Soil Dynamics
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Lecture 40
Analysis of Machine Foundations (Elastic Half Space Method – Part 4)

Hello friends, today we will continue our discussion on Analysis of Machine Foundation using Elastic Half Space Method.

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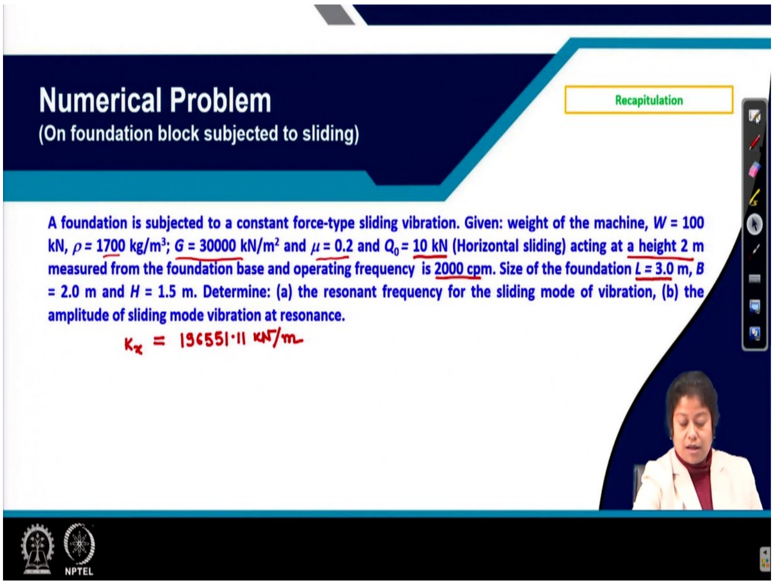

So, today first we will try to solve one numerical problem, before that what I will do, I will show what is the solution for that numerical problem, which we have discussed last class. So, if you recall last class we try to solve these numerical problem, I am just reading the problem first, it was said that a machine foundation is subjected to constant force that force type sliding vibration. A machine foundation is subjected to a constant force type sliding vibration

A few data are given, what are those data? Weight of the machine which is W here, which is equal to 100 kilo Newton, then, rho, rho means density of the soil on which the foundation is resting is given that is 1700 kg per meter cube, that dynamic shear modulus of the soil that is g here, which is equal to 30,000 kilo Newton per meter square is mentioned.

Poison's ratio mu is equal to 0.2 and the amplitude of horizontal sliding which is 10 kilo Newton is mentioned. However, it is also added that this horizontal force is acting at a high 2 meter from the base of the foundation and its operating frequency is also provided. Also the size of the foundation which is in L, B and H are given. So, L which is length is 3 meter here, weight is 2 meter and height 1.5 meter.

We are asked to find out the resonant frequency or the sliding mode of vibration, the amplitude of sliding mode of vibration at resonance, not any other frequency, but only at the resonance. So, last class I told how to solve these numerical problems I have written down the steps. So, today what I will do, I will tell you the final answer for the problem. So, instead of just saying the final answer, I would like to write the value for important parameters also, for an example, in this case, first you will find out Kx which is the spring constant for horizontal vibration and that is equal to 196551.11 kilo Newton per meter.

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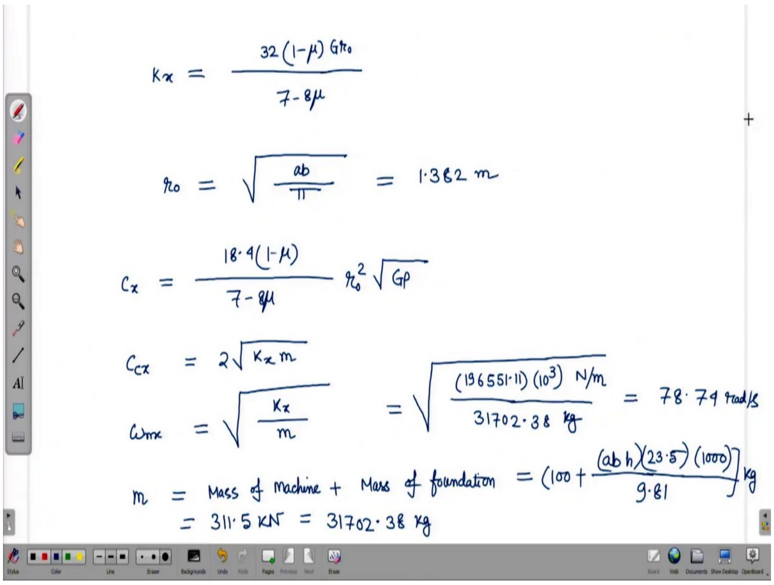

