Soil Dynamics
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Lecture 27
Determination of Dynamic Properties of Soils (Seismic Refraction Survey)

Hello friends, today we will discuss on Seismic Refraction Survey. So, this seismic refraction survey is one of the field tests, which we can conduct in the field to determine the dynamic properties of soils.

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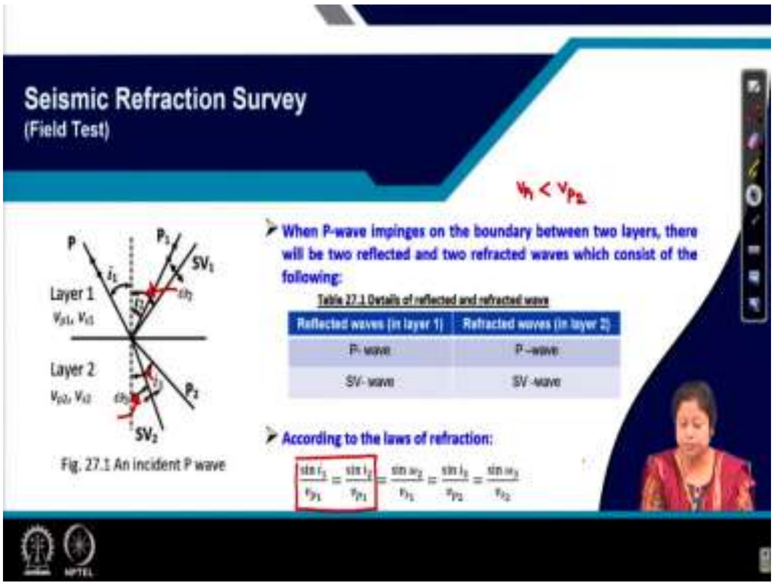

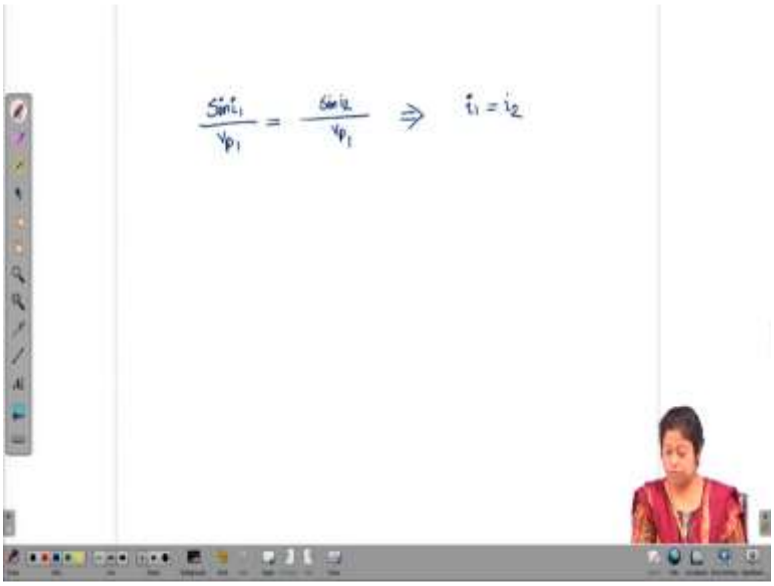

So, here you can see when our P wave impinges that boundary between layer 1 and layer 2, there are two waves, two reflected waves say P1 and SV 1 and two refracted waves say P2 and SV 2 are generated. What is SV here? SV is shear wave where the direction of wave and the direction of the movement of the particle are in vertical plane. So, in this figure you can see P wave is striking the boundary at an angle i1 with the vertical plane you can or with the vertical line you can see here.

So, i1 is the incident angle for this P wave. Now, when this wave is reflected back, that time it is making an angle i2 and for SV 1 the SV 1 is making an angle, this is omega 2 which is for the SV 1, this angle. Similarly, when the wave is refracted from the boundary to the layer 2, that time P2 wave is making an angle i3 and so, i3 is this angle whereas, the SV2 wave is making an angle omega 3 here. Now, according to the laws of refraction, we call it also as a Snell's law. What we can write? Sine of this angle i1 divided by the velocity of this P wave that is VP 1 is equal to sine of angle i2, which the reflected P wave makes divided by VP 1, which is the velocity of the P1 wave.

Likewise, we can write that ratio is also equal to sine of omega 2 divided by VS 1 is again equal to sine i3 divided by VP 2 and that is also equal to sine of omega 3 divided by VS 2. Here, one thing we can note that, VP 1 is the velocity of the P wave in layer 1 whereas, VP 2 is the velocity of the P wave in layer 2. Also, velocity of P wave in layer 1 is lesser than the velocity of the P wave in layer 2. What is VS 1 here? VS 1 is velocity of the SV wave in layer 1. Whereas VS 2 is velocity of the SV wave or shear wave in layer 2. Now, from the from this relation what we can conclude?

From this relation that means sine i1 divided by VP 1 is equal to sine i2 divided by VP 1, this tells us that i1 is equal to i2. So, I can write on the board, from the relationship sine i1 divided by VP 1 is equal to sine i2 divided by VP 1 gives us, sorry it is VP 1, these tells us that i1 is equal to i2. Actually we all know it but just for sake of completeness, I am repeating this thing.

