

Soil Dynamics
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Lecture 29
Determination of Dynamic Properties of Soils
(Numerical Problems on Seismic Reflection & Refraction Tests)

Hello friends, last two classes we have discussed the theoretical background of seismic reflection and refraction tests in soil conducting these two tests either reflection or refraction test we can find out the velocity of the P wave in different layers we can also find out the thickness of the soil layers and the inclination if any of the boundary of two or three layers that we have studied.

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Now, today we will discuss the next part which is numerical problem on seismic reflection and refraction test.

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Numerical Problem-1

Referring to Fig. 29.1 the results of a refraction survey are as follows. The distance between A and D is 60 m. Determine (a) v_{p1} and v_{p2} , (b) z' and z'' and (c) θ .

Point of disturbance A	
Distance from A (m)	Time of first arrival (ms)
0	0.0
5	6.0
10	12.5
15	17.6
20	24
30	30.2
40	34.3
50	38.5
60	42.6

Point of disturbance D	
Distance from E (m)	Time of first arrival (ms)
0	0
5	5.8
10	11.4
15	17.7
20	23.4
30	35
40	39.1
50	41.4
60	43.8

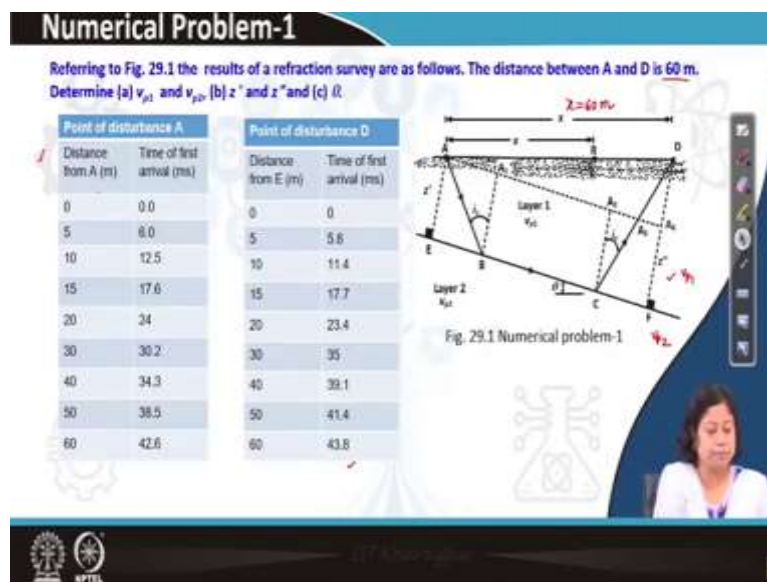
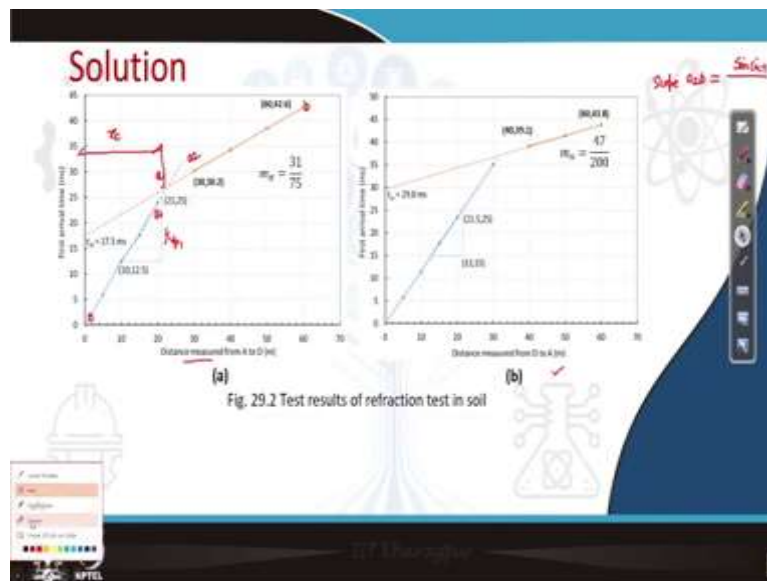
Fig. 29.1 Numerical problem-1

please note part (c) is asked to find out theta of the Numerical Problem- 1

So, let us take the first problem which is shown in this figure also. So, here it is said that in figure to 29.1 the results of a refraction survey are given, the distance between A and D is 60 meter that means here x is equal to 60 meter, our objective is to determine the velocity of the P wave in layer 1 that means v_{p1} here and the velocity of the P wave in layer 2 that means v_{p2} .