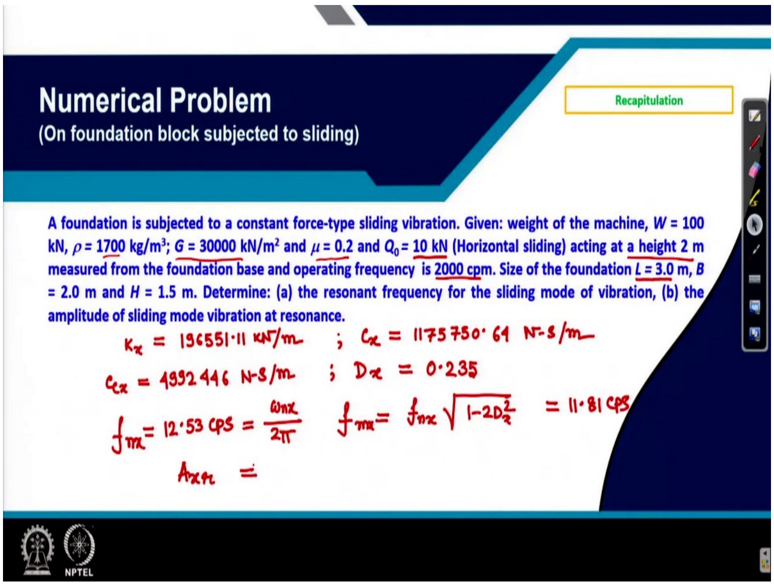

How we get it? I am just writing here. So, Kx is equal to 32 times 1 minus mu times Gr 0 divided by 7 minus 8 times mu. What was r0? That we have already calculated I think r0 we can get by these equal addition which was I can find it was 1.382 in meter. So, with this you can get the value of the Kx which is given here. Next step was to find out Cx, which is the coefficient of dashpot and that you can calculate by using this equation actually and what was the what is the value for Cx then? It is 1175750.64 and unit is Newton's second per meter.

After knowing Kx and Cx or after calculate Kx and Cx, we can calculate critical coefficient of critical damping using this equation. So, the value of Ccx is 4992446 in Newton's second per meter. So, from this you can calculate damping ratio for horizontal sliding which is the ratio of Cx to Ccx and it is 0.235. Now, next step is to find out that natural frequency. So, natural frequency omega nx is equal to square root of Kx divided by m. Now, what is m here? m is mass of machine plus mass of foundation.

So, mass in kg is coming approximately 311.5 in kilo Newton if you will convert it then you will get the mass. So, which is coming 31702.34 in kg. So, now from this we can calculate omega nx which is Kx divided by omega mass. So, Kx now Kx in Newton per meter n is here in kg. So, from these you can get frequency in cycles per second which is coming 12.53 cycle per second.

So, here I can write the final answer for omega nx is 12.53 Cps you can convert it in cycle per minute by multiplying with 60. Next is to find out Fm which is…I just made a mistake, omega nx is not cycle per second I have directly calculated here if so, better I should not do that just give me one minute time. So, basically you may get this is 78.74 in radians per second. Now, if you will divide it by 2 pi you will get f nx, so this is f nx which is 12.53 which you can calculate by simply this way.

Now, from this we can calculate Fm which is the resonant frequency and it is coming 11.81 cycles per second.

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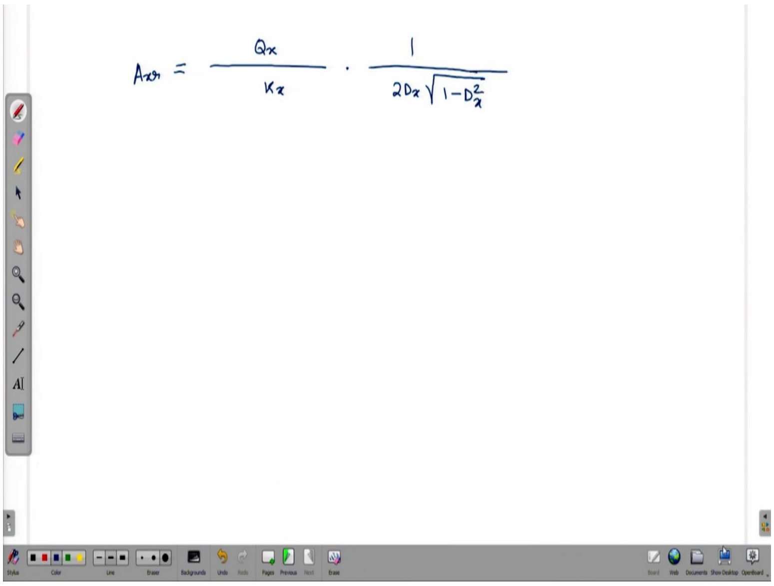