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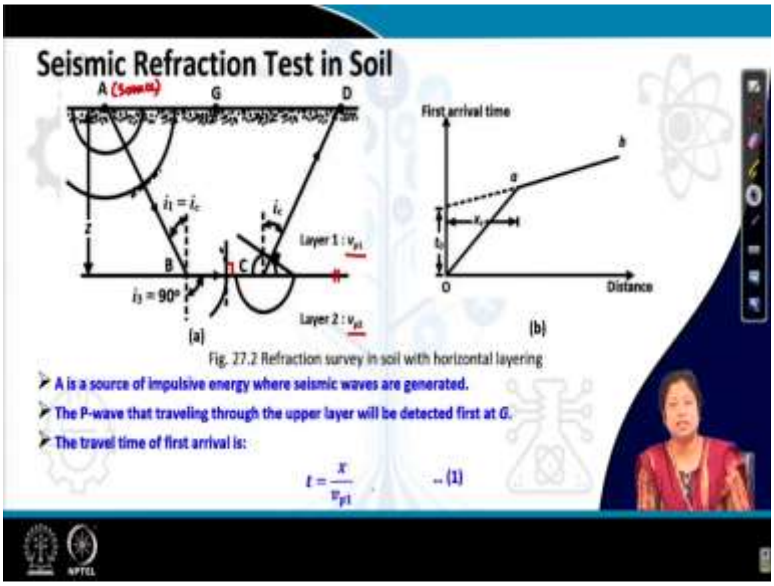

Now, in figure 2 we can see A is source of energy, where we are creating these turbines, then what will be happen? You can see P wave from a will start to propagate and that the wavefront is spherical in shape. So, this spherical wavefront can reach to the ground surface at different location there is one or a few wave fronts, which can strike the boundary of the two layers, this is our boundary of between layer 1 and layer 2. So, there are a few wave fronts which strike the boundary, this boundary.

Also, we can note in this figure that as long as the wavefront is not striking this boundary, till that time we can measure on the ground surface when we are receiving the wave in some location away from the source. So, a is your source here. So, away from the source when we are receiving it, initially the wave which is propagating through the upper layer that means layer 1 only will arrive fast. Now, what is happened when it is striking the boundary, there is one wavefront, let us take this one here. It strikes that boundary in such a way if we draw our tangent at that point of intersection to the boundary line then that tangent is perpendicular to the boundary line.

So, here we can see this tangent is perpendicular to the boundary line. And that time what will be happen, the wave which the ray AB will now follow the path BC that means, it will move parallel to the boundary with the velocity of VP 2, that means the velocity of the layer 2. Now, let us see what is the velocity of the direct wave propagating through upper layer? So, the velocity of P wave propagating through upper layer is VP 1 you can see here therefore, the travel time of the direct wave is if I will write it as t, then t is equal to x divided by VP 1, where x is the distance which we can measure along ground surface.

So, in this case, let us take G is any point which is not far away from the source, where that direct wave propagating through upper layer can arrive fast. So, for that case that travel time for the P wave can be calculated by x by using this equation 1, which says t is equal to x divided by VP 1.

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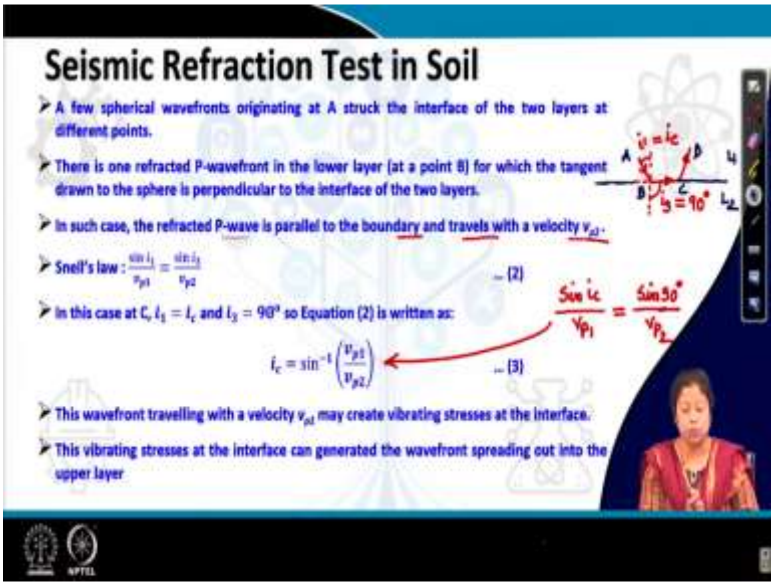

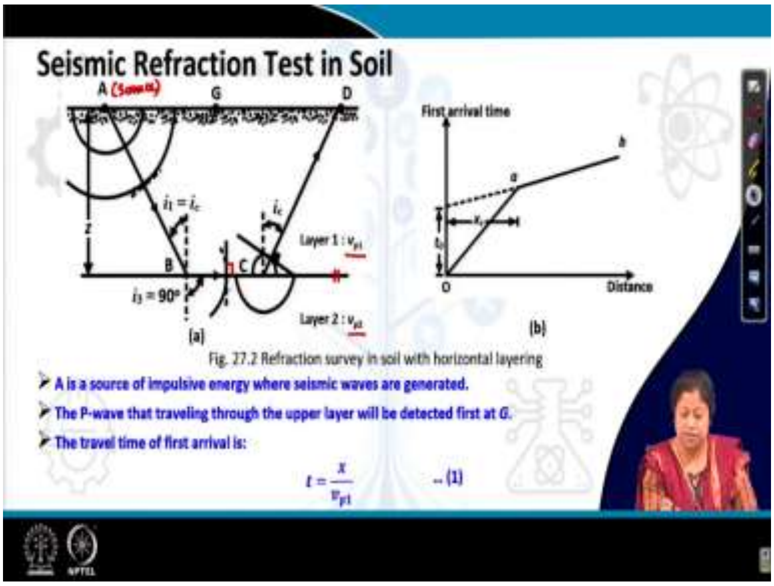

Now, as I said already, there are a few spherical wave fronts originating at a struck the interface of the two layers at different points. So, among these wave fronts, there is one refracted P wavefront in the lower layer at a point B for which the tangent drawn to the sphere is perpendicular to the boundary of the two layers, that also we have already seen. So, I am not showing the figure once again here. So, in that case, the P wave is refracted P wave that means, is parallel to the boundary and travels with a velocity VP 1.

Now, as per laws of refraction or Snell's law for that case, what we can write sine of i1 divided by VP 1 is equal to sine of i3 divided by VP 2. Now, if you recall that wave is like this, this is the boundary say of the two layers after some point it will follow this path. So, now, what is I want here, this angle is i1. What is i3 here? This angle is i3. Now, eventually this i3 is equal to 90-degree. So, when i3 is equal to 90-degree, that time i1 is referred as critical incident angle and we can write i1 as iC also.