Also we are asked to find out the magnitude of z' and z'' which are the thickness of layer 1 and layer 2 respectively and θ which is the inclination angle of the boundary of the two layers. So, when A is the source and D is the receiver of the first arrival of wave P wave of course that data is given in the table on the left hand side and when D is source of disturbance and receiver is placed at A that time whatever data we have recorded that is given in the table on the right hand side.

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So, for this type of problem first we need to plot the curve which is you can see here. So, first I have plotted the curve x on the horizontal axis x represents the distance measured from the source to the receiver. So, for the left hand side table source is A and receiver is placed at D so the distance measured from A to D in meter is given here, you can see so first graph is related to table 1.

And for table, second table where source is at D and that sorry, receiver is placed at A that time we get the information which are presented in figure b. So, you can see here now we get two straight lines for, from each table. So, if I will give name here so this blue colored line I can give the name here this is o , this is a_1 , let us take this is a_2 and this is b .

Here $o a_1$ and $a_2 b$ enter, these two straight lines are meeting at point a . So, what is the significance of a here? a is the critical distance or the point at which the direct wave from a and the refracted wave following the path $a b c d$ reaches, reach together at the same time. And if you see the time is also we can measure from this curve and that time is close to 21.5 to 22, sorry the distance is 21.5 to 22 meter from a as per this plot.

Now, here what we need to know in order to calculate the velocity of the P wave in layer 1 we need to find out the slope of the line $o a_1$. Likewise for other calculation like to know the value of $i c$ which is the incident angle of the ray on the boundary and θ which is the inclination angle itself of the boundary so for these two parameters we need to know the slope of the line a to b .

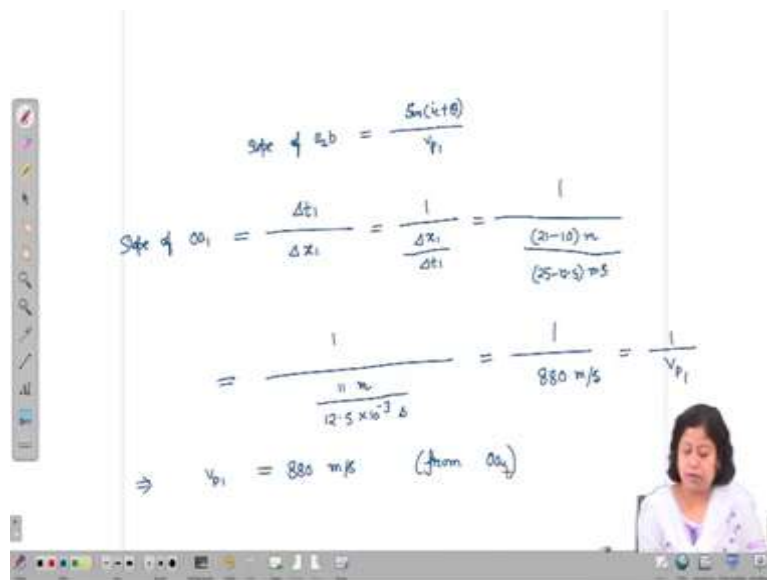
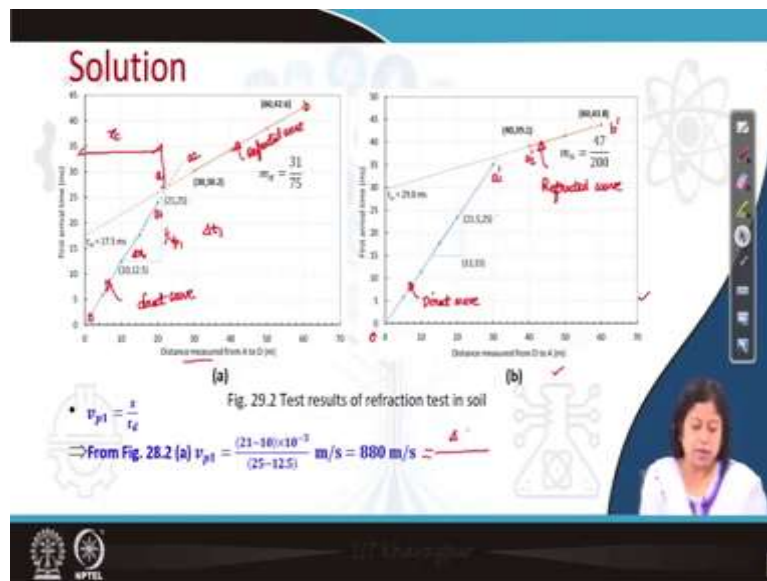
So, $x c$ I can show now in this figure what is the distance of $x c$ if you, so this is $x c$. Now, from the slope of the straight line $o a_1$ we can calculate $v p 1$. What is that? So, $v p 1$ is the velocity of the P wave in layer 1 which we can calculate from the slope of the straight line $o a_1$. From the slope of the straight line $a_2 b$ we can calculate so from slope $a_2 b$ we can calculate \sin of $i c$ plus θ I think it will be better if I will write it on the whiteboard. So, please allow me to write on the whiteboard, I am erasing this.