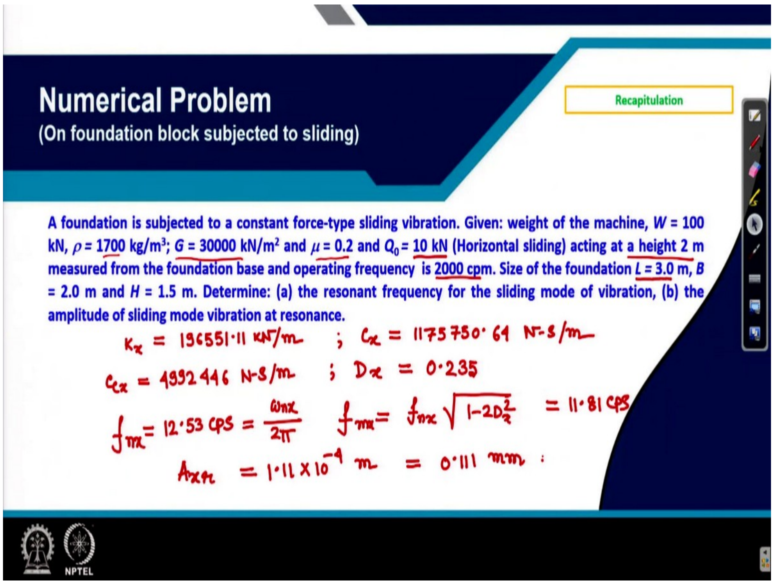

Then, you can calculate Axr which is the amplitude of sliding mode vibration at resonance and which equation in this case you will use for Axr I think I wrote it last class also. So, this equation will be used there 1 minus dx square. So, if you use this equation what you are getting is your qx is 10 kilo Newton and Kx also you can write in kilo Newton per meter when using the equation written on a whiteboard, then you will get the value of Axr which is coming 1.11 into 10 to the power minus 4 meter or you can write it as 0.111 in millimetre, this is the approximate value and the final answer for this numerical problem.

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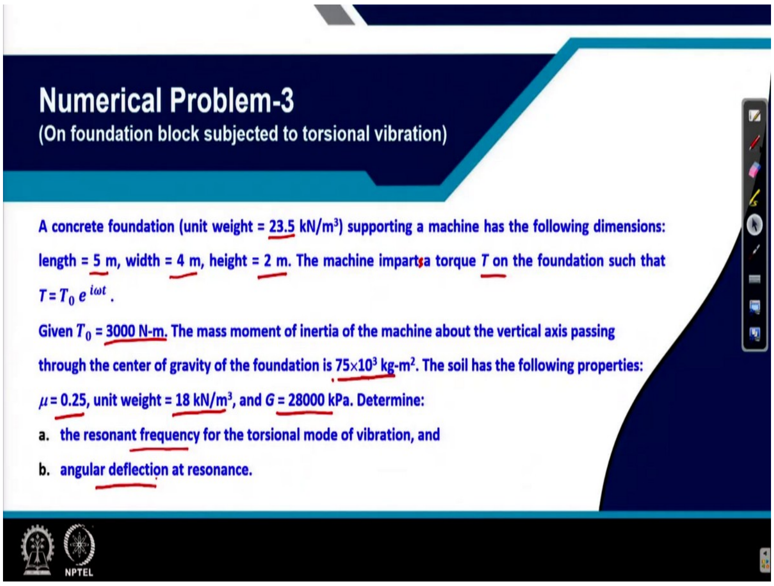

Now, today, we will solve a new problem, I am giving it as problem number 3. So, in this problem, what is said? A concrete foundation the unit weight of the concrete is mentioned which is 23.5 kilo Newton per meter cube supporting a machine has the following dimensions. So, the length of the foundation is 5 meter, width 4 meter and height is 2 meter. The machine imparts a torque T on the foundation such that T is equal to T0 times e to the power i omega t. So, what is T0? T0 is 3000 Newton meter, the mass moment of inertia of the machine about the vertical axis passing through the cg of the foundation is this one. So, the mass moment of inertia for the machine is provided, not the mass moment of inertia for the foundation about the axis passing through the cg of the foundation.

So, we need to find out the mass moment of inertia of the machine and the foundation together about the vertical axis passing through the cg and soil properties are also provided here you can see, poison’s ratio 0.25 unit weight of the soil bulk unit weight of course, is 18 kilo Newton per meter cube, G is 28,000 kPa.

So, what do we need to find out? We are asked to find out the resonant frequency for the torsional mode of vibration and we are asked to find out the angular deflection at resonance. So now, we will see how to solve these numerical problem step wise.

