

Soil Dynamics
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Lecture 30

Determination of Dynamic Properties (Block Vibration Test)

Hello friends. Hope you are all doing well. So, today we will discuss another field test to determine the dynamic properties of soils.

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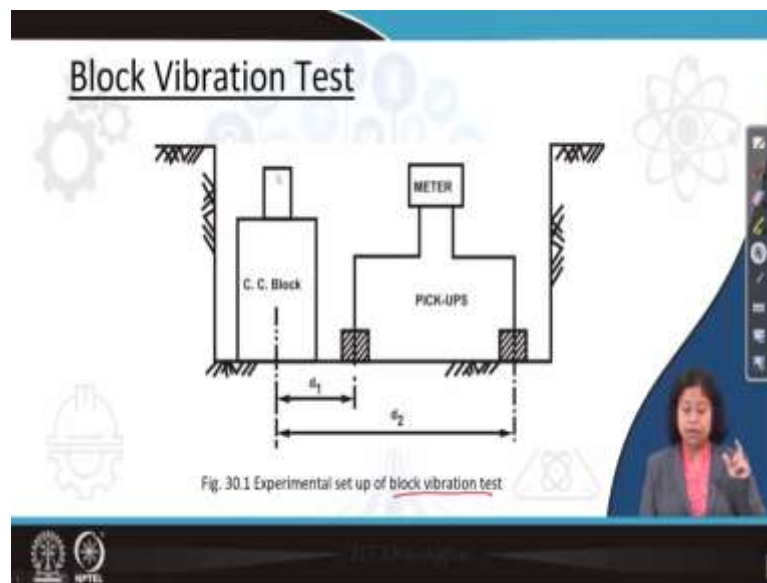
Introduction

- Block vibration test is generally carried out in the field to determine the dynamic properties of in-situ soil.
- The block vibration test can be conducted as forced vibration test or free vibration test.
- For this test IS 5249: 1992 is followed.
- Generally, a test pit of 3 m × 6 m size or other suitable dimension is required to be made.
- The size of the test pit depends upon the test block.
- A plain cement concrete block of M-15 concrete is generally constructed in the test pit.
- The selection of the block size depends upon the sub-soil condition.
- For ordinary soil, the block size is 1 m × 1 m × 1.5 m.
- For dense soil, the block size is 0.75 m × 0.75 m × 1.0 m.

We will first study the block vibration test which is generally carried out in the field to determine the dynamic properties of in situ soil. The block vibration test can be conducted as forced vibration test or free vibration test. Here, we will discuss the forced vibration test. So, for this test we will follow IS 5249 published in 1992. For this test generally a test pit of size 3 meter by 6 meter or other suitable dimensions is required to be made at the site.

The size of the pit mainly depends upon the test block. A plain cement concrete block of M 15 grade concrete is generally used to construct the test block in the pit. The selection of the block size depends upon the sub soil condition. For ordinary soil generally we take the block size 1 meter by 1 meter by 1.5 meter that means the plan area is 1 meter by 1 meter whereas the depth of the block is 1.5 meter. However, for dense sand or dense soil the block size is changed to 0.75 meter by 0.75 meter by 1 meter in depth.

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Block Vibration Test

- In block vibration test a plain cement concrete block (made of M-15 concrete) is used.
- The size of the block depends upon the soil type as discussed in previous slide.
- As per IS 5249: 1992 the block size should be so adjusted that the mass ratio is always higher than unity.
- The minimum curing period of the concrete block should be at least 15 days.
- Foundation bolts should be embedded into the concrete block at the time of testing for fixing the oscillator assembly.
- The vibration pick-ups is generally fixed at the top of the block for sensing the vertical motion as shown in Fig. 30.1.
- The vibration exciter is required to mount on the block such that it generates purely vertical sinusoidal vibrations and line of action of vibrating force passes through the centre of gravity of the block.
- The exciter is operated at a constant frequency.