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So, what I said is that slope of $a_2 b$ is equal to \sin of $i c$ plus θ divided by $v p 1$, once again $v p 1$ is the velocity of the P wave in layer 1.

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Now, come to the second figure, this figure b is for that test when D is the source of disturbance and receiver or geophone is placed at A. So, that time we first get the first arrival time for direct wave so this is for the direct wave I can show here also this is for direct wave. And this portion for the refracted wave, here also I am writing this is for refracted wave.

So, now here if I know the slope of, I am giving the name also here o a 1 dash this is a2 dash b dash. So, now if I know the slope of o a 1 dash then I can say about the velocity of P wave in layer 1. Theoretically the slope of o a 1 and the slope of o a 1 dash these two are same, if, because of the measuring error sometime we cannot stop the stopwatch exactly at the time of arrival of the wave, there may be some fraction of second delay for that reason if there is any

change or some other reason then what we can do, we can take the average of these two slopes and report it as the velocity of P wave in layer 1.

So, now let us calculate the velocity of P wave that means the slope of $o a 1$ and $o a 1$ dash. So, using the data which is given here first one for $o a 1$ if you say, if you see it is first coordinate is given 10 and 12.5, 10 in meter and 12.5 in millisecond and the second data point has the coordinate 21 meter 25 millisecond. So, using these two data points we can find out the velocity of the P wave in layer 1.

Now, so basically I am just simplifying this thing once again here basically what we have done is we have, if I will write it also as this is the, if I will write it Δt then probably it will match with some other symbol so I should not use Δt here so let us take this is our $\Delta t 1$ I am writing here so this is nothing but $\Delta t 1$ and this distance is nothing but let us take $\Delta x 1$. So, from this we can get the slope that is $\Delta t 1$ divided by $\Delta x 1$.

So, I can write here slope of $o a 1$ is $\Delta t 1$ divided by $\Delta x 1$ which is equal to 1 divided by $\Delta x 1$ divided by $\Delta t 1$. Now, what is Δx in this figure? In this figure $\Delta x 1$ is 21 minus 10. So, 21 minus 10 is $\Delta x 1$ that is in meter. What is $\Delta t 1$? $\Delta t 1$ is 25 minus 12.5.

So, I am writing here 25 minus 12.5 that is in millisecond or I can express it also in second for that I need to I am writing it here itself so or I can write it as 11 meter divided by 12.5 into 10 to the power minus 3, now it is converted in second. So, then what will be the answer? 1 divided by 11 divided by 12.5 times 10 to the power minus 3. So, I get it as 880 that is in meter per second this is the slope.

Now, from slope it is equal to 1 divided by $v p 1$, so from this we can say $v p 1$ is 880 meter per second which we get from the line $o a 1$. So, here you can see what I have found out that is written directly in this slide. The same way now we can find out the velocity of the P wave from the straight line $o a 1$ dash, let us do that also.

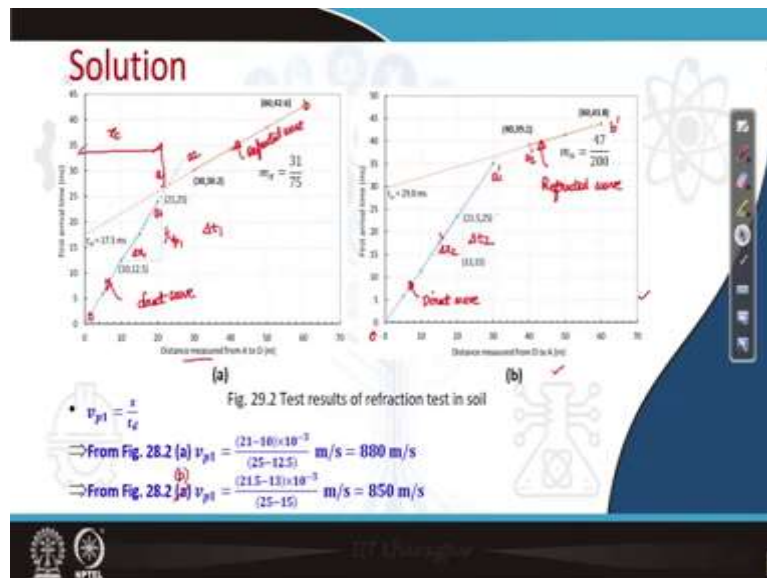
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$$\text{Slope of } o a 1 \text{ dash} = \frac{\Delta t_2}{\Delta x_2} = \frac{1}{\frac{\Delta x_2}{\Delta t_2}} = \frac{1}{\frac{(21.5-13)\text{m}}{(25-15)\text{ms}}}$$