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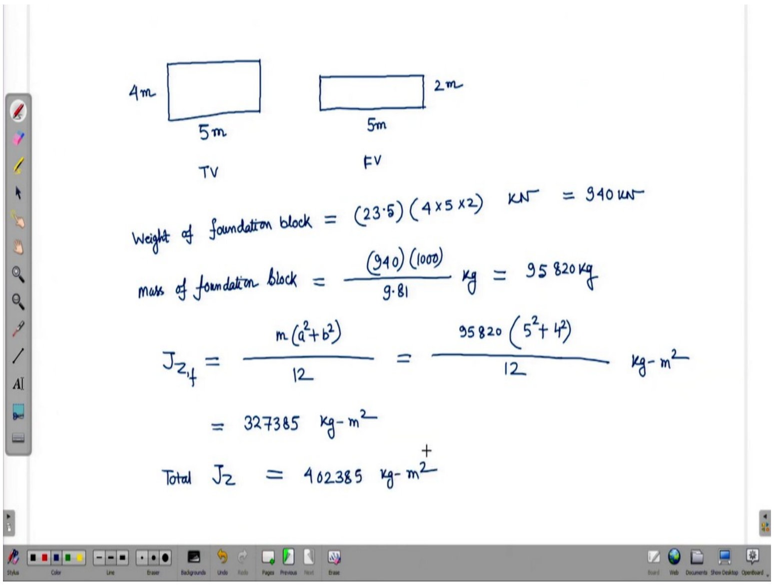

So, let us start. First, I will draw a schematic diagram plan that means top view and the front view of the rectangular foundation. So, length is 5 meter and width is 4 meter, so width is 4 meter this is top view. Now, what is the front view? Height is 2 meter and if you see a rest of the things are same so I can draw it also like length and height this is front view. Then, what is the total weight of this foundation block weight of foundation block, that is unit weight of concrete times volume of this foundation block it is in kilo Newton.

So, it is coming 940 kilo Newton. Next is to convert these weight to mass of foundation block mass of foundation block means 940 times 1000 divided by 9.81 that is in kg. So, how much we will get? Let me calculate, we are getting 95,820 in kg. Now, next term which we will find out is Jz for the foundation about an axis passing through cg. So, we will use this equation then here m is mass 95,820 kg A is length of the foundation which is 5, B is the width of the foundation which is 4 meter divided by 12.

So, coming how much it is coming 327385 in kg meters square, that means, we are getting 327385 unit kg meter square. Now, total Jz is how much in this case? Total Jz is if you see already the value for the machine is given which is 75,000 so I can add 75,000 to this is for foundation so I am writing it for f so coming 402385 kg meter square. Now, we need to calculate the equivalent radius for this case.

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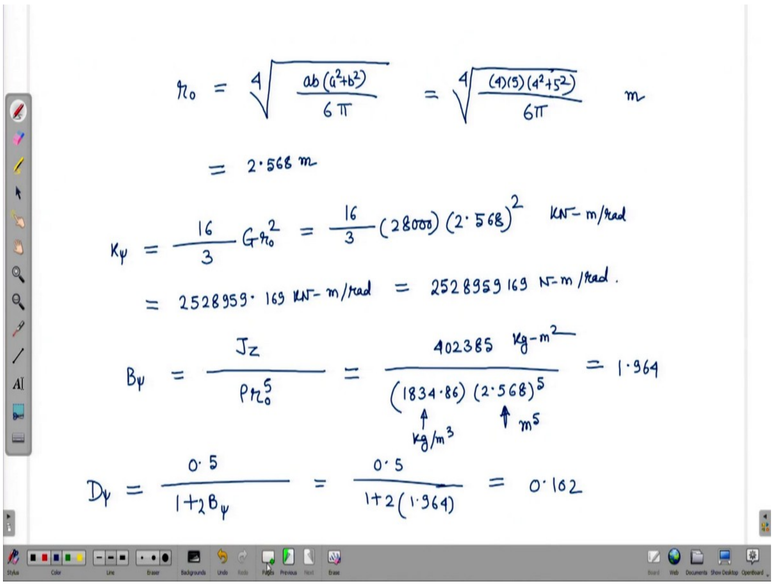

So now, we will find out r0 which is the equivalent radius. So, equivalent radius r0 is equal to fourth root of ab times a square plus b square divided by 6 pi. So, ab is already given 4 times 5 times 4 square plus 5 square here divided by 6 pi and it is coming in meter. So, we are getting here 2.568 meter, this is r0. Next task is to find out stiffness of the spring which is used to represent the soil and that we can calculate using this equation where G is shear modulus dynamic shear modulus and r0 is already calculated, what will be the unit? Its unit is since I have taken G in kilo Newton per meter square, so its unit is kilo Newton meter per radian and it is coming 2528959.169 that is in kilo Newton meter per radian or we can convert it in Newton.