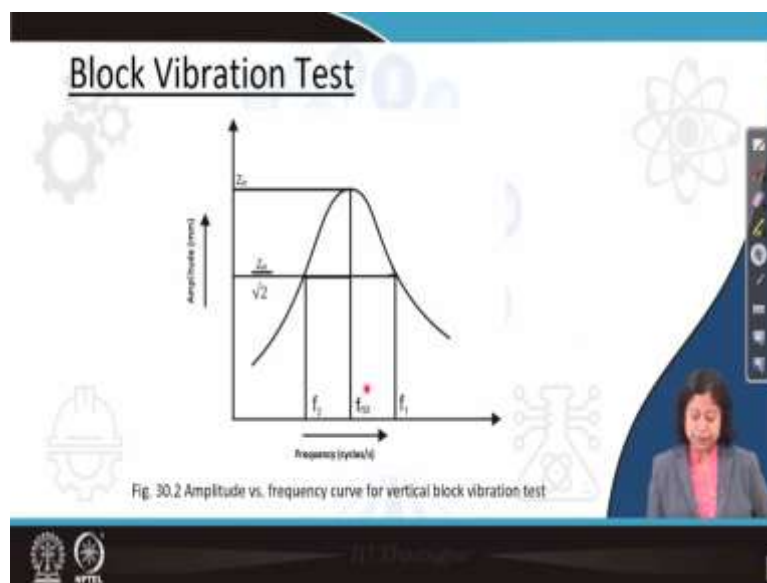
So, in figure 1 you can see typical experimental setup for block vibration test. So, let us see the procedure of block vibration test. In block vibration test a plain cement concrete block made of M 15 grade concrete is used. The size of the block depends upon the soil type as we have already discussed. As per IS 5249 published in 1992 the block size should be adjusted so that the mass ratio is always higher than unity. The minimum curing period of the concrete block which will be tested should be at least 15 days.

That means we need to allow minimum 15 days for the curing of the concrete block. Next is the foundation bolts should be embedded into the concrete block at the time of testing for fixing the oscillator assembly. The vibration pickups is generally fixed at the top of the block

for sensing the vertical vibration as shown in figure 1. So, here you can see pick up mounted on the block.

The vibration exciter is required to mount on the block such that it generates the pure sinusoidal wave vibration and line of action of the vibration or vibrating force passing through the CG which is center of gravity of the block. So, if I go back to the figure so the line of action of the sinusoidal load should pass through the CG of the block. The exciter is operated at a constant frequency in block vibration test.

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So, in this figure you can see the variation of the amplitude with the frequency. So, with the increase of frequency what is happened amplitude also increases up to the maximum value of z_M say in this case and then it starts to decrease you can see the trends with increasing frequency. Now, we need to find out the maximum amplitude and the corresponding frequency which is f_n in this case.

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Block Vibration Test

- Co-efficient of elastic uniform compression of soil
- It is calculated by Using the following Equation (1):

$$C_u = \frac{4\pi^2 f_{nz}^2 M}{A} \quad \dots (1)$$

where, f_{nz} is natural frequency of the system, M is the total mass of the foundation block, exciter and motor, and A is the contact area of the foundation block with the underlying soil.

- Damping ratio of soil
- It is calculated by using following Equation (2):

$$D = \frac{f_2 - f_1}{2f_{nz}}$$

The slide features decorative icons of a gear, an atom, a lightbulb, and a flask. A vertical toolbar is visible on the right side.

Now, from this curve we can calculate two important parameters for the soil. One is coefficient of elastic uniform compression of soil which is calculated by using this equation C_u ; C_u is the coefficient of elastic uniform compression of soil that is expressed by $4\pi^2 f_{nz}^2 M$ divided by A . What are the meanings of these parameters? Here f_{nz} is natural frequency of the system.