$$= \frac{1}{\frac{8.5}{10 \times 10^{-3}} \text{ m/s}} = \frac{1}{850 \text{ m/s}} = \frac{1}{v_{p1}}$$

from $o a 1 \text{ dash}$ $v_{p1} = 850 \text{ m/s}$

The average velocity of P-wave in layer-1 = $\frac{880 + 850}{2} \text{ m/s} = 865 \text{ m/s}$



So, now slope of o a 1 dash is equal to what is the slope of this line here we can calculate delta t 2 and delta x 2, so in terms of delta t 2 and delta x 2 first I am writing the expression so delta t 2 divided by delta x 2. Now, so slope of o a 1 dash is equal to 1 divided by delta x 2 divided by delta t 2.

Now, what is the value of delta x 2? In this figure delta x 2 is equal to 21.5 minus 13. So, 21.5 minus 13 is our delta x 2. Now, see the delta t 2. How much is delta t 2 here? Delta t 2 is 25 minus 15. So, 25 minus 15 in millisecond is our delta t 2 or I can express the entire thing in meter per second which will be 8.5 divided by 10 into 10 to the power minus 3, now it is expressed in second.

So, how much you are getting here is 1 divided by 850, 1 divided by 850 meter per second and we know that is equal to 1 divided by v_{p1} once again. So, from curve o a 1 dash we get v_{p1} is equal to 850 meter per second. So, there are two curves, one is o a 1 and the other one is o a 1 dash which we got by interchanging the source and receiver points and from that what we get are v_{p1} for first case is 880 meter per second whereas for the second case v_{p1} is 850 meter per second.

So, the average velocity of P wave in layer 1 is average of these two values that is in meter per second so it is coming 865 in meter per second. So, now here let us see so you can see here just correct this statement this is from b.

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Calculations

\rightarrow Average velocity of P-wave in layer-1: $v_{p1} = \frac{880+850}{2} \text{ m/s} = 865 \text{ m/s}$
 $\rightarrow i_c = \frac{1}{2} [\sin^{-1}(v_{p1}m_d) + \sin^{-1}(v_{p1}m_a)]$

Solution

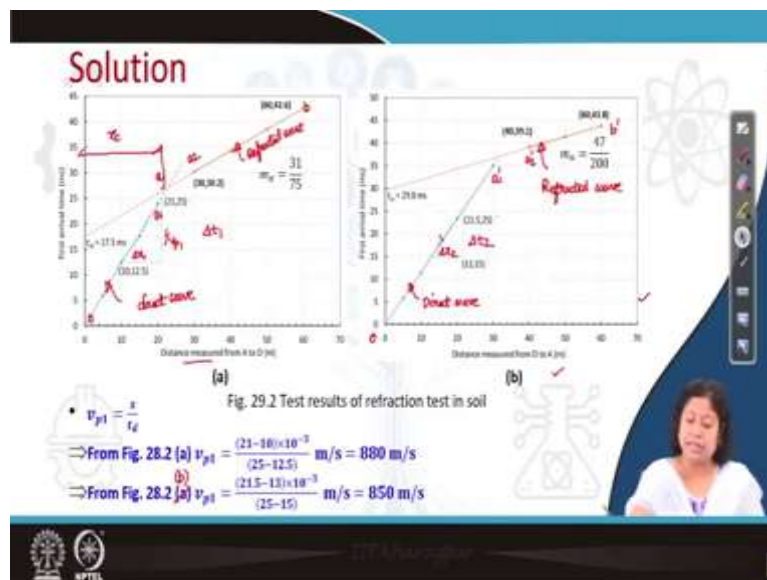
$v_{p1} = \frac{x}{t_d}$
 \rightarrow From Fig. 28.2 (a) $v_{p1} = \frac{(21-10) \times 10^{-3}}{(25-12.5)} \text{ m/s} = 880 \text{ m/s}$
 \rightarrow From Fig. 28.2 (b) $v_{p1} = \frac{(21.5-12) \times 10^{-3}}{(25-15)} \text{ m/s} = 850 \text{ m/s}$