So now, we know K psi value next we need to find out B psi, which is the ratio of sorry, which is the ratio of Jz or you can write it as Jzz also earlier I have used the symbol Jz so I am writing the same symbol Jz divided by rho r0 to the power 5. So, Jz already calculated 402385 in kg meter square, rho you can see the unit weight of concrete soil is given which is 18 kilo Newton per meter cube if we convert at Newton per meter cube and then divide by g we will get the density.

So, density is coming 1834.86, r0 is already calculated which is 2.568 to the power 5. So, density is in kg per meter cube and this is in meter to the power 5. So, finally, we are getting a number because you can see the final value is dimensionless. So, it is coming 1.964. So, B psi value is 1.964 from this, we can calculate D psi which is 0.5 divided by 1 plus 2 times B psi so we are getting 1 plus 2 times B psi means 1.964 which is coming approximately equal to 0.102.

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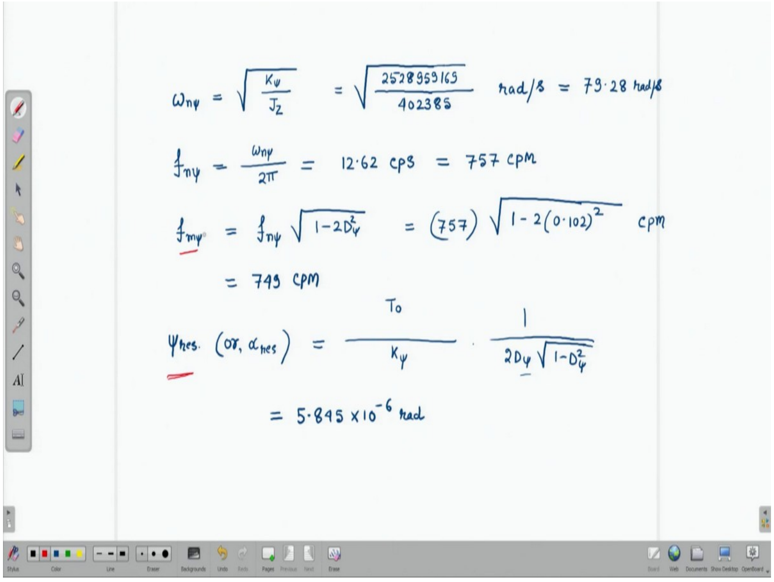

So, next we need to find out the natural frequency for torsional mode of vibration. So, for that we can write omega n psi is equal to square root of K psi divided by Jz. So, what is K psi here? K psi is I am writing the value which we have already calculated it is 2528959169. So, earlier, first we have calculated the value of K psi in kilo Newton meter per radian now I have converted it in Newton meter per radian and what is Jz here? Jz is 402385 it is in kg meter square. So, from these we are getting omega n psi in radian per second and it is coming it is coming 79.28 in radians per second.

Now, this is the circular frequency, so from this we can find out omega, sorry from omega n psi we can find out if n psi which is basically omega n psi divided by 2 pi and the value of F n psi is coming 12.62 in cycle per second, we can convert it in cycle per minute and then we will get 757 cycle per minute.

Now, from natural frequency what do we need to find out, we need to find out the resonance frequency which is F n psi and F n psi can be calculated by using the equation if n psi times square root of 1 minus 2 D psi square. So, D psi represents the damping ratio for the system subjected to torsional mode of vibration. So, what we will get here 757 times square root of 1 minus 2 times 0.102 square which is coming it is in cpm. So, from this we are getting 749 in cpm.

So, in this way we can calculate the resonant frequency or frequency at resonance. Now, we need to find out that angular displacement at resonance. So, angular displacement or we can call angular deflection also as part the problem statement that is that I can represent by psi resonance, sometimes we write it also in place of that we represented by alpha also. So, you can write in place of psi you can write alpha also at resonance. So, that is equal to T0 divided by K psi this will be multiplied with 1 divided by 2 times D psi times square root of 1 minus D psi square.

Now, here we know the value of T0 we know the value of K psi because we have already calculated K psi in previous page and also we have calculated D psi, so from this if I will use the values of T0 K psi and D psi what we will get, the angular deflection or angular displacement at resonance is 5.845 times 10 to the power minus 6 in radian. So, in this way we can calculate angular deflection at resonance we can calculate resonance frequency which are asked in the problem. So in this way we can solve this type of problem.

