Soil Dynamics
Professor. Paramita Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture No. 41
Analysis of Machine Foundations
(Linear Elastic Weightless Spring Method – Part 1)

Hello everyone, today we will discuss on Analysis of Machine Foundations considering elastic Linear Elastic Weightless Spring Method today is part one of this discussion. Earlier, we have done the analysis of machine foundation using elastic half space theory, where we considered that we represented basically the foundation soil system by mass attached to a spring and dashpot where mass replaces the mass of the foundation and the machine and spring and dashpot replacing the properties of the soil on which foundation is resting.

Now, in linear elastic weightless spring method, we will not consider the dashpot. Here, soil will be represented by the spring only, and we already know if we do not consider the mass of the spring, then whatever be the natural frequency of the system that will change when we consider the mass of the spring. Therefore, in this case, we will not consider the mass of the spring, we will only consider the mass of the foundation and the machine together.

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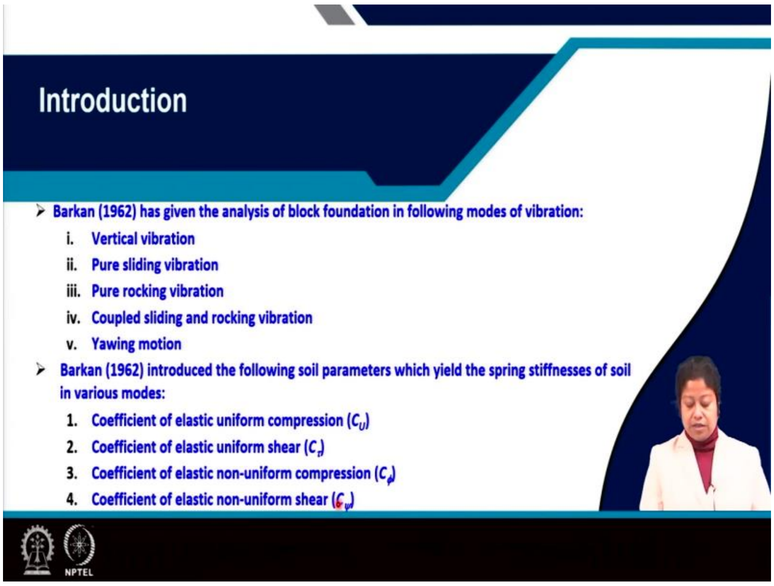

So, the method which we are discussing today is proposed by Barkan in 1962. What he said, he given the analysis of block foundation in the following modes of vibrations, what are those modes? Vertical vibration, pure sliding vibration, then pure rocking vibration, coupled sliding and rocking vibration and yawing motion. Now, Barkan also introduced a few soil parameters, which are required to find out the spring stiffness for the soil in various modes of vibration.

First one is coefficient of elastic uniform compression, symbolically we represented by capital C, small u, you can see here. Then coefficient of elastic uniform shear, symbolically we write it as capital C tau. Then, coefficient of elastic non-uniform compression, symbolically we write it as capital C phi and coefficient of elastic non-uniform shear, symbolically we represented by C psi.

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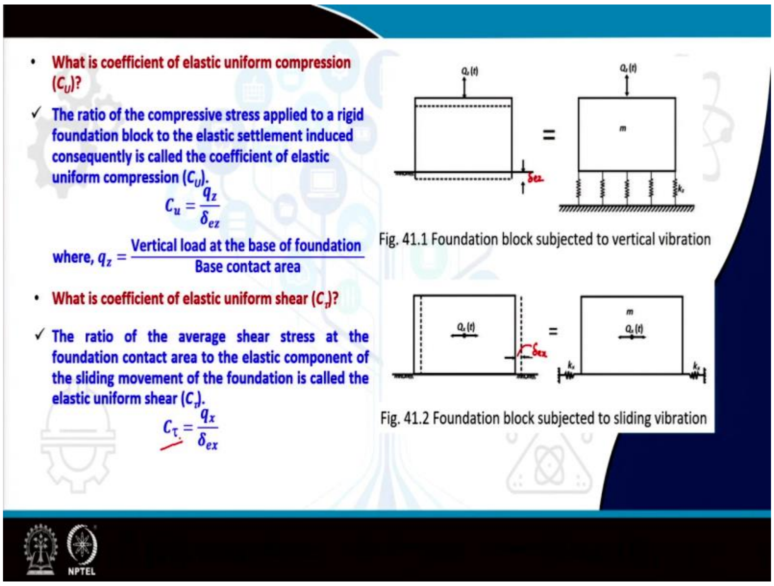