So, that time using Snell's, law what we can write, I basically I can write here then sine of iC, divided by VP 1 is equal to sine of angle 90-degree, which is i3 divided by VP 2. So, from this we can get iC, which is sine inverse of VP 1 divided by VP 2. Now, this wavefront which is traveling with a velocity VP 2 that means, this wave may create vibrating stresses at the interface. Now, because of these vibrating stresses at the interface, what we can see the wavefront spreading out into the upper layer, that means if I can use another colour here.

So, if this is our boundary, this is layer 1, this is layer 2. Now, when the vibrating stresses are generated along this boundary that will cause this wavefront to spread out into the upper layer that means in layer 1 and as a consequence now, these P wave will follow the path. I think in that figure, I give the number AB. Let us see the figure number ABCD. So, ABCD. So, now the travel time for the P wave following the path ABCD is required to know.

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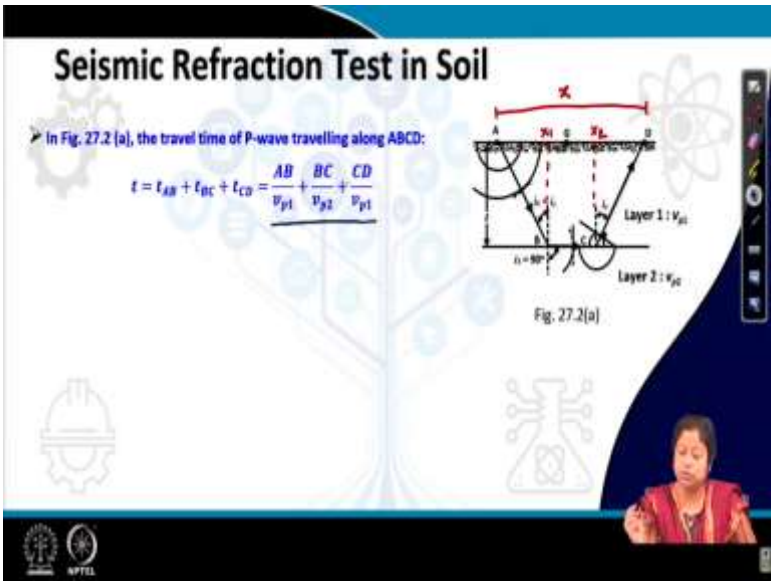

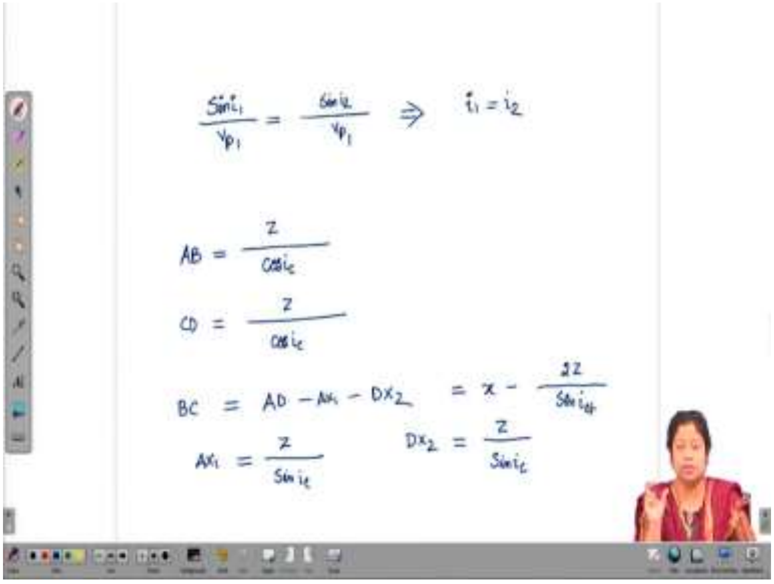

So, what is the travel time for the wave ABCD for path AB wave is traveling with the velocity VP 1, for the path BC the wave is traveling with a velocity VP 2 and four the path CD wave is traveling with a velocity VP once again. So, the travel time, total travel time for the P wave traveling along ABCD is AB divided by VP 1 plus BC divided by VP 2 plus CD divided by VP 1, I hope it is alright to all of us.

Now, we need to know what is AB, what is BC, what is CD? Before that let us take the distance AD is represented by x. Now, go back to the board. So, first we need to know AB, AB means if the thickness of the upper layer is z then we can calculate AB. So, if you now see this figure how much is AB? AB is equal to z divided by cosine of iC. So, here I am writing AB is equal to z divided by cosine of iC.

Now, eventually AB and CD are equal. So, CD is also equal to z divided by cosine of iC. What is the length of BC, if we note this figure, then, I am giving a few names here let us take is this one is x1 and this one is x2. Then BC is equal to AD minus A x1 minus D x2. So, BC is equal to AD minus A x1 minus D x2. Now, here what is A x1 and what is D x2? If you see the figure carefully, then you will see A x1 is equal to z divided by sine of iC, because for the triangle AB x1 we can see this angle A x1 B is 90-degree. so, it is a right-angle triangle.

So, easily we can find out the length A x1 from this triangle. So, A x1 is equal to z divided by sine of iC. Similarly, D x2 is also equal to z divided by sine of iC, then what is BC? BC is then equal to AD, which is equal to x minus 2z divided by sine of iC. So, this is the length of BC. So, now BC is also known. So, from this we can now calculate the total travel time.