Whereas M is the total mass of the foundation block exciter and the motor together. A is the contact area of the foundation block with the underlying soil. Now another important parameter which we can find is damping ratio of soil. Damping ratio you can call it or you can call it as damping factor as well. So, for that we can use this formula directly. Please make a few correction what we have discussed about C_u .

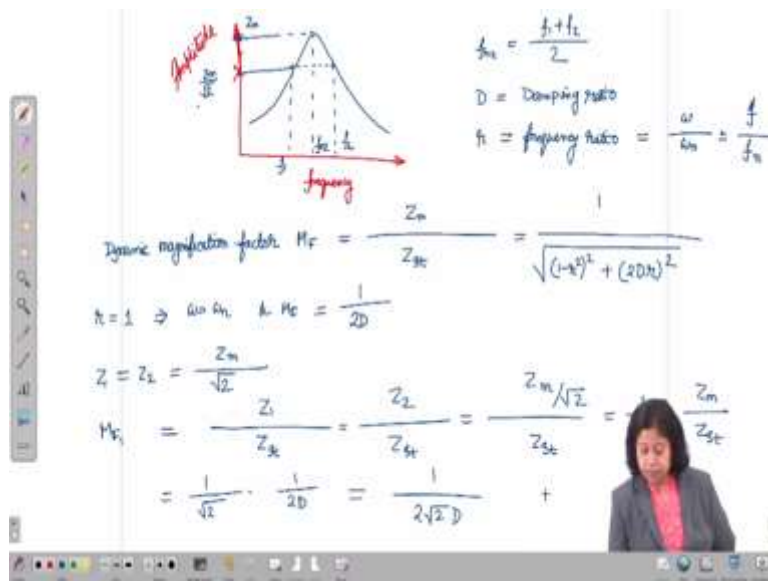
This is not for foundation block because the word foundation block may create trouble for all of us. It is better to use the word test block and here also it is test block then the question what will be C_u ; C_u means coefficient of elastic uniform compression of soil when we are designing the foundation block. Foundation means here the actual foundation for the machine. So, that time how we will calculate C_u .

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$$C_u = C_u \sqrt{\frac{A}{A_1}}$$

So, let us take that C_u is C_u , C_u is equal to alright where already we have seen what is A and A_1 is the contact area of the actual foundation to the soil. Now, generally if the value of the A_1 exceeds 10 square meter that time if we need to calculate C_u then we take the value of C_u considering A_1 is equal to 10 square meter. Now, what is the f_2 and f_1 ? If I will show you here.

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So, I can draw amplitude versus frequency curve horizontal axis is representing frequency here whereas vertical axis represents amplitude. Here you can see now I am just going back to the white board the same curve I am plotting here. Now, here this is the maximum

amplitude which we can write as so this maximum amplitude is Z_m . Now we will choose f_1 and f_2 in such a way that first let me mark $f_m z$.

Now we will choose f_1 and f_2 in such a way that the amplitude corresponding to both f_1 and f_2 should be equal to Z_m divided by root 2. So, first here I will mark the position of Z_m divided by root 2. Now, this give two operating frequency smaller one is f_1 and the larger one is f_2 . Eventually we can see that $f_m z$ we can get the average of approximately we can get this.

Now, if I know $f_1 f_2$ and the corresponding amplitude that is Z_m divided by root 2 where Z_m is the maximum amplitude corresponding to f and z that means the natural frequency of the entire system then how we will calculate the damping ratio of soil which is represented by capital D that we will now see. We already know that dynamic magnification factor which we can define by M_F capital M with a subscript F is equal to sorry here we are using Z .

So, Z_{maximum} divided by Z_{static} . Now, from this what we can write in we can also express it as 1 divided by square root of $1 - r^2$ whole square plus two times D times r whole square where D is the damping ratio. I am writing here D is damping ratio and r is frequency ratio. So, if operating frequency of a machine is ω and natural frequency of the system is ω_n then r is equal to ω divided by ω_n or in terms of frequency in cycles per second we can write f divided by f_n .