The first question is what is coefficient of elastic uniform compression? Let us see its definition. It is the ratio of the compressive strength applied to a rigid foundation block to that elastic settlement, which is induced consequently in the foundation is called the coefficient of elastic uniform compression. So, here you can see Cu is equal to qz divided by delta ez.

Now, what is here in this if you see the adjacent figure 41.1 you can see, a foundation block is subjected to vertical vibration here. So, the elastic settlement is this much delta ez. And the uniform pressure distribution at the base of this foundation is qz. So, we can calculate the qz using applied load divided by the base area of the foundation block.

Next is what is coefficient of elastic uniform shear. So, it is the ratio of the average shear stress at the base of the foundation or we can see at the foundation contact area to the elastic component of the sliding movement which is occurred in horizontal direction of the foundation is called the elastic uniform shear. So, let us see the diagram. So, in this diagram qx is the applied horizontal vibration or force.

Now, it causes an elastic settlement this one magnitude delta ex. So, the elastic uniform shear which is officiant of elastic uniform shear which is C tau here is equal to qx divided by delta ex, delta ex is the elastic component of the sliding movement. What is qx here? qx is the shear stress at the base of the foundation or we can call it as the shear stress at the contact surface between foundation and soil and how we can calculate it? We calculate qx dividing the horizontal load acting at the base of foundation by the base contact area.

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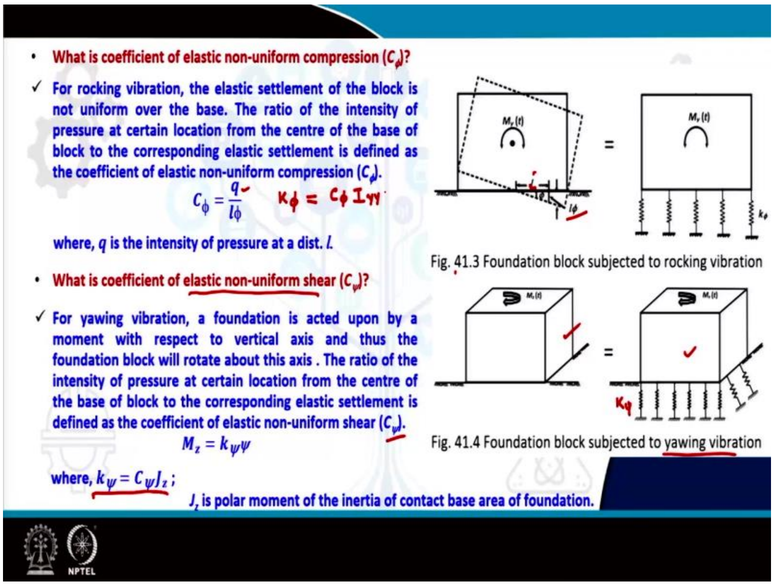