Fig. 29.2 Test results of refraction test in soil

So, we are taking finally the average value it is matching to 865 meter per second. Now, we need to know the value of i_c which is the angle of incidence for the P wave at the boundary of the two layers. So, how we can calculate i_c ? For i_c we can use this equation or relationship directly, so for that we need to know the slope of the straight line $a_2 b$ and for μ we need to know the slope of the straight line. So, let us see this figure once again so slope of $a_2 b$ that is represented by m_2 is 31 divided by 75.

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$$\begin{aligned} \text{Slope of } a_2 b &= \frac{31}{75} \text{ ms/m} = \frac{31}{75000} \text{ s/m (ms)} \\ \text{Slope of } a_2 b' &= \frac{47}{200} \text{ ms/m} = \frac{47}{200000} \text{ s/m (ms)} \\ i_c &= \frac{1}{2} \left[\sin^{-1} \left(\frac{31}{75000} \times 865 \right) + \sin^{-1} \left(\frac{47}{200000} \times 865 \right) \right] \\ &= 16.34^\circ \\ \theta &= \frac{1}{2} \left[\sin^{-1} \left(\frac{31 \times 865}{75000} \right) - \sin^{-1} \left(\frac{47 \times 865}{200000} \right) \right] \\ &= 4.61^\circ \end{aligned}$$



Calculations

- Average velocity of P-wave in layer-1: $v_{p1} = \frac{600+850}{2} \text{ m/s} = 865 \text{ m/s}$
- $i_c = \frac{1}{2} [\sin^{-1}(v_{p1}m_d) + \sin^{-1}(v_{p1}m_u)]$
- $\theta = \frac{1}{2} [\sin^{-1}(v_{p1}m_d) - \sin^{-1}(v_{p1}m_u)]$
- $t_{id} = \frac{2z' \cos i_c}{v_{p1}}$

I am just writing here slope of a 2 b is 31 divided by 75, this is in millisecond per meter or if we can write it in second per meter also then it will be 31 divided by 75000 in second per meter. Likewise we can write slope of a 2 dash b dash that means slope of this curve and if you use the data which is shown here that means 43 minus 43.8 minus 39.1 divided by 60 minus 40, then you will get 47 divided by 200.

So, I am directly writing it 47 divided by 200 and it has a unit of millisecond per meter I can express it in second per meter so this is our m d and this is our m u. Then now here in this equation we know v p 1 which is 865 meter per second we know m d which is also mentioned so from these two and also we know m u we can calculate i c. So, let us see i c.

So, i c is equal to half of sin inverse m d, m d means 31 divided by 75000 times v p 1 which is 865 plus again sin inverse 30, 47 this time which is m u divided by 2 lakhs times 865. And I am trying to express it in degree, so sin inverse, so it is coming 16.34 in degree.

We can also calculate theta which is in place of positive sign here if we write minus then we can calculate theta, so I am just writing the formula for theta sin inverse 31 times 865 divided by, so it is coming let me check, so it is coming 4.61 degree, so in this way we can calculate i c and theta. So, let us see here.

Next is to find out Z dash and Z double dash. For this we can see here what is the value of t i d and t i u, t i d is the intercept of a 2 b to the vertical axis and t i u is the intercept of the a 2 dash b dash to the vertical axis, so t i d is 17.5 millisecond and t i u is 29.8 millisecond. So, I am writing this value also.

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Please read t_u equals to 29.8 ms

Solution

Fig. 29.2 Test results of refraction test in soil

- $v_{p1} = \frac{x}{t_d}$
- From Fig. 28.2 (a) $v_{p1} = \frac{(21-10) \times 10^{-3}}{(25-12.5)} \text{ m/s} = 880 \text{ m/s}$
- From Fig. 28.2 (b) $v_{p1} = \frac{(21.5-13) \times 10^{-3}}{(25-15)} \text{ m/s} = 850 \text{ m/s}$