So, in our case we will define frequency ratio using f_1 and f_2 . Now, when the amplitude is Z_m divided by root 2 that time what is happened. Before seeing that first let us see what is the magnification factor when r is equal to 1. So, frequency ratio r is equal to 1 means what it means ω is equal to ω_n and magnification factor becomes almost equal to 1 divided by $2D$ it becomes actually equal to 1 divided by $2D$.

Now this is when r is equal to 1 that means amplitude is Z_m . Now we have another amplitude let us take Z is equal to or I can write it as Z_1 and Z_2 actually both are same. Z_1 and Z_2 corresponding to f_1 and f_2 so this equals to Z_m divided by root 2. So, what I can write a new magnification factor M_{F1} that when the amplitude is Z_m divided by root 2 or Z_1 that time what is the magnification factor that I am trying to write here.

So, Z_1 by Z_{static} or I can write it also as Z_2 by Z_{static} that is equal to what I can write in place of Z_1 or Z_2 , I can write Z_m divided by root 2. Now in Z_{static} remains as it is so it is

now becoming 1 divided by root 2 times Z m divided by Z s t. Now Z m divided by Z s t means 1 divided by 2 D so I can write here then M F 1 is equal to 1 divided by root 2 times 1 divided by 2 D. So, we are getting 1 divided by 2 times root 2 D.

So, this magnification factor is corresponding to the amplitude Z m divided by root 2 that means for this case. Now, at that point let us take frequency ratio we can write frequency ratio at that point is instead of writing r, I can write as capital R 1 or capital R also I can write.

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R is the frequency ratio corresponding to f_1 or f_2 or $\frac{Z_m}{\sqrt{2}}$ amplitude.

$$M_r = \frac{1}{2\sqrt{2}D} = \frac{1}{\sqrt{(1-R^2)^2 + (2DR)^2}}$$

$$R_1 = \frac{f_1}{\omega_n}$$

$$R_2 = \frac{f_2}{\omega_n}$$

$$\Rightarrow (1-R^2)^2 + 4D^2R^2 = 8D^2$$

$$\Rightarrow R^4 - 2R^2 + 1 + 4D^2R^2 - 8D^2 = 0$$

$$\Rightarrow R^4 - 2R^2(1-2D^2) + (1-8D^2) = 0 \quad \text{--- (I)}$$

$$R_1, R_2 = \frac{1}{2} \left[2(1-2D^2) \pm \sqrt{4(1-2D^2)^2 - 4(1-8D^2)} \right]$$

$$= (1-2D^2) \pm \sqrt{(1-2D^2)^2 - (1-8D^2)}$$

$$\omega_n = \frac{f_1 + f_2}{2}$$

D = Damping ratio

$$f_n = \text{frequency ratio} = \frac{\omega}{\omega_n} = \frac{f}{f_n}$$

Dynamic magnification factor $M_r = \frac{Z_m}{Z_{st}} = \frac{1}{\sqrt{(1-R^2)^2 + (2DR)^2}}$

$$M_r = 1 \Rightarrow \omega = \omega_n \text{ \& } M_r = \frac{1}{2D}$$

$$Z_1 = Z_2 = \frac{Z_m}{\sqrt{2}}$$

$$M_r = \frac{Z_1}{Z_{st}} = \frac{Z_2}{Z_{st}} = \frac{Z_m/\sqrt{2}}{Z_{st}} = \frac{1}{\sqrt{2}} \cdot \frac{Z_m}{Z_{st}}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2D} = \frac{1}{2\sqrt{2}D}$$

So, capital R, I am going to the next page, so capital R is the frequency ratio corresponding to f_1 or f_2 or we can say corresponding to amplitude Z m divided by root 2. So, what is the

mathematical form of R that first we need to find out in this case. So, $M F 1$ we already get as 1 divided by $2 \sqrt{2}$ times D . So, I can write in place of first $M F 1$ as 2 times 1 divided by 2 times square root of 2 times D which is equal to the mathematical expression for the magnification factor which is $1 - \text{capital } R^2 \text{ whole square} + 2 D \text{ capital } R \text{ whole square}$.

