

**Soil Dynamics**  
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**Lecture 26**

**Determination of Dynamic Properties of Soils (Seismic Reflection Survey)**

Hello everyone, today, we will discuss our new topic which is related to the Determination of Dynamic Properties of Soil by conducting field tests. There are different field tests which can be performed to determine the dynamic properties of soil.

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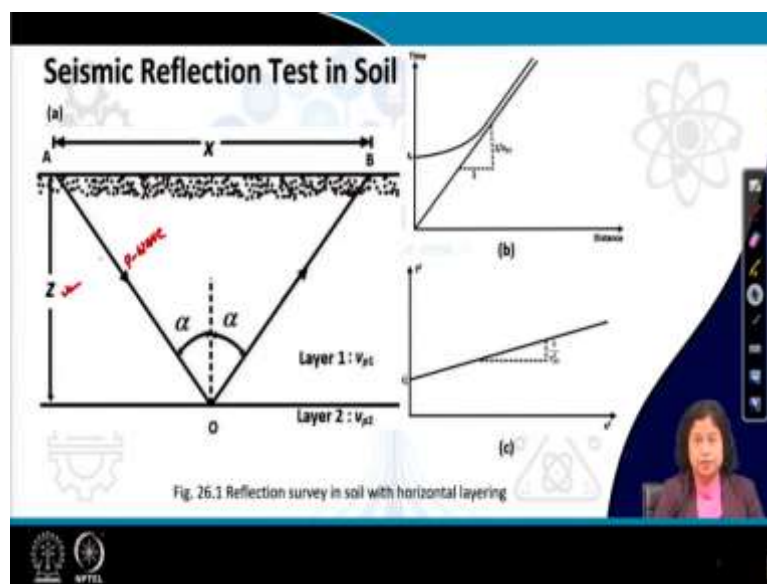
**Determination of Dynamic Properties of Soils**  
(Field Tests)

- Field tests are conducted to determine the properties of soils in-situ.
- There are some field tests which can be conducted from the ground level whereas for some other fields drilling boreholes or advancement of the probe into the soil is essential.
- Surface tests are less expensive and can be conducted quickly.
- Field tests can be classified into two groups- say, low strain test and high strain test.
- Examples of low strain test- seismic reflection survey, seismic refraction survey, steady state vibration test, spectral analysis of surface waves (SASW), seismic cross-hole test etc.
- Examples of high strain test- standard penetration test, cone penetration test, dilatometer test etc.

Field tests are conducted to determine the properties of soil in situ. There are some field tests which can be conducted from the ground level, whereas for some other field test drilling of boreholes or advancement of the probe into the soil is essential. Generally, the surface tests are less expensive and can be conducted quickly. The field tests can be classified into two groups, say, low strain test and high strain test.

Examples of low strain test, seismic reflection survey, seismic refraction survey, steady state vibration test, spectral analysis of surface waves or SASW, seismic cross hole test etcetera. Whereas, the examples of high string test are standard penetration test, cone penetration test, dilatometer test etcetera. So, our today's class is on seismic reflection survey, which is one of the low string field test.

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So, in this figure, we can see how we do the seismic reflection test in soil, this is a surface test that means, for this type of test, we do not need to drill the bore holes, we do not need to drill the bore holes. So, here you can see point A is the point where generally disturbance is created. Now, because of this disturbance, what will be happened, P wave will propagate either in this direction, you can see this is the direction of P wave propagation, also we can see surface wave will propagate from A to B directly along the ground surface.

So, if there is a geophone at B, B maybe any point that means,  $x$  is a variable here, then what will be happened, depending upon the position of B we will measure or we will trace either the surface wave or the P wave first. So, let us see how it will work. In this case, I have taken two layered soil. So, when P wave will propagate from A to the soil inside layer one, then what will be happen, when it will reach to the interface of layer one and layer two that time the P wave will be reflected back and will follow this path.

So, this is the path of the reflected P wave, and it will reach B at some time  $t$  and that time we will be able to measure, we will able to trace it. So now, using these kind of seismic reflection test, what we can get, we can get information about the thickness of layer one which is  $Z$  here.