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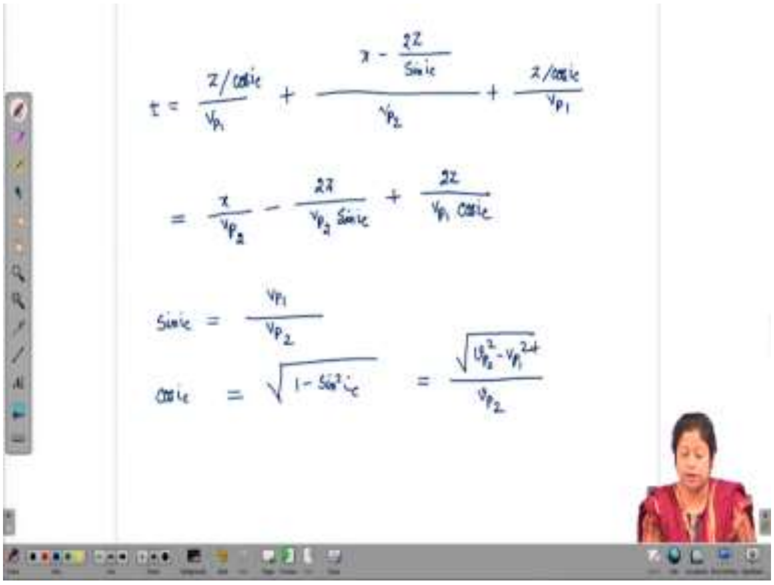

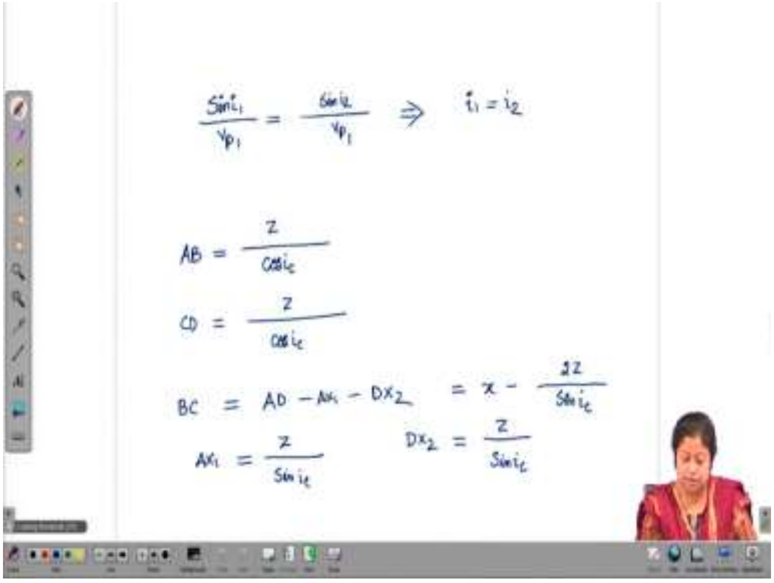

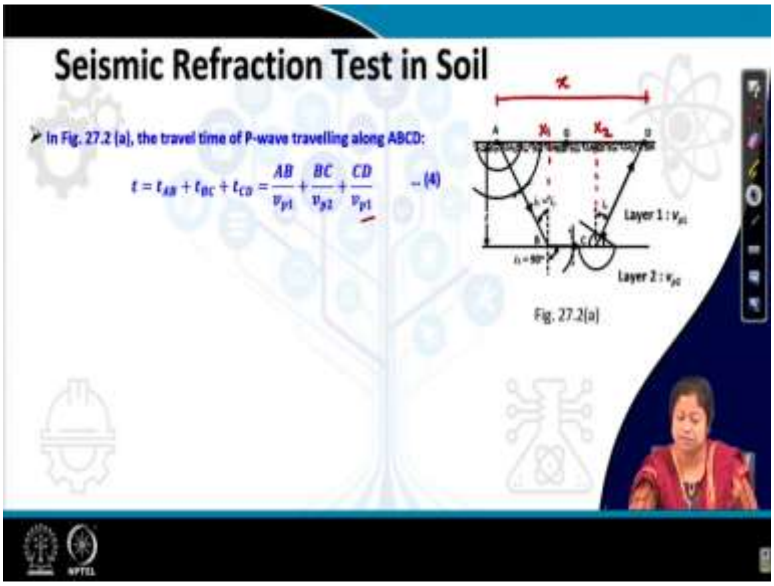

So, total travel time for the wave ABCD is AB, AB means z divided by cosine iC, that should be divided by the velocity of the P wave in layer 1. So, I can write here then z divided by cosine iC divided by VP 1. So, I am writing it this way z divided by cosine iC divided by VP 1, this is the travel time for the ray AB. Now, for the wave BC it is z x minus 2z divided by sine iC. So, this is BC divided by VP2. So, I can write it as x minus 2z divided by sine of iC divided by VP 2 plus once again for the path CD it is z divided by cosine iC divided by VP 1.

So, then we can write it as x divided by VP 2 minus 2z VP 2 sine of iC plus 2z VP 1 times cosine of iC. Now, we have one law which is the laws of refraction, what it is saying that sine iC is equal to VP 1 divided by VP 2. So, here now we can use then what will be cosine iC? Cosine iC will be equal to then, better I should write it in next line rather than writing it this way. So, we know sine iC is equal to Vp 1 divided by VP 2, then cosine Ic is equal to square root of 1 minus sine square iC which is equal to VP 2 times square root of VP 2 square minus VP 1 square.

So, we have these components now. Now, what we will do? Let us see, I am just going back to the previous page just to check whether there is any mistake from my end or not. So, now, we are interested to find out the travel time for the wave traveling along the path ABCD. So, for path AB travel time is AB divided by VP 1, because it is traveling with a velocity VP 1 in upper layer, when it is traveling along the boundary BC that time it has velocity VP 2. So, the travel time is BC divided by VP 2 for the path BC.

Again, for the path CD, it is traveling with a velocity VP 1. So, the travel time is equal to CD divided by VP 1, that we can see here. Now, we need to know what is the value of AB, BC and CD in terms of the parameter which we can measure from the experiment. So, from that experiment, we can measure this distance along the ground surface, let us take it as x. Now, let us see how we can write AB, BC and CD using x and z. So, for that, I am writing these two points as x1 and x2 in this figure. So, what is our AB, BC and CD? Let us see.