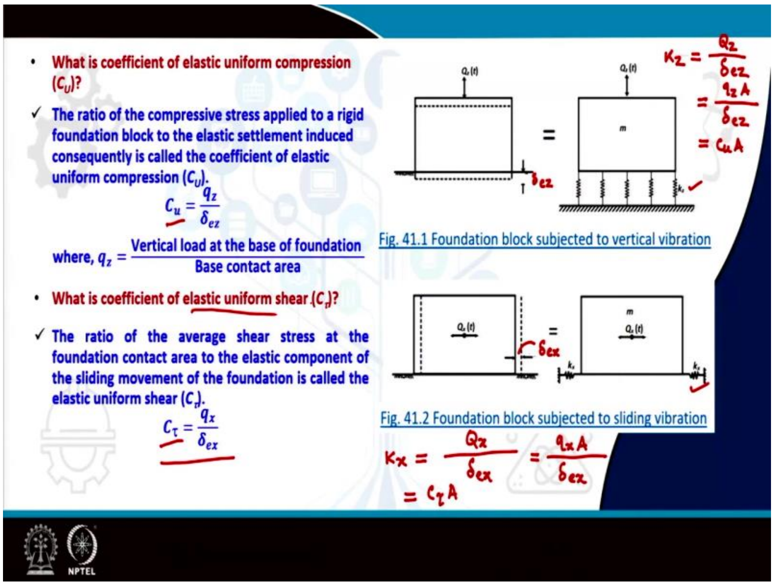

So, the next question is what is coefficient of elastic non-uniform compression? So, basically what is happening when a foundation is subjected to rocking vibration? It is tilted on either left or right side. So, during rocking vibration, the elastic settlement of the block is not uniform over the base. Also, if you see this figure, the pressure distribution at the base of the foundation block is also not uniform. So, the ratio of the intensity of the pressure at a certain location from the center of the base of foundation to the corresponding elastic settlement is defined as the coefficient of elastic non-uniform compression.

So, let us see the figure 41.3. In this figure, what we can see the foundation when tilted towards right that time at a distance l from the center of the foundation, what you can see, from the center of the foundation, you can see the magnitude of the pressure is q and the elastic settlement can be measured if we know the amount of tilting. So, tilting is it is tilted at an angle phi. If so, then the inelastic settlement is l phi. So, the coefficient of elastic non-uniform comprehension in this case is equal to q divided by l times phi, where q is the intensity of the pressure at a distance l from the center of the base of foundation block.

Next question is what is coefficient of elastic non-uniform shear? So, now elastic non-uniform shear comes into the picture when the foundation block is subjected to torsional vibration. So, for yawing vibration our foundation is acted upon by a movement with respect to the vertical axis, which is eventually the z axis and the foundation blocks will rotate about this vertical axis or z axis. The ratio of the intensity of pressure at certain location from the center of the base of foundation to the corresponding elastic settlement is defined as the coefficient of elastic non-uniform shear.

So, let us see this bigger. Here you can see the foundation block is subjected to torsional vibration or we can call it yawing vibration. So, as a consequence, we can measure the pressure at certain location and corresponding elastic settlement from which we can finally, find out the coefficient of elastic non-uniform shear.

Now, you can see these foundation soil system can be represented by mass resting on or attached to the spring. So, if spring stiffness is k psi in this case, then how we can write Mz? This Mz is equal to k psi which is spring constant times psi which is the angle of rotation. Now, these k psi can be calculated from the coefficient of elastic non-uniform shear by using this relationship. Here, Jz it is the polar moment of inertia of contact base area of this foundation block.

Now, the same thing you may find out for C phi, C tau and Cu as well. So, what is C phi? C phi already is defined here. So, if you are interested to find out K phi that is equal to C phi times I if it is rotating about yy, then I can multiply it Iyy. Similarly, if I will go back to the previous slide here. So, here what we can do? Here, we can find out Kz and Kx as well, how what is Kz exactly? Kz is stiffness constant. So, that is equal to load divided by settlement. So, if load let us take Py settlement sorry, in this case, it is about z axis sorry along z axis so, I cannot write these.

So, Kz is equal to Qz divided by the settlement which is delta ez. Now, what is Qz? Qz is equal to small qz times A divided by delta ez, we have already seen that small qz divided by delta ez is equal to Cu which is coefficient of elastic uniform compression. So, I can write here Cu times A. So, in this way we can represent the or we can determine the spring constant Kz by using elastic uniform the coefficient of elastic uniform compression.

Similarly, for the foundation block subjected to sliding we can calculate Kx. So, in this case Kx is equal to Qx, capital Qx divided by delta ex. Now, capital Qx means small qx times A, A is the content area of the foundation block to the soil divided by so, qx times A divided by delta ex. This is our Kx. Now, small qx divided by delta ex that we have already seen is equal to C tau which is the coefficient of elastic uniform shear. So, in this case Kx is equal to C tau times A. So, in this way we can determine the spring stiffness by knowing the coefficient of elastic uniform shear C tau.