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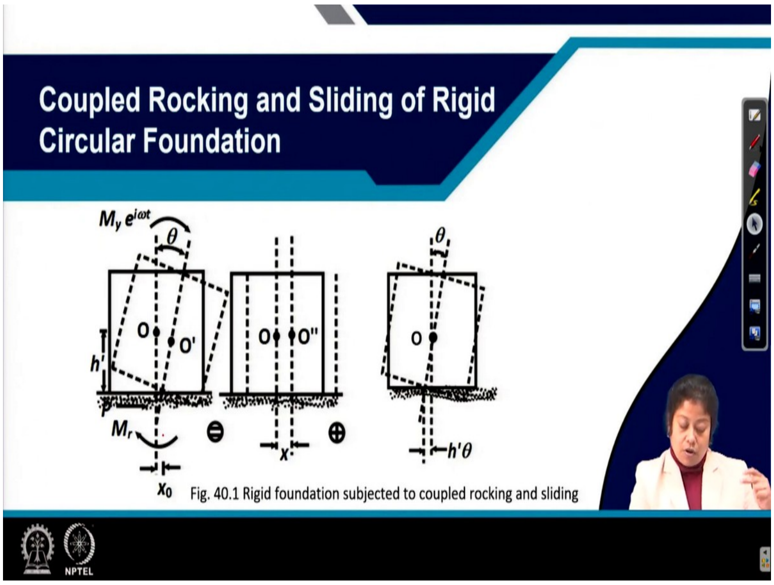

Now the there is one important thing which is when what will be happened when the foundation or rigid circular foundation will be subjected to coupled rocking and sliding. So here you can see the figure 1 when it is subjected to rocking you can see its cg eo is shifted from o2 to o dashed and the angular rotation in this case is theta, then what is happened by the soil to the base of the foundation a resistance moment m r is developed and also a resistance horizontal force opposing the sliding also we can see.

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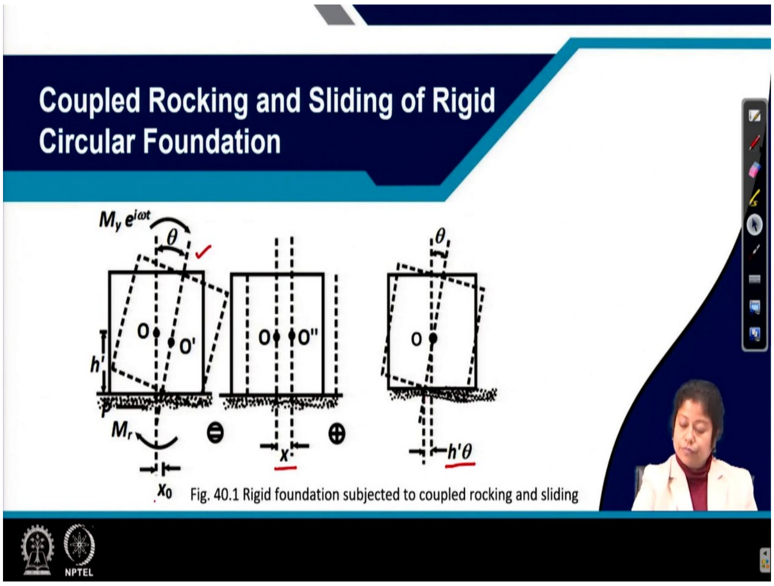

Now, we can actually represent this figure one by the by summing the two cases, one pure sliding and the second one is pure rocking. In case of pure sliding you can see the cg is shifted horizontally from o to o double dashed in case of rocking the axis vertical axis is rotated by an angle theta. Then, for that the base is shifted horizontally by an amount h dashed theta because of the sliding it is subjected the base is subjected to horizontal displacement equals to X only. Then, what is the resultant displacement? Resultant displacement is x minus h dashed theta.

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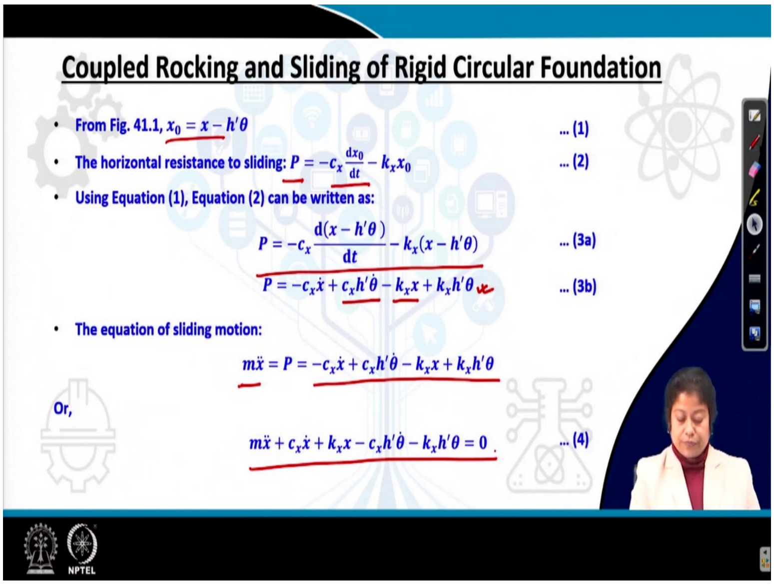