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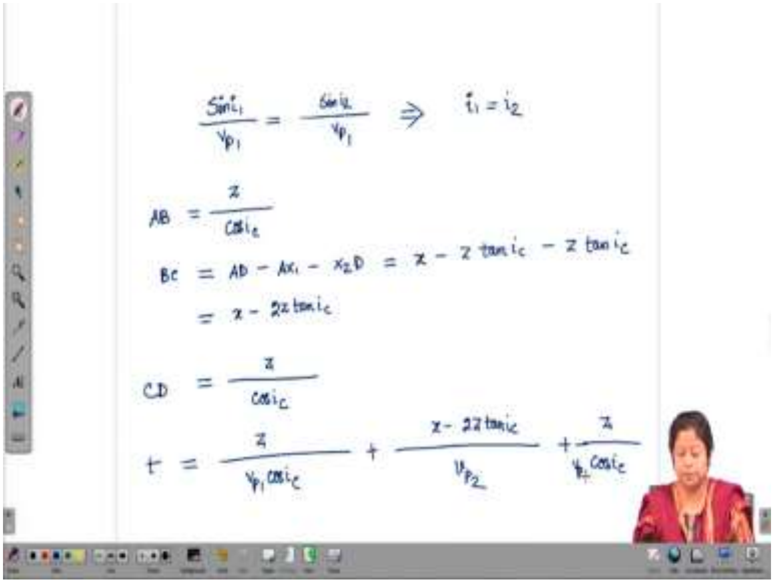

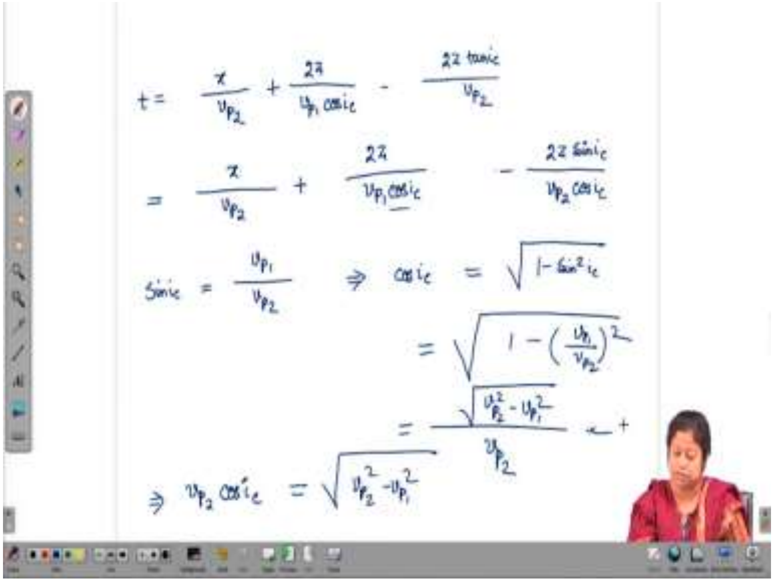

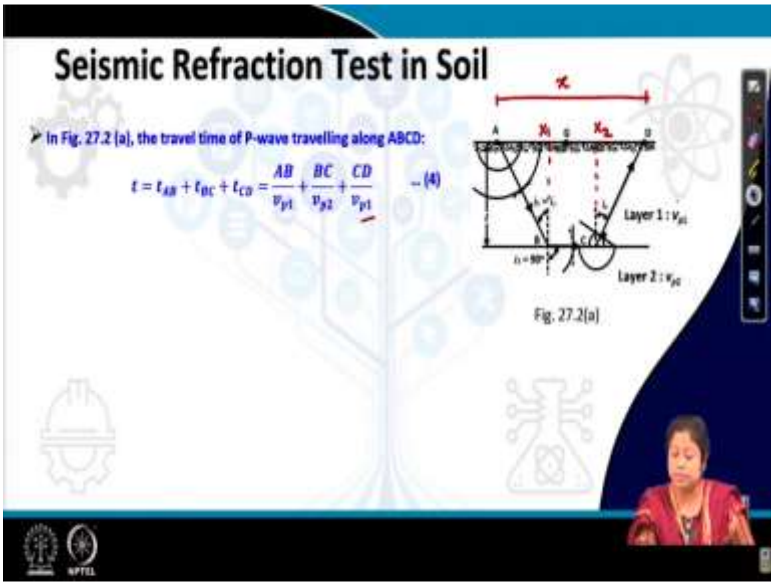

So, AB is equal to if you refer to the triangle AB x1, then AB is equal to B x1 divided by cosine of iC, B x1, means z. So, I can write it as z divided by cosine of iC. What is the value for AD, sorry BC? BC is equal to AD minus A x1 minus D x2. So, BC is equal to AD minus A x1 minus x2 D. What is AD? This is equal to x. Now, A x1 is equal to how much we can see again the same triangle AB x1. So, A x1 divided by C x1 is tangent of angle iC, from that we can get A x1 is equal to B x1 times tangent of angle iC and B x1 is z.

So, minus z tangent of angle iC for x2 D also the same thing. So, I can write here z times tangent of angle iC. So, finally for BC then we are getting x minus 2z times tangent of angle iC, this is BC. Now, CD is equal to AB, that as per this diagram. So, CD is equal to say divided by cosine iC. Then, what is the total travel time? Total travel time then is equal to z divided by VP 1 times cosine iC, this is for t AB, then t BC, for t BC we can write x minus 2z tangent of iC divided by VP 2. Then, plus t CD is equal to z divided by cosine of iC times VP 1.

Now, then what we can write, we can write it as x divided by VP 2, then we can write also as plus 2z divided by 2z divided by VP 1 times cosine iC. And then we can write minus 2z tangent of iC divided by VP 2. Now here in place of tangent of iC, I can write it once again as first I am writing in place of tangent of iC, I am writing sine iC divided by cosine iC. Now, this one what I can write it as I am writing same, which we have written here.

Now, what is cosine iC? If you recall, we get that sine iC is equal to VP 1 divided by VP 2. So, from this we can get cosine iC, which is equal to square root of 1 minus sine square iC which is nothing but 1 minus VP 1 by VP 2 whole square. So, finally, we are getting VP 2 times square root of VP 2 square minus VP 1 square, this is for cosine iC. Now, the thing is that in I can do more simplification just for the third term. So, VP 2 cosine iC directly we can write VP 2 square minus VP 1 square. So, here we have a term VP 2 cosine iC where we can write directly square root of VP 2 square minus VP 1 square. Now, here we have only cosine iC in the second term. So, they are in place of cosine iC we will write this expression.