Now, from this what we can write? We can write $1 - R^2 \text{ whole square} + 4 D^2 R^2$ which is equal to $8 D^2$ then I can expand this term as R to the power 4 minus $2 R^2$ plus $1 + 4 D^2 R^2$ minus $8 D^2$ is equal to 0 . From this what I can write? I can write R to the power 4 then minus $2 R^2$ times $1 - 2 D^2$ plus $1 - 8 D^2$ is equal to 0 .

Now, we can see it is a fourth order equation of R . So, we will get two positive value for this R let us take R_1 and R_2 because we know R_1 is equal to f_1 divided by $f_n Z$ and R_2 is equal to f_2 divided by $f_n Z$. So, from this now I will try to write this expression in terms of R_1 and R_2 . So, the two roots are R_1^2 and R_2^2 for this equation I can give it a number roman 1.

So, if R_1^2 and R_2^2 are the two roots of this equation 1 then what we can write? We can write R_1^2 and R_2^2 should be equal to half of we can write here this is equal to here half so minus, minus plus $2 R^2$ sorry, no $2 R^2$ only $2, 2 1 - 2 D^2$ square then plus minus square root of $4 1 - 2 D^2 \text{ whole square} - 4 \text{ times } 1 - 8 D^2$.

So, $1 - 8 D^2$ square. I can simplify this also as $1 - 2 D^2$ plus minus square root of $1 - 2 D^2 \text{ whole square} - 1 - 8 D^2$. I can simplify it also once again. So, we have seen how to find out the two roots R_1^2 and R_2^2 .

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$$\begin{aligned}
 R_2^2 &> R_1^2 \\
 R_1^2, R_2^2 &= (1-2b^2) \pm \sqrt{(1-2b^2)^2 - (1-8b^2)} \\
 &= (1-2b^2) \pm \sqrt{1-4b^2+4b^4-1+8b^2} \\
 &= (1-2b^2) \pm \sqrt{4b^2+4b^4} = (1-2b^2) \pm \sqrt{4b^2(1+b^2)} \\
 &= (1-2b^2) \pm 2b\sqrt{1+b^2} \\
 R_2^2 &= (1-2b^2) + 2b\sqrt{1+b^2} & R_1^2 &= (1-2b^2) - 2b\sqrt{1+b^2} \\
 R_2^2 - R_1^2 &= 4b\sqrt{1+b^2}
 \end{aligned}$$