So you can write this expression that x 0 is equal to x minus h dashed theta where h dashed is the height of the cg from the base. Now, the horizontal resistance to the sliding can be represented by P which is equal to the damping force and the stiffness force, then here in place of x 0 if we will write x minus h dashed 3 which is written in equation 1 then what we can get, we can get this expression. Now, we can write this expression in more simple way in in place of dx dt we are writing x dot which is the velocity in x direction and if it will be multiplied to sorry cx which is the coefficient of damping for sliding vibration then we will get the damping force which is a kind of resistance force.

Also we will get this resistance force but in this case it is not resistance force you can see, what are the other resistance force, that is Kx times x and this is towards the direction of the sliding. So, from these we can write the equation of sliding inertia force is equal to the unbalanced resistance unbalanced force in x direction, so since there is no external force acting in horizontal direction, so only P will be on the right hand side, we can take everything on the left hand side and write this equation as you can see in equation 4.

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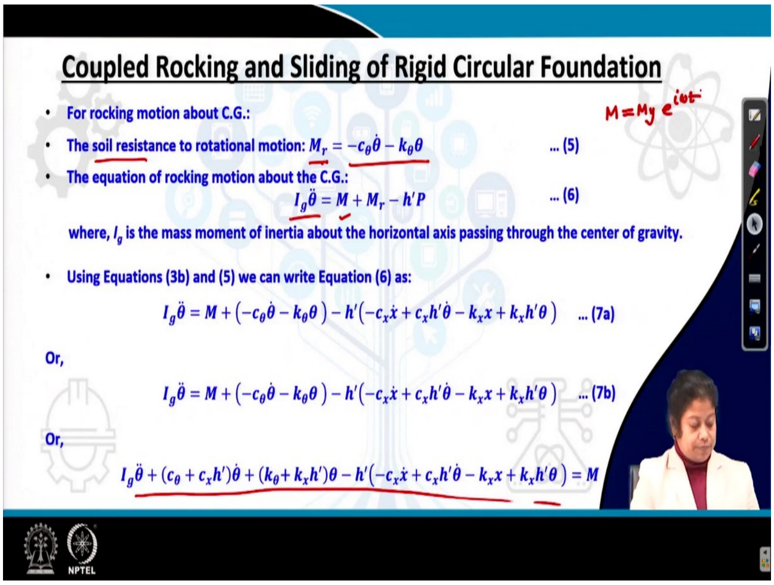

Similarly, the equation of rocking motion about the cg can be written for that we need to find out the resistance moment from coming from the soil reaction which is Mr and can be calculated by using equation 5 here. The equation of rocking motion about cg, then we can write as the inertia component on the left hand side and the external moment M plus Mr minus h dashed P. So p is the resistance at the base, so at the cg it causes a moment which is h dashed p and considering the direction here we are taking negative sign.

So, what is Ig here? Ig is the mass moment of inertia about the horizontal axis passing through the cg of the foundation. Now, if we combine if we consider equation 3B and 5 and use it in equation 6, equation 3B for the expression of P equation 5 for the expression of Mr and that we write in equation 6 we will finally get this one, what is capital M here? Capital M is the external moment or rocking vibration in which the foundation block is subjected. So, in this case I think we have taken M is equal to my times e to the power I omega t.

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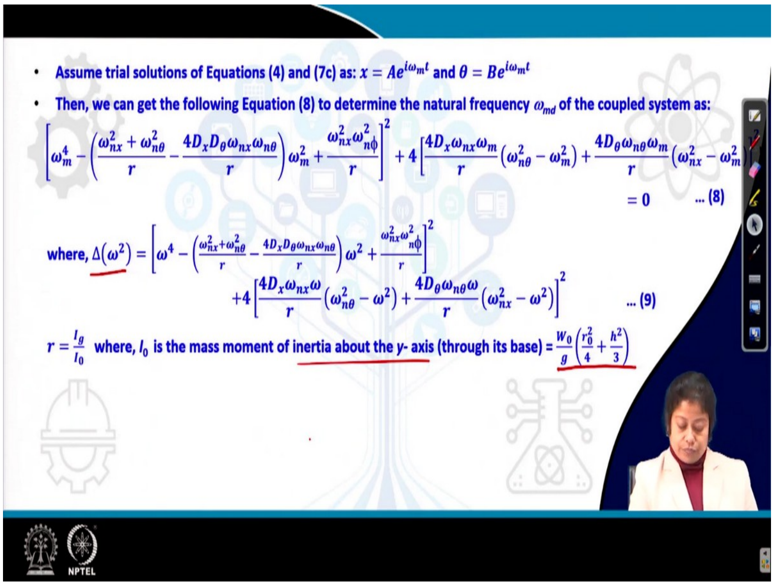