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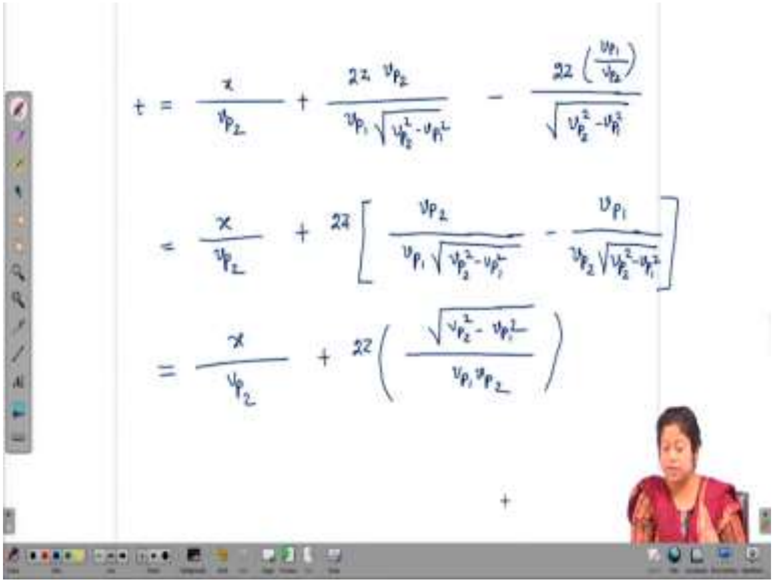

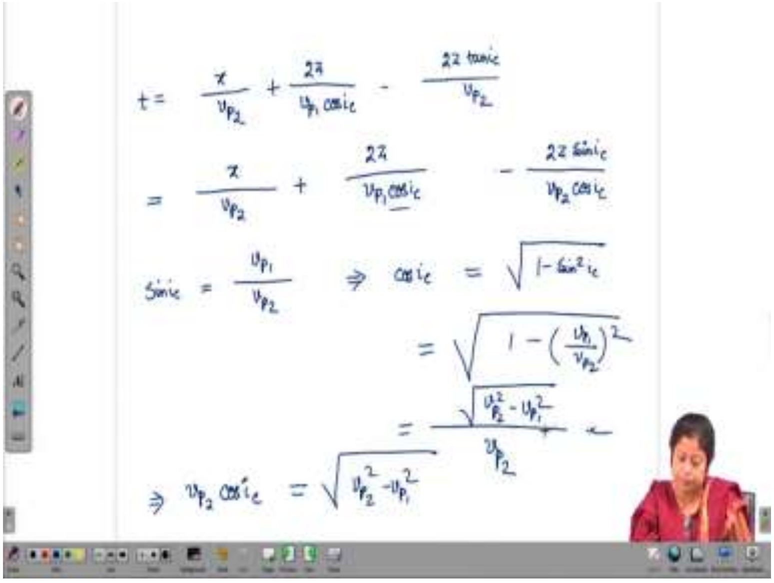

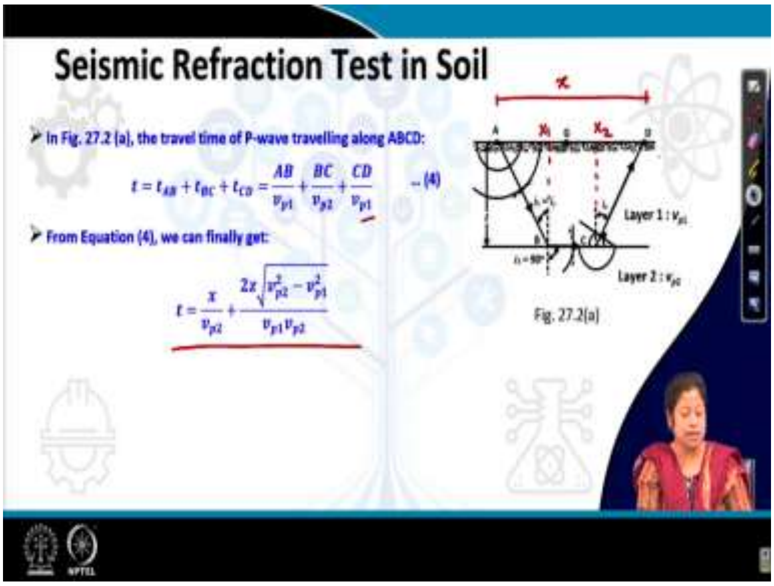

So, let us do this thing. So, the total travel time is equal to x divided by VP 2 plus. So, 2z divided by VP 1 times this, in place of cosine iC, I will write this expression. So, 2z VP 1 times VP 2 square minus VP 1 square here it is I think VP 2. Yes, so, 2z VP 2. Likewise, the third term I need to write here which is minus 2z sine iC. So, in place of sine iC, I can hear itself right the expression which is VP 1 divided by VP 2 this divided by VP 2 square minus VP 1 square, in place of VP 2 cosine iC, I am writing this expression directly.

So, now if I will arrange it first term will remain as it is here. What about the second and third term? For second term, I am writing these here only I have taken out 2z because that is common in second and third term. So, this is the expression, here you can note the expression very carefully. From these we will get in the numerator VP 2 square minus VP 1 square and in the denominator, we have square root of VP 2 square minus VP 1 square. So, finally I can write the numerator as square root of VP 2 square minus VP 1 square and in the denominator, I can write VP 1 times VP 2.

So, this is our final expression for total travel time, which follows the path ABCD. So, let us see here, you can see the total travel time for the P wave traveling ABCD is given, which we have already calculated.