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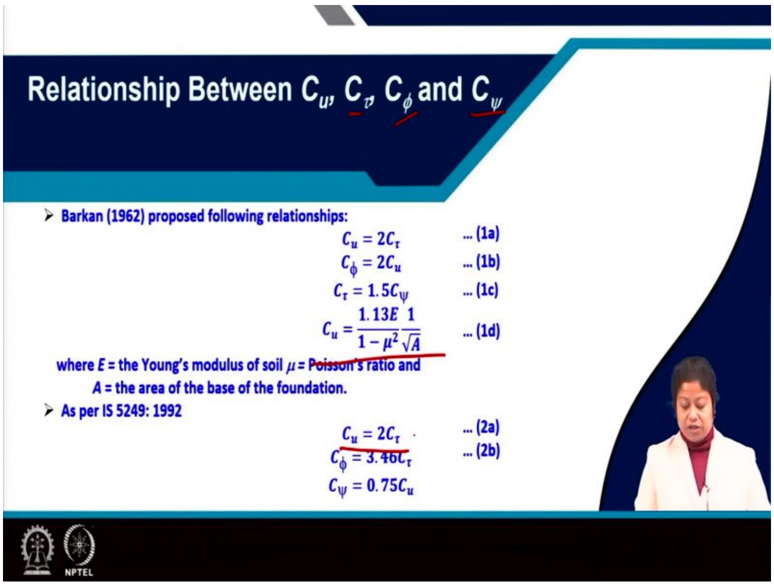

Now, we need to know the relationship between these different coefficients of elastic parameter that means, relationship between Cu which is coefficient of elastic uniform compression, C tau which is coefficient of elastic uniform shear, C phi which is the coefficient of elastic non-uniform compression and C psi which is the coefficient of elastic non-uniform shear.

So, Barkan in 1962 proposed following relationships you can see here Cu is equal to 2 times C tau that means, Cu is greater than C tau. C phi is equal to 2 times Cu and C tau is equal to 1.5 times C psi. Cu which is the coefficient of elastic uniform compression can be determined by using this equation, where E is dynamic elastic modulus, mu is the Poisson ratio and A is the contact area of the foundation to the soil.

So, if we know E value of the soil, if we know mu value for the soil and if we know the contact area of the foundation to the soil, then we can calculate Cu, if we know that value for Cu, then we can also find out C tau, we can find out C phi. And finally, from C tau we can calculate C psi.

Now, let us see what our IS code says. So, as per IS code 5249, published in 1992, Cu is equal to 2 times C tau that means, coefficient of elastic uniform compression is equal to 2 times the coefficient of elastic uniform shear. C phi which is the coefficient of elastic non-uniform compression is equal to 3.46 times C tau and C psi, which is the coefficient of elastic non-uniform shear is equal to 0.75 times Cu which is coefficient of elastic uniform completion.

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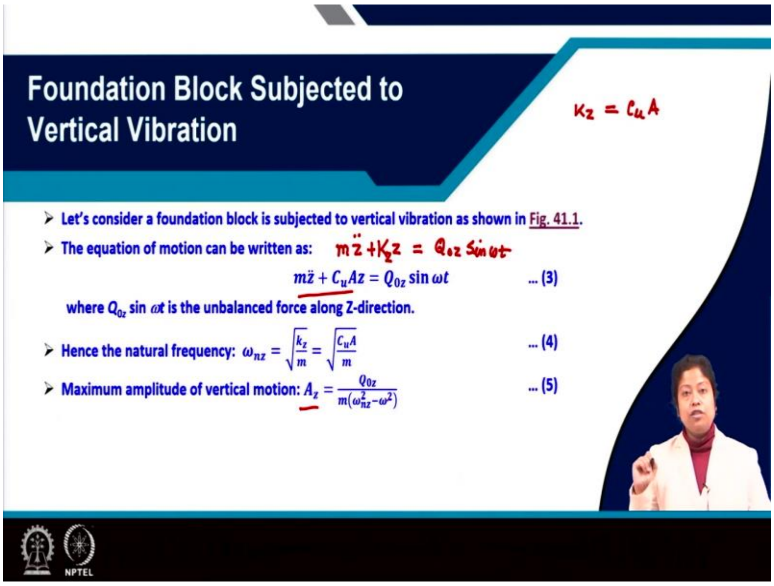