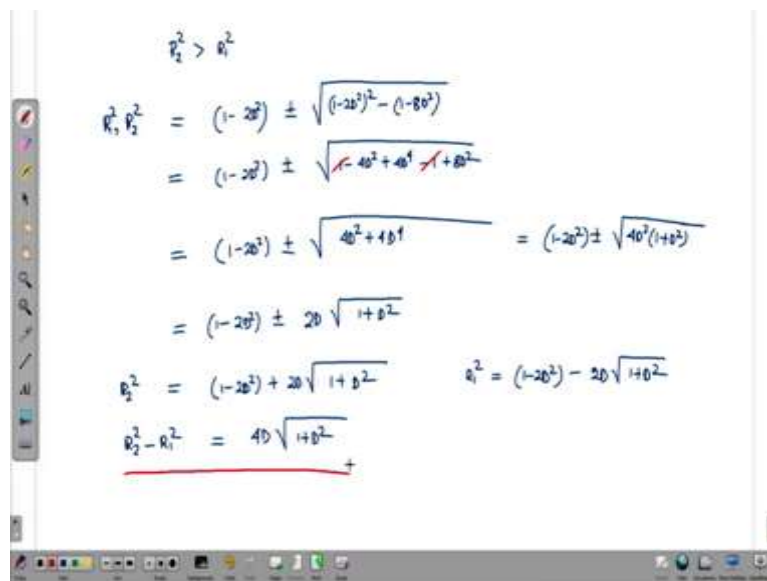
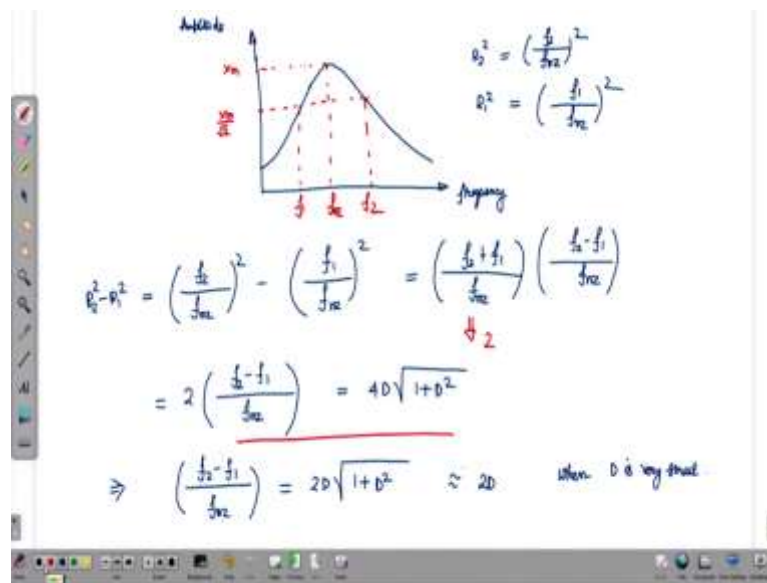
In this case always this R 2 square is greater than R 1 square this is the first thing then now what we can write R 1 square or R 2 square that is equal to 1 minus 2 D square plus minus. So, when R 2 square is greater than R 1 square that means for R 2 square we need to consider here plus and for R 1 square we will consider minus sign. However, now we can write the expression for R 1 square and R 2 square.

So, that is 1 minus 2 D square plus minus square root of 1 minus 2 D square whole square that we have written in the previous page minus 1 minus 8 D square. Now as I said I need to expect this 1 minus 2 D square whole square. So, for the expression within square root will be 1 minus 4 D square plus 4 D to the power 4 minus 1 plus 8 D square. Now here what we can see here there is 1 plus 1 and here there is 1 minus 1.

So, these two will be cancelled out other than that we have 8 D square minus 4 D square. So, that we can write as 4 D square. So, next line will be 1 minus 2 D square plus minus square root of 4 D square plus 4 D to the power 4 which is equal to 1 minus 2 D square plus minus now whatever written within square root there I can write 4 D square common so 4 D square times 1 plus D square that is within square root.

Then in the next line what I can write 1 minus 2 D square plus minus 2 D times square root of 1 plus D square. So, here what is R 2 square then? R 2 square is 1 minus 2 D square plus 2 D times square root of 1 plus D square whereas R 1 square is equal to 1 minus 2 D square minus 2 D times square root of 1 plus D square then from this we can write R 2 square minus R 1 square which is equal to R 2 square minus R 1 square is equal to 4 D times 1 plus D square.

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Now, if I will go to the next page. First please recall this curve. So, what we can see here we have this kind of curve or I can make that tip little bit smooth like this or just this is not looking smooth so I am just erasing, like this. So, what we have taken this peak is if it is x_m then we have considered a point x_m divided by root 2 and the corresponding frequencies are f_1 and f_2 and the frequency corresponding to x_m is denoted as $f_n z$.