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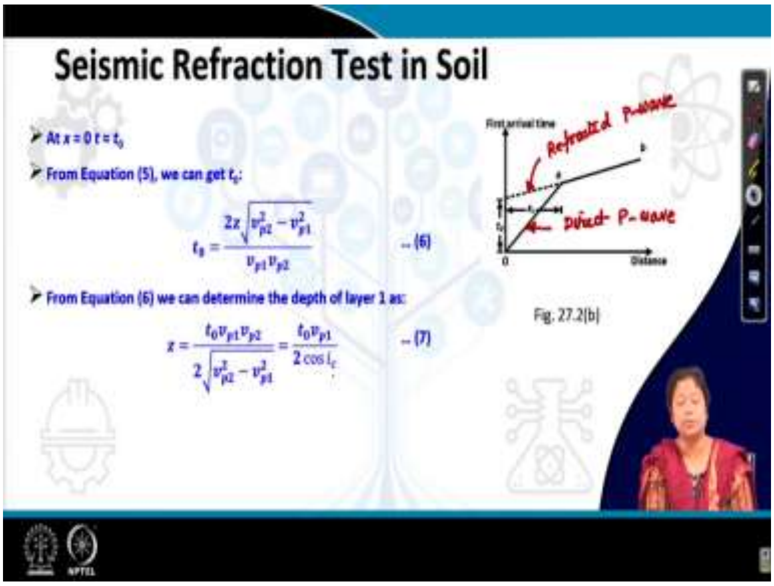

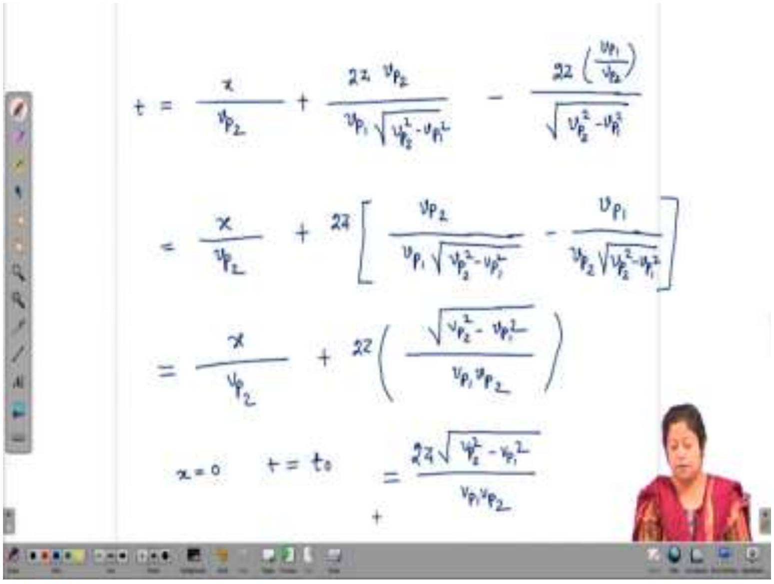

So, now, in this graph what we can see, if we are plotting the two equations, one for the direct wave that means, which is propagating through upper layer only and for the second wave which is propagating initially through upper layer, then refracted at the boundary following the path BC and then again spread out to the upper layer following the path CD which is to the ground surface. Then, we will get to different straight line. So, this is for the refracted P wave and this straight line for the direct P wave.

Now, in this figure t0 is the time for the refracted P wave at x is equal to 0. So, we have the expression for this is for the expression for the refracted the P wave. So, in this expression when x is equal to 0 t becomes t0 and that is equal to 2z square root of VP 2 square minus VP 1 square divided by VP 1 times VP 2. So, this is our t0. So, t0 in this way we can find out. Now, another thing you can note here, A is the point where the curve for the equation of the refracted P wave and the direct P wave intersect.

What does it mean? Basically, it is the distance on the ground surface where both the waves that is refracted wave following the path ABCD and the direct wave following, propagating through on the upper layer arrives at arrive at the same time, both the wave arriving at the same time. So, that distance is called the critical distance, beyond that distance what is happened refracted wave reaches before the direct wave, the reason is that refracted wave get a velocity for the strech BC VP 2 which is higher than the velocity VP 1, which was the velocity for the direct wave throughout. Because of this reason only after some distance refracted P wave reaches before the direct P wave.

So, we can get that distance also. Before that you can see from equation 6 we can also get the thickness of the upper layer that is z. So, just you need to do this calculation z is equal to from these you can see VP 1 times VP 2 times t0 divided by 2 times square root of VP 2 square minus VP 1 square. Now, here you can use in place of square root of VP 2 square minus VP 1 square divided by VP 2 in place of that you can use cosine iC also.

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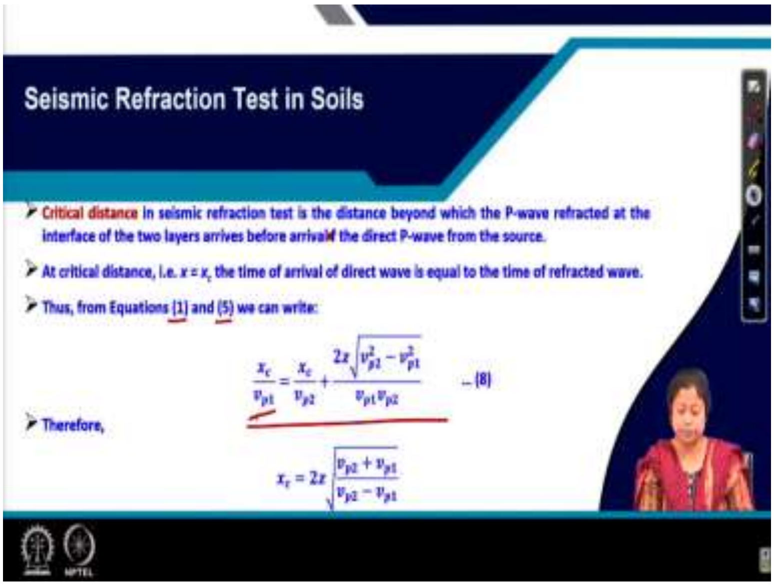

Now, as I said the critical distance is the distance where arrival of the direct I am reading the definition of critical distance. Critical distance in seismic refraction test is the distance beyond which P wave refracted at the interface of the two layers arrives before arrival of the direct P wave from the source of course, that means, xC is that distance when refracted wave and the direct P wave reach at the same time. So, we can equate the equations 1 and 5 to get xz in equation 1, we know that travel time is x divided by VP 1, here x is equal to xC.

