with m and Mm these are related to the geometry or properties of the foundation block.

So, next is equation 10 which is represent the amplitude of the rotation about y axis. So, here you can see that expression also. Now, after knowing Ax and A phi. Finally, we need to find out what is the total vertical displacement and what is the total horizontal displacement or the maximum horizontal displacement and maximum vertical displacement?

So, for maximum vertical displacement Av what we need to do? From the equation of motion in z direction, we can find out Az which is the amplitude of displacement in z direction because of the vertical vibration. So, if we know Az if we know A phi, which is the amplitude of rotation then because of the rotation how much vertical displacement occurs that we can calculate, which is shown here small a by 2 times A phi. So, the sum of these two will give us the total maximum vertical displacement or we can say amplitude of vertical displacement.

Now, what about the maximum horizontal displacement? That can be calculated by this equation. So, what is h dashed here? In this case, h dashed is the height of the top of foundation above the combined CG. Combined CG means combined CG of machine and foundation together. So, this is means h dashed replaces the height of the top of the foundation from the combined CG.

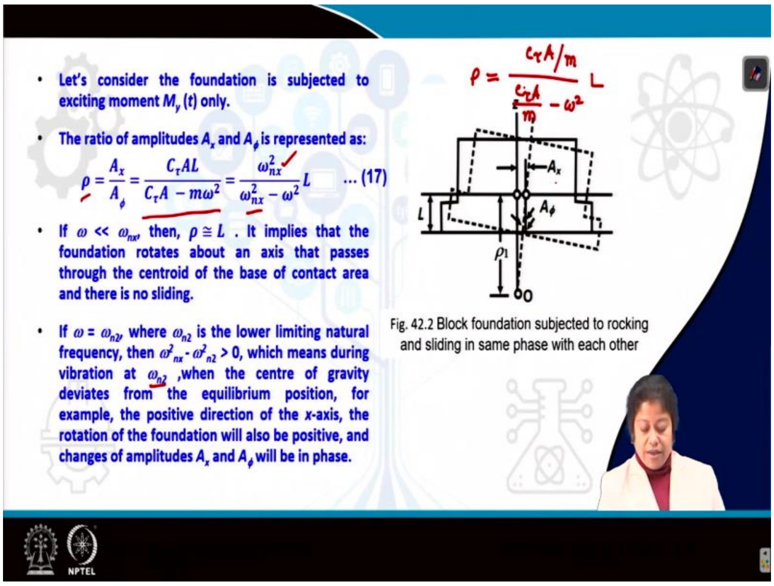
(Refer Slide Time: 24:56)




There are two other cases. In second case what is happened, only the moment about y axis is present the sliding vibration is absent. So, that time the magnitude of Q 0y should be 0. So, whatever expression earlier we got for Ax and A phi in the same expression if we will write in place of Q 0y if we write 0 then we can get the expression for Ax and Ay in absence sorry Ax and A phi in absence of the horizontal vibration or the sliding force. So, in this way here we get Ax and A phi.

The third case is the case when moment about y direction is absent only the horizontal force or we can call it as sliding is present. That means, if we in that solution have Ax and A phi, which we obtain for the coupled system in that expression, if we write in place of M 0y, if we write the value is 0, then we can get the expression for Ax, which you can see in equation 15. And similarly, we can get the expression for A phi, which you can see in equation 16.

(Refer Slide Time: 27:07)