Now, let us see how we will analyze the foundation block which is subjected to vertical vibration. So, here you can see foundation block figure 41.1 You can see a foundation block is subjected to vertical vibration qz which is a function of t. Now, for this when we represent this foundation soil system by a mass and spring that time what will be its equation of motion? Its equation of motion is Mz two dot that means acceleration in z direction times Kz in place of Kz we can write actually, I am just writing one steps before this, we know the equation of motion for a mass subjected to connected with a spring when it is subjected to vertical vibration, that time the equation of motion is m times z two dot plus kz is equal to Q 0z sin omega t? What is Q 0z here? Q 0z is the amplitude of the external force.

So, in this case instead of k, I can write Kz times z. So, what is Kz here? It is the spring constant when the mass is subjected to vertical vibration or vibration along z direction. Already we have seen Kz can be calculated by using the coefficient of elastic uniform compression Cu? What do we have seen? Kz is equal to Cu times A so, that is used in this equation and we get these form. So, obviously, the right hand side is the unbalanced force along z direction. So, from this equation of motion we already know how to get the natural frequency for the undamped system.

So, natural frequency omega n z for the undamped system is equal to square root of kz by m, kz means Cu times A. So, from this we can calculate the natural frequency of the foundation soil system. Now, using equation 5, we can calculate the maximum amplitude of vertical motion which is kz. Here omega is the operating frequency of the machine and omega nz is undamped natural frequency of the foundation soil system.

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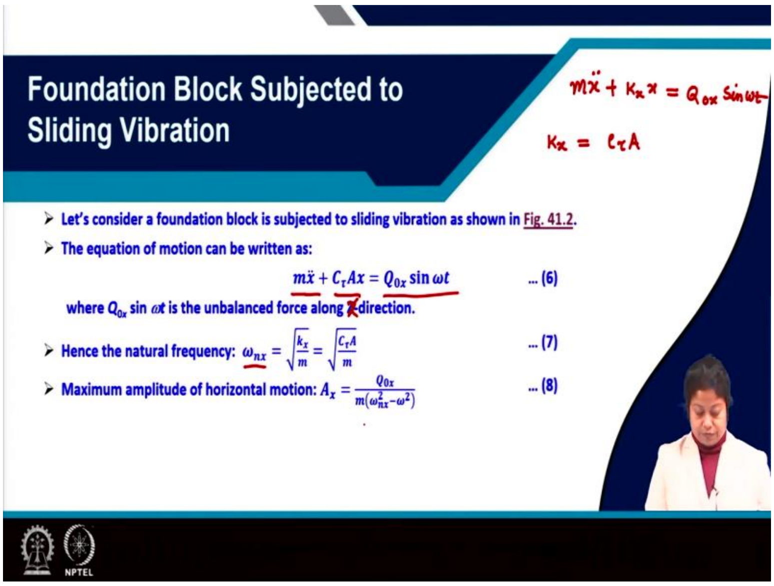

Let us see what will be happened when a foundation block is subjected to sliding vibration. So, let us consider a foundation block is subjected to sliding vibration as shown in figure 41 point. So, basically this figure. This is the case when a foundation block is subjected to sliding vibration. Now, the, in this case what is the equation of motion? Generally, we can write in this case m times x two dot that means, the acceleration in x two dot means, acceleration in x direction plus stiffness forces in this case Kx times x, which is equal to unbalanced force in x direction. So, that is Q 0x times sin omega t.

Now, we already have seen that Kx can be written as C tau times A. So, in place of Kx, if we will write C tau times A, what we will get? We will get m x two dot plus C tau A times x is equal to Q 0x times sin omega t. So, here Q 0x sin omega t is the unbalanced force along x direction not z. This is for x direction. So, in this case the natural frequency is omega nx, natural frequency for the undamped system and that can be expressed as square root of kx by n.