Now, here what is then R_2^2 square minus R_1^2 square if you see R_2^2 square means referring to these figure what we can write? We can write f_2 divided by $f_n z$ whole square. Here you need to level the axis this is frequency and this axis is amplitude that we have already seen also. So, R_2^2 square is already written. What is R_1^2 square? As per this figure R_1^2 square is equal to f_1 divided by $f_n z$ square.

f_1 divided by $f_n z$ whole square that is R_1 square. Now then we can also write R_2 square minus R_1 square is equal to f_2 divided by $f_n z$ whole square minus f_1 divided by $f_n z$ whole square which is equal to f_2 plus f_1 divided by $f_n z$ times f_2 minus f_1 divided by $f_n z$. Now, today only just a few minutes before what we have seen about f_2 plus f_1 divided by $f_n z$ that is equal to 2, only 2.

So, I am just writing once again, this is equal to 2 then what I can write R_2 square minus R_1 square is equal to 2 times of f_2 minus f_1 whole divided by $f_n z$. Now, this expression is also equal to what we get in the previous slide that is R_2 square minus R_1 square is equal to $4D$ times square root of 1 plus D square then I can write it here also $4D$ times square root of 1 plus D square.

Now then from this expression what we can write? We can get the value of f_2 minus f_1 divided by $f_n z$ and that value f_2 minus f_1 divided by $f_n z$ is equal to $2D$ times square root of 1 plus D square or what I can write we can write this is approximately equal to $2D$ when D is very small; very small means you can give a try when D is equal to 0.25 that means (())(33:51) has 25 percent damping that time.

What will be the value of square root of 1 plus D square that will be equal to 1.03 or something like that, that is the reason we are neglecting the term which is written under square root. Now, from this what we can write finally.

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The image shows a whiteboard with a vertical toolbar on the left and a Windows taskbar at the bottom. The whiteboard contains the following handwritten mathematical derivation:

$$\frac{f_2 - f_1}{2f_n} = D \Rightarrow D = \frac{f_2 - f_1}{2f_n}$$

Block Vibration Test

- Co-efficient of elastic uniform compression of soil
- It is calculated by Using the following Equation (1):

$$C_u = \frac{4\pi^2 f_{nz}^2 M}{A} \quad \dots (1)$$

where, f_{nz} is natural frequency of the system, M is the total mass of the foundation block, exciter and motor, and A is the contact area of the foundation block with the underlying soil.

- Damping ratio of soil
- It is calculated by using following Equation (2):

$$D = \frac{f_2 - f_1}{2f_{nz}}$$

Now what we can write from this $f_2 - f_1$ divided by 2 times of f_{nz} is equal to D . Generally, we write it as D is equal to $f_2 - f_1$ divided by 2 times of f_{nz} . In this way we can calculate the damping ratio of the soil by conducting block vibration test. So, here you can see the same expression is mentioned directly that damping ratio D is equal to $f_2 - f_1$ divided by 2 times of f_{nz} .

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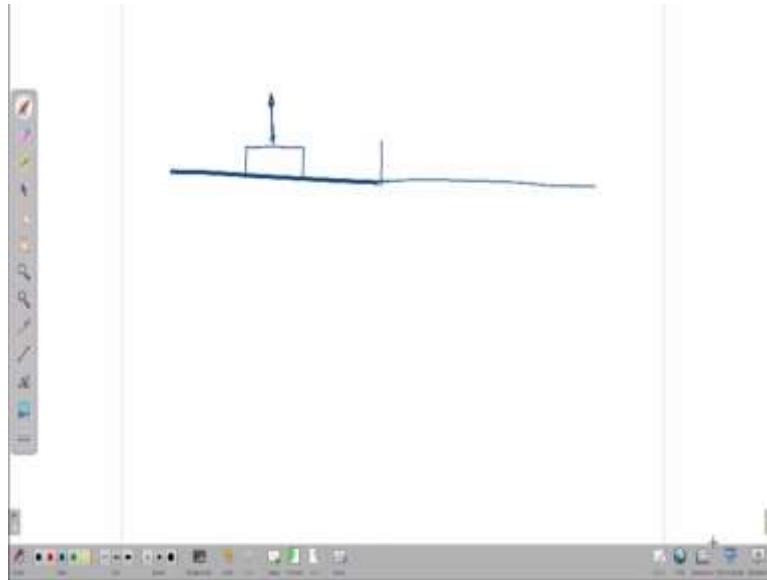
Steady State Vibration Test