So now, if we assume the trial solution as shown here, then we will we can write equation 8 as you can see here in this form. So omega m is the natural frequency for the machine foundation system subjected to coupled rocking and sliding. So here, what is r? r is a new term which is the ratio of the Ig to I0, I0 is the mass moment of inertia about y axis through passing through the base, whereas Ig passing through the cg of the foundation, we already have seen how to calculate Ig for I0 we will use this equation.

Another thing we already know how to calculate omega nx how to calculate omega n theta, so I am not repeating that, if we know the operating frequency omega if we know omega nx and omega n theta from that we can calculate delta omega square also.

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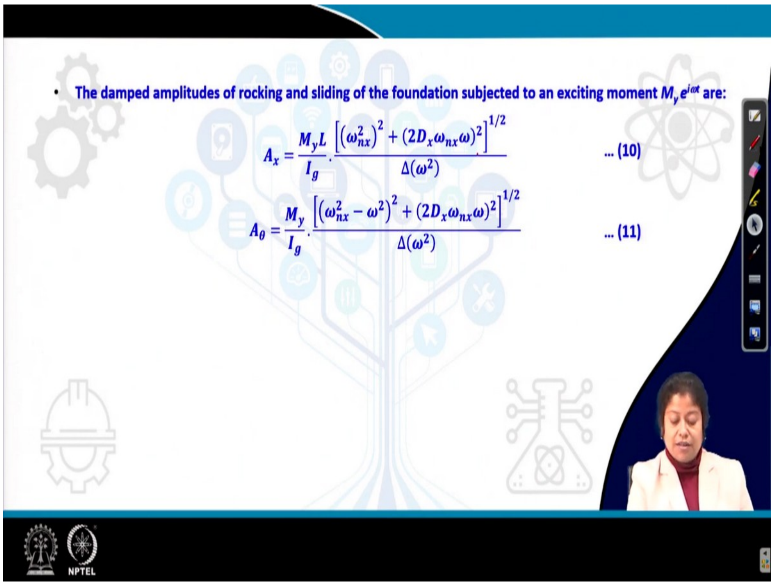

So, if we solve the previous equation 8, then we will find out omega m once we know omega m from that we can calculate the Ax and A theta.

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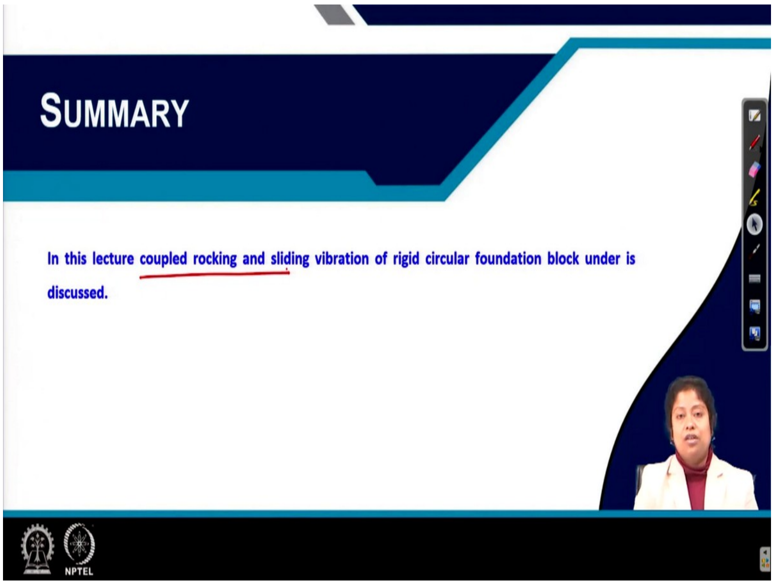

So, in this way we can find out the solution for the foundation block subjected to coupled sliding and rocking vibrations. So in this lecture we have solved numerical problems, where foundation block is considered to be subjected to torsional vibration also we have discussed how we will form the equation of motion when the foundation is subjected to coupled rocking and sliding vibrations and then how to get the solutions.

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So here is the reference which I have used for this class. Thank you.