So, kx can be replaced by C tau A. So, the final form of the natural frequency is square root of C tau A divided by m. Please remember here that the natural frequency omega nx here is the circular natural frequency. So, the unit of omega nx is radian per second. So, if we need to express it in cycles per second, we need to divide omega nx by 2 pi. Next is maximum amplitude of horizontal motion which is Ax here. So, Ax is equal to Q 0x divided by m times omega n x squared minus omega squared. Omega nx is the natural frequency whereas, omega is the operating frequency of the machine.

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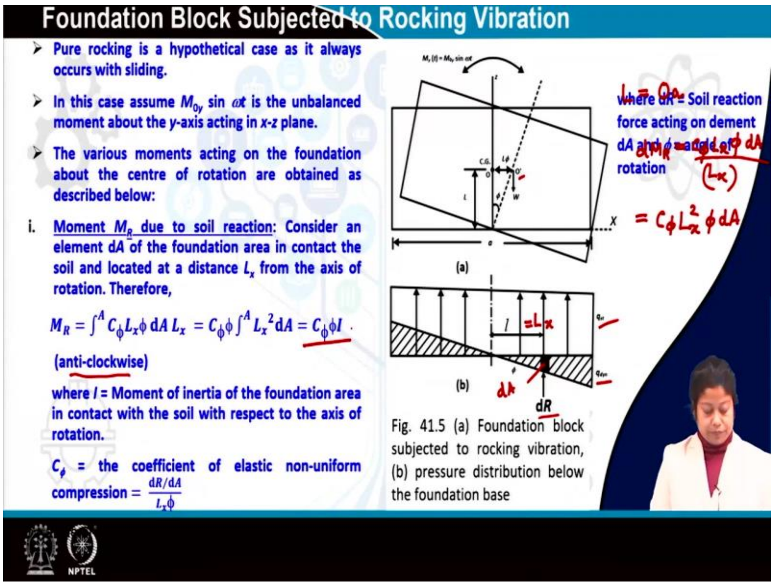

Now, let us see the third mode of vibration, which is rocking vibration that means, in this case, the foundation block is subjected to rocking vibration. So, in figure a, we can see a foundation is subjective to rocking vibration and consequently it is tilted by an amount pi. Now, here what you can see, because of this tilting. it is CG. Originally, CG was located at O, which is shifted to O dashed now. So, the distance between O and O dashed is n times pi, where n is the height of the CG from the base of the foundation that means, basically L means Oa, L is equal to Oa as per this diagram.

What is the w here? Here w is the total weight of the foundation block so, because of the tilting, these tilting is occurred during dynamic loading. So, because of the tilting, you can see the pressure distribution which is represented Q (())(30:17) or dynamic. So, that dynamic pressure distribution is non-uniform over the base of the foundation. However, because of the static load, which may be because of the dead weight or some other life weight or live load, whatever pressure distribution occurred at the base of the foundation, that is not non-uniform that is uniform, but if we consider the pressure distribution because of the dynamic load that is here non-uniform; you can see.

So, now at a distance l from the CG, if we consider a soil element of dA so, there what is the resistance force, that is d capital R. So, before starting to derive the equation of motion, first thing we need to remember is that pure rocking is a hypothetical case as it always occurs with sliding. Now, in this case, we are assuming that the unbalanced moment about y axis acting in x, z plane is M 0y times sin omega t, M 0y is the amplitude of the unbalanced moment.

Now, the various moments acting on the foundation about the center of rotation are obtained as described below. So, what are those? First one is, moment MR due to soil reaction. So, here we can as I said we can consider a soil element dA of the as shown here, which is located at a distance Lx from the axis of rotation. So, the small l is equal to capital Lx, then, MR can be calculated by integrating the moment of resistance for the small element.

So, (ele) or I can say elementary moment MR, d MR so, how much is d MR in this case? In this case, d MR is equal to I can write here d MR is equal to C phi times Lx times phi times area of the element times Lx, Lx here is the distance. So, these part is the resistance times distance from the A. So, finally, what we are getting is C phi times Lx squared times phi times dA.