So, I am writing xC divided by VP 1 for equation 5, we have already calculated how to get the travel time. So, in that equation x is equal to again xC. So, we are getting this relation. Now, from this we can calculate the value of xC which is equal to you can see 2z divided in times square root of VP 2 plus VP 1 divided by VP 2 minus VP 1.

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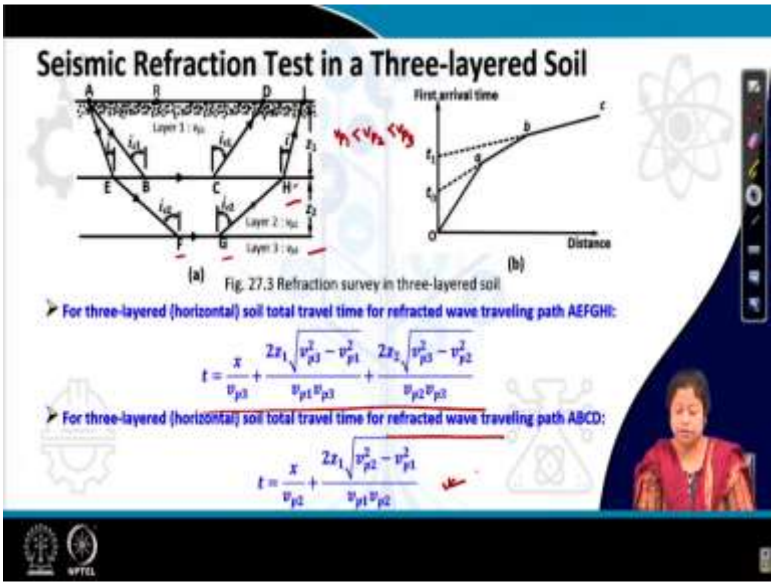

Next is if there are three layers that means, this is your layer 1, this is layer 2, and this is layer 3, in this case relationship of P wave velocity is like this VP 1 is low or then VP 2 and VP 2 is also lower than VP 3. So, in this case for three layered soil the travel time for the refracted wave following the path AEFGHI is expressed by this equation. The same approach which we have done for the two layered soil, the same way you can get the expression for the travel time for three layered soil. And for this three-layer soil total travel time for the refracted wave traveling along ABCDE is this one, this is already we have derived.

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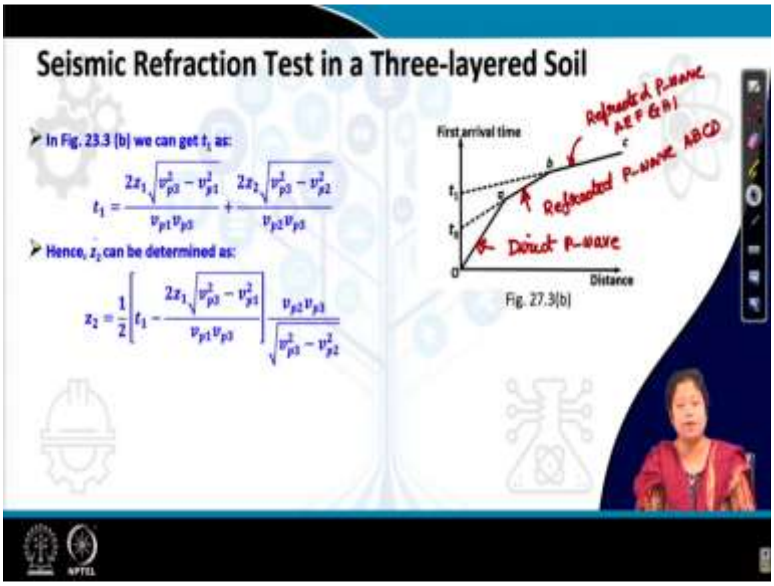

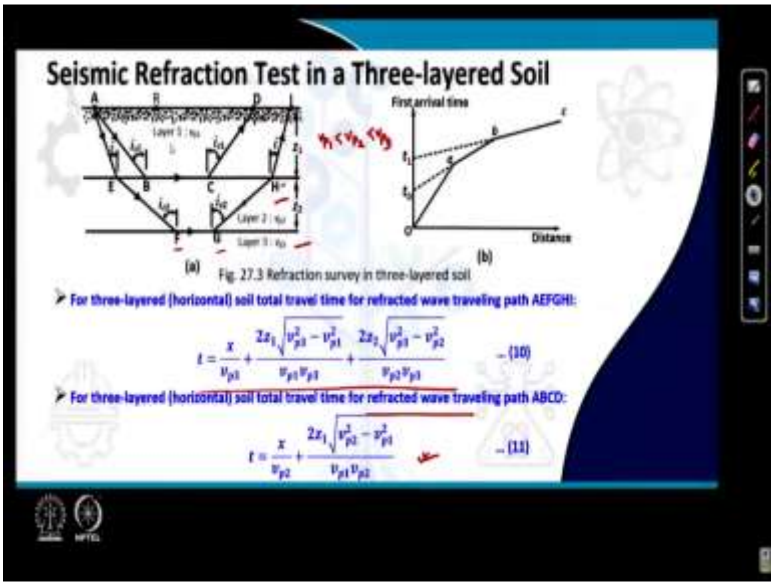

So, we can plot graph. So, this is for refracted P wave, following that path I can just show you sorry, following the path AEFGHI. This is the equation for refracted P wave follows the path ABCD and this is the direct wave. So, t0 already we know how to get and if we extend the back the straight-line BC that will intersect the travel axis vertical axis at point t1. So, t1 how we can get in the previous equation for the travel time of refracted P wave following the path AEFGHI, if in that equation we will use x is equal to 0 we can get t1and from t1 we can calculate z2 also in terms of z1, t1 and the VP 1, VP 2 and VP 3.

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So, come to the summary of today's class. In this lecture basically we have studied the seismic refraction test in soil, which is two layered soil and here we have considered on the horizontal layering. In the second case we have studied the seismic refraction test in three layered soil considering horizontal layering only. The next class we will see the incline layering.

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So, these are the references, which I have used in today's class. Thank you. We will meet once again in next class.