- In this test a circular plate is placed on the ground surface.
- Then the plate is vibrated by sinusoidal loading.
- This vibration will send out the Rayleigh waves.
- This Rayleigh wave causes the vertical vibration of the ground surface predominantly.
- The velocity of the Rayleigh wave is calculated as:

$$v_R = f \lambda_R \quad \dots (7)$$

- f is the frequency of the wave and λ_R is the wavelength.
- The wavelength (λ_R) can be measured by wave number as shown here:

$$\lambda_R = \frac{x}{n} \quad \dots (8)$$



Now, another test is steady state vibration test. So, what is happened in steady state vibration test? In this test generally a circular plate is placed on the ground surface then the plate is subjected to vertical sinusoidal loading, then what is happening? This vibration will send out the Rayleigh waves and this Rayleigh wave causes the vertical vibration of the ground surface predominantly.

So, what I am saying I am just trying to draw here we have taken one circular plate test it on the ground surface. So, this is our ground surface let us take this is the circular plate which is subjected to vertical vibration. So, what will happen from here sinusoidal wave will generate and what else will be happened this ground surface will be vibrated because of this sinusoidal wave.

Now this Rayleigh wave it is already mentioned that the Rayleigh wave causes the vertical vibration of the ground surface predominantly. Now, how we calculate the velocity of the Rayleigh wave? For that we need to know the wavelength of the Rayleigh wave and the frequency. So, if I can draw a figure here so if the Rayleigh wave is like this then what is the wavelength here? Wavelength is this much this is λ_R ; R stands for Rayleigh wave.

So, if we know the wavelength and also if we know the frequency then using this relationship that frequency times wavelength is equal to the velocity of the wave we can calculate the velocity of Rayleigh wave. Now the question how we can measure the wavelength λ_R . So, we can measure the number of wave or we can call it wave number for a particular distance x .

Suppose, wave number is n then we can calculate the wavelength which is x divided by n you can see in this equation 8. Here just I have forgotten to write the axis so horizontal axis is representing displacement, please correct it, please write it here and the vertical axis represents the amplitude of the wave.

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Steady State Vibration Test

- The velocity of shear wave can also be determined from Equation (9)

$$v_R = v_s \quad \dots (9)$$

- Generally Rayleigh waves travel through soil within a depth of one wavelength.
- So, the v_s calculated here will represent soil condition at an average depth of $\lambda_R/2$.

The slide also features decorative icons of a gear, a tree, a flask, and a person, along with a vertical toolbar on the right side. Logos for IIT Bombay and NPTEL are visible at the bottom left.

We have already seen that the velocity of the Rayleigh wave is almost equal to the velocity of the shear wave. So, we can calculate, we can report the velocity of the shear wave from the velocity of the Rayleigh wave using this relationship. Now, generally Rayleigh wave travels through soil within a depth of one wavelength that means from the ground surface to the depth λ_R we can expect the Rayleigh wave because it is a surface wave. So, the velocity of the shear wave which is v_s here represents the soil property for a soil having average depth $\lambda_R/2$ where λ_R is the wavelength of the Rayleigh wave.

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SUMMARY

In this lecture field tests related to determine the dynamic properties of soils are discussed. For this following two tests are studied

- Block vibration test
- Steady state vibration test

So, come to the summary of today's lecture. In this lecture field test related to determine the dynamic properties of soils are discussed. What are those field test? Block vibration test and steady state vibration.

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REFERENCES

1. Principles of Soil Dynamics by B M Das, PWS-KENT Publishing Company (available Indian Edition: by B M Das and Luo Zhe, Cengage Learning India Private Limited; Third edition)
2. IS 5249:1992

So, these are the references which I have used for today's lecture, one textbook and IS code 5249 1992. Thank you.