Now, if we will integrate these d MR, then we will give the total moment of resistance or reaction coming from the soil and it is when the foundation is tilted in right side that time or clockwise direction that time the moment of resistance is acting in anti-clockwise direction. So, the direction is also mentioned here as anti-clockwise. So, what is in the next step, what we have done? We have taken C phi times phi outside this integration and inside the integration only Lx squared times dA is left and these integration we will do over the area based area A, contact area A which will give us this expression that C phi times phi times I.

What is I here? I is moment of inertia of the foundation area in contact with the soil with respect to the axis of rotation that means with respect to y axis here and C phi is the coefficient of elastic non-uniform compression.

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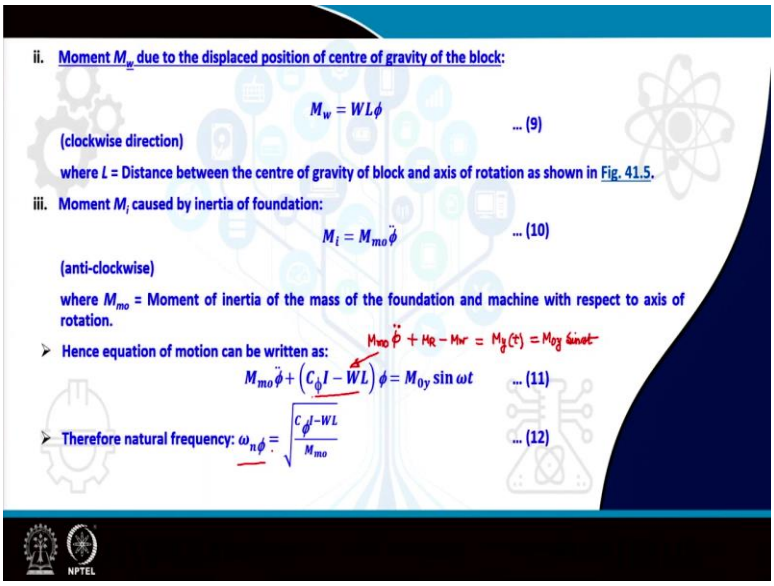

Next is moment Mw due to the displaced position of the center of gravity of the block. So, because of the displaced position of center of gravity of the block that means O is shifted to O dash in the earlier figure, what is happen? The weight of the foundation block w is also causing moment about O. So, how much is that? That is w times L phi, whereas L phi is the distance between O and O dashed and these will act in clockwise direction when the foundation block is tilted clockwise.

The third moment is Mi which is caused by the inertia of the foundation block and that is equal to M mo times by two dot and obviously, it is acting in anti-clockwise direction provided the external moment My is acting clockwise direction. So, under these circumstances what is the equation of motion? You can see equation of motion can be written as M mo times phi two dot plus C phi times I minus WL this whole thing is multiplying by phi and that is equal to the M 0y times sin omega t.

So, how we are getting these? Basically, whatever written on the left hand side can be written as M mo times phi two dot plus what is the moment because of the soil resistance that is a MR minus moment because of the weight of the foundation block that is Mw. So, this left hand side is always equal to right hand side which is My, what is My? My is a function of t which represents the moment about y axis. So, we can also write it as M 0y times sin omega t from these we can get this equation which is shown here, equation 11.

So, from equation 11, then we can calculate the natural frequency which is omega n phi and that is equal to square root of C phi times I minus W times L whole divided by M mo. So, what is M mo here? M mo is moment of inertia of the mass of the foundation and the machine together with respect to the axis of rotation. What is I here? I is moment of inertia of the foundation area in contact with the soil with respect to the axis of rotation. In this case, axis of rotation is y axis. C phi we can calculate the which is the soil property and w is the total weight of the foundation and the machine together.

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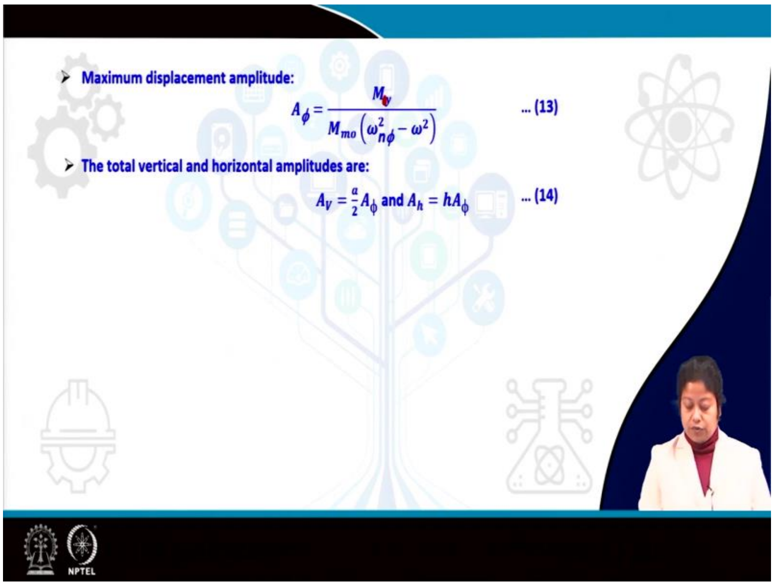

So, the maximum displacement of course in this case it is rotation is how much My divided by M mo times this one. This is the amplitude. So, basically here it will be M 0y. Let me check the figure once again, yes M 0y we have to consider here. And from this we can calculate that total vertical and horizontal amplitudes as shown in equation 14.

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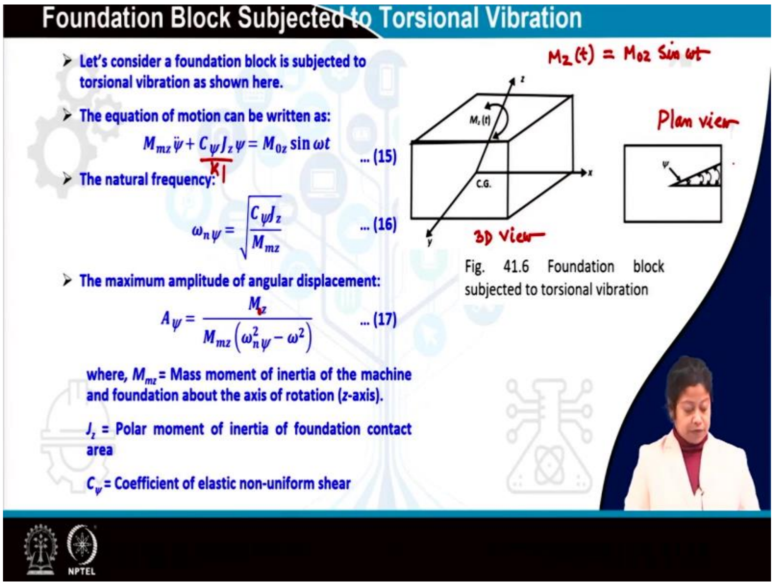

Now, the fourth motor vibration torsional vibration, so, in this case the block is subjected to torsional vibration about vertical axis that means z axis, so, here you can see the plan area. This is the plan area or plan view and this is 3D view. So, from this you can find out the equation of motion here in place of K psi already I have written C psi times jz. So, from this equation of motion, we can calculate the natural frequency as shown in equation 16. Then, we can find out the maximum amplitude of angular displacement which is A psi in this case.

Here also we need to write M 0z in place of M z if we are writing M zt is equal to M 0z times sin omega t, where omega is the operating frequency and M mz is the mass moment of inertia of the machine and foundation system together about the axis of rotation which is the vertical axis or z axis in this case. What is Jz? Jz is the polar moment of inertia of the foundation contact area. C psi is the coefficient of elastic non-uniform shear and here you can see the shear stress distribution also at the base of the foundation.

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So, let us come to the summary of today's class. In this lecture the following topics like foundation block subjected to vertical vibration, subjected to sliding, subjected to rocking vibration, subjected to yawing vibration or torsional vibration are discussed. So, with this, I am stopping here. These are the references that I have used in today's class. Thank you.