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### Seismic Reflection Survey

➤ From Fig. 26.1, the travel time of P-wave in layer 1 is:

$$t = \frac{AO + OB}{v_{p1}} = \frac{2 \sqrt{x^2 + \left(\frac{z}{2}\right)^2}}{v_{p1}} \quad \dots (1)$$

➤ From Equation (1), the depth of the layer-1 can be written as:

$$z = \frac{1}{2} \sqrt{(v_{p1}t)^2 - x^2} \quad \dots (2)$$

➤ Here  $t_0$  can be determined as:  $z = \frac{v_{p1}t_0}{2}$



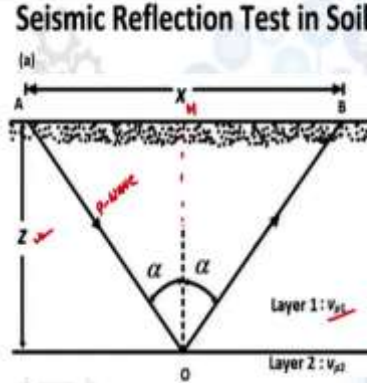
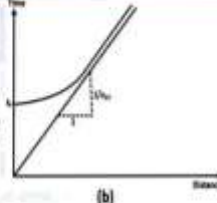


Fig. 26.1(b)



### Seismic Reflection Test in Soil

(a) 

(b) 

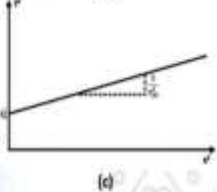

(c) 

Fig. 26.1 Reflection survey in soil with horizontal layering



Travel time for reflected P-wave traveling along AOB

$$= t_{AO} + t_{OB} = \frac{OA + OB}{v_p}$$

$$= \frac{2OA}{v_p} = \frac{2OB}{v_p} = \frac{2\sqrt{z^2 + (\frac{x}{2})^2}}{v_p}$$

$$OB = \sqrt{z^2 + (\frac{x}{2})^2}$$

$$t = \frac{2\sqrt{z^2 + (\frac{x}{2})^2}}{v_p} \Rightarrow t^2 v_p^2 = 4(z^2 + \frac{x^2}{4})$$

$$t^2 = \frac{1}{v_p^2} (4z^2 + x^2) \quad \text{where } y = t^2$$

$$y = \frac{4z^2}{v_p^2} + \frac{1}{v_p^2} x \quad x = x^2$$

So, let us do some calculation for that, if you go back to figure one once again. So, what is the travel time of the reflected P wave? The time taken to travel the path AO plus the time taking to the path to travel the path OB. So, I can write it on whiteboard, total travel time for reflected P wave traveling along AOB that is equal tA O plus tO B. So, that should be equal to the distance OA divided by VP 1 plus OB divided by VP, 1 because in layer one the velocity of the P wave is VP 1. Here, you can see the velocity of the P wave is VP 1.

Now, what is the distance OA, how we can measure this distance? So, you can see the calculation same calculation in next slide. So, t is the travel time of P wave in layer one, that means, it is traveling AO plus OB that divided by VP 1, which is the velocity of the P wave in layer one. Now, we have already studied in physics that the inclination angle and this reflected angle both are same, that means both are alpha.

So, OA and OB these length will also be same, then what we can write instead of writing OA plus OB, I am just writing it as 2 times OA, or I can write it as also 2 times OB divided by VP 1. Now, what is OA or OB. This distance is x by 2, let me give a name here I can write this this is one this is the midpoint. So, let us give it M. So, if AB is the distance between A and B is x then the length of the stretch AM is x by 2 and OM is already given I mean it is unknown, but as per this figure it is it.

So, from this we can write OB is equal to square root of z square plus x by 2 whole square. So, that is then I am writing here itself, z squared plus x by 2 whole square divided by VP 1. So, you can see same thing is written here. So, from this we can give it a name equation 1. So, now from equation 1 we can get, we can express actually z in terms of t from equation 1,

then we can write  $z$  is equal to half of square root of  $VP_1 t^2 - x^2$ . So, if maybe it is your equation 2. So, in this way, we can get the thickness of layer one.

Now, if we plot time versus distance, then what we will get, we will get two curves, initially we will see at lower time we will see this curve which that means basically for the same distance travel if you want to travel the same distance let us take this distance, then the wave which is coming along A to B, along the ground surface that will arrive fast and the reflected wave this is the curve for the reflected wave.