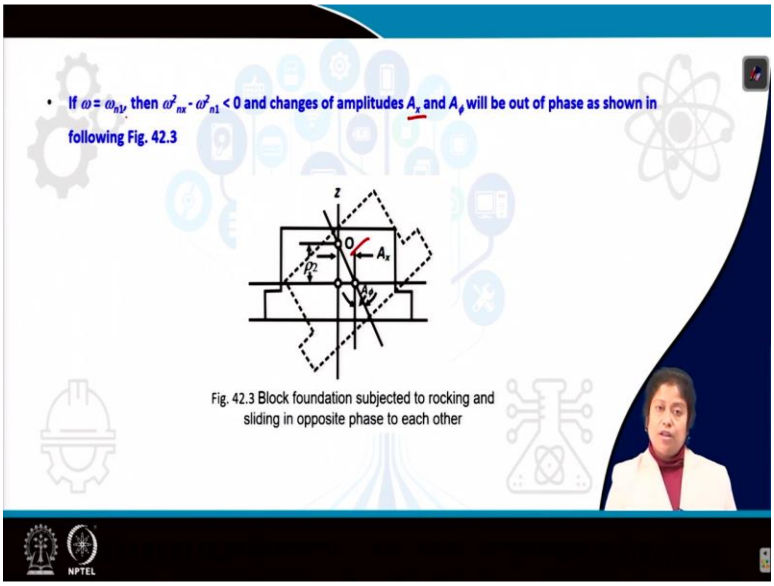

Now, let us see what is happened when the foundation block is subjected to only these exciting moment My t, here My t means My is a function of t time. We can introduce a new term rho which defines the ratio of Ax to A phi. Now, we know in case of Qy t 0 that time or in absence of the horizontal force, we know that expression for Ax and also we know the expression for A phi. So, from those 2 expressions, we can find out rho which is the ratio of Ax to A phi and we get this expression C tau AL divided by C tau A minus M times omega square.

So, here if I divide the numerator and denominator both by M which is the combined mass of the foundation and the machine, then what we will get? We will get an expression like I am just writing here something like this C tau A divided by m here we will get C tau A divided by m minus omega squared times L. So, C tau A divided by m that is nothing but omega nx squared. So, here you can see, I have written omega nx squared likewise, denominator also we can write omega nx squared minus omega squared and this whole thing is multiplied with L.

Now, what will be happened if omega which is the operating frequency of the machine is very, very less than omega nx? Then you can see here, if omega is very, very less than omega nx, then equation 17 says that rho is almost equal to L. What is L here? L is the height of the CG from the base or the foundation block. The third case when omega which is the operating frequency of the machine is equal to omega n2, that time, what is omega n2. Omega n2 is the lower limiting natural frequency. So, in that case omega nx squared minus omega n2. So, in equation 17 in place of omega we can now consider omega n2.

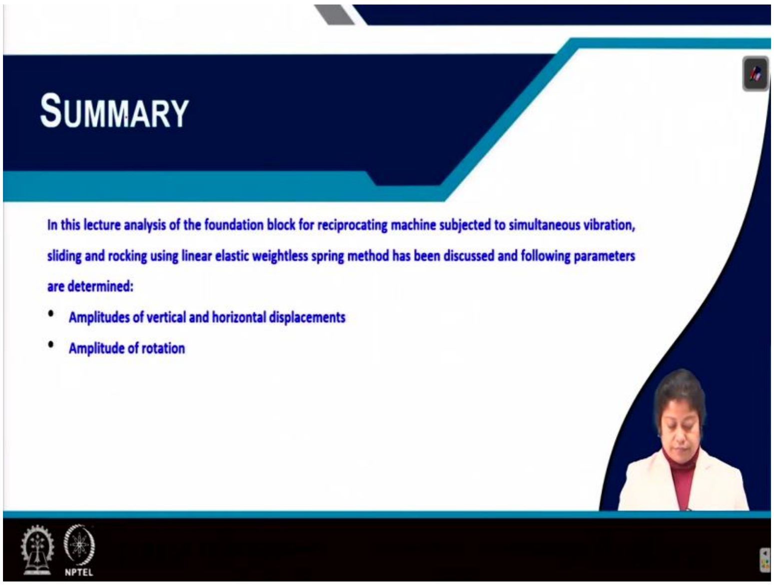
So, what will be the value of omega nx square minus omega n2 square? That will be greater than 0 what does it indicate? It indicates that, when the machine is operating at omega n2 frequency the CG deviates from the equilibrium position that means, CG shifted from O to O dashed that in which direction? In the positive x direction that means, the figure which we have shown and also the rotation of the foundation will be positive that means, Ax and A phi these two are in same phase. So, here you can see the diagram when Ax and A phi both are in same phase. So, Ax and A phi both are positive here.

(Refer Slide Time: 32:26)




Now, another possibility is like this when omega is equal to omega n1. So, what is omega n1 here? The higher limiting frequency. So, that time omega nx squared minus omega n1 square should be, the difference between omega nx square and omega n1 square should be less than 0 and changes of amplitude that means, Ax and A phi will be out of phase as you can see in the figure. So, one is positive and one is negative, if Ax is positive that means, along positive x direction, then that A phi it is not in the same face with Ax. So, when we can observe this kind of situation? When omega that means, the operating frequency of the machine is very close to the hand limit of the frequency, natural frequency or equal to omega n1 itself.

(Refer Slide Time: 33:57)





So, let us come to the summary of today's class. Today we have discussed how to analyse the foundation block for the reciprocating machine when it is subjected to simultaneously vertical vibration, sliding and rocking vibrations. And we have seen how to find out amplitudes of vertical and horizontal displacements, how to find out amplitude of rotation. So, here you can see the summary. This is the IS code which can be used for design of reciprocating machine foundation. Thank you.