Calculations

- Average velocity of P-wave in layer-1: $v_{p1} = \frac{880+850}{2} \text{ m/s} = 865 \text{ m/s}$
- $i_c = \frac{1}{2} [\sin^{-1}(v_{p1} m_d) + \sin^{-1}(v_{p1} m_u)] = 16.34^\circ$
- $\theta = \frac{1}{2} [\sin^{-1}(v_{p1} m_d) - \sin^{-1}(v_{p1} m_u)] = 4.61^\circ$
- $t_{d'} = \frac{2z' \cos i_c}{v_{p1}} \Rightarrow z' = 7.89 \text{ m}$
- $t_{u'} = \frac{2z'' \cos i_c}{v_{p1}} \Rightarrow z'' = 13.43 \text{ m}$
- $v_{p2} = 3074.61 \text{ m/s}$

$\frac{v_1}{v_2} = \sin i_c \Rightarrow v_2 = \frac{v_1}{\sin i_c}$

So, here I can write t_{id} which is 17.5 millisecond and t_{iu} is 29.8 millisecond. So, t_{id} I can directly use this equation for t_{id} which is $2Z \cos i_c$ divided by v_{p1} I am writing this equation directly so from this we can calculate Z dash which is v_{p1} times t_{id} divided by 2 times of $\cos i_c$, here we know the value of i_c we know the value of t_{id} and also v_{p1} so using that we can get, we can get the value of Z dash, likewise we can get the value of Z double dash also using this equation.

So, how much is coming Z dash and Z double dash let me calculate 17.5 into 10 to the power minus 3 divided by 2 times $\cos i_c$. So, it is coming 7.89 meter and the second one is coming 13.43 in meter. So, let us see the final answer, 7.89 you can see and t_{iu} is sorry Z 2 dash is 13.43 in meter from that we can calculate v_{p2} , for v_{p2} what we need to do, we know v_{p1} divided by v_{p2} is equal to \sin of i_c so from this we can get v_{p2} which is v_{p1} divided by \sin of i_c and that is coming 17 sorry, 3074.61 in meter per second.

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Let us take another numerical problem, in this problem we have carried out reflection survey and the distance between source and receiver is given which is 80 meter, the travel time of the P wave reflected P wave from A to C is given which is 0.038 second and the travel time of the reflected P wave from A to B is given which is t_B 0.026 second, we are asked to find out theta and Z dash velocity of P wave of layer 1 is also given which is 400 meter per second.

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Solution

$$\sin \theta = \frac{v_{p1} \Delta t}{2x} = 1.72 \checkmark \Rightarrow \theta = 1.72^\circ \quad \text{(Ans.)}$$
$$Z' = \frac{v_{p1} t}{2} = 6.4 \text{ m} \checkmark \Rightarrow Z' = 6.4 \text{ m}$$
$$\Delta t = t_c - t_b$$
$$x = 80 \text{ m}$$
$$v_{p1} = 400 \text{ m/s}$$
$$\bar{t} = \frac{t_c + t_b}{2}$$

So, sin, for sin theta we can write sin theta is equal to v_{p1} times delta t divided by $2x$ we know what is delta t, here delta t is equal to t_c minus t_b delta t is equal to t_c minus t_b so if I will write the difference of t_c and t_b then I will get delta t and x is already mentioned it is 80 meter so from that we can calculate the sin theta.

So, it is coming approximately, so let us see it is coming 1.72, yes, I am also getting the same answer. Now, let us see Z' so we know the value of v_{p1} , I can just write here, it is 400 meter per second. So, using this sorry 400 times, what is t bar here, t bar is the average of t_c and t_b .

So, here we are getting 6.4 meter. In this way we can calculate the value of theta, sorry I just here it is not sin, so please correct here it is theta is equal to 1.72 in degree so I am just writing it one second better way 1.72, 1.72 degree and this equation gives us Z' which is 6.4 meter, this is our final answer.

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SUMMARY

In this lecture following topics related to dynamic properties of soils are discussed:

- Numerical problems on seismic refraction and reflection tests for the determination of the thickness of layers, inclination of layering from seismic reflection survey, velocities of P-wave in layers etc.

So, from today's class we have learned how to find out the velocity of the P waves in different layers by conducting from the results of the refraction survey and also how to find out the inclination of the boundary of the two layers by conducting refraction survey and reflection survey and how to find out the thickness of the layers using any of these two tests.

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REFERENCES

1. Principles of Soil Dynamics by B M Das, PWS-KENT Publishing Company (available Indian Edition: by B M Das and Luo Zhe, Cengage Learning India Private Limited; Third edition)

So, this is the reference which I have used for today's lecture. Thank you we will meet once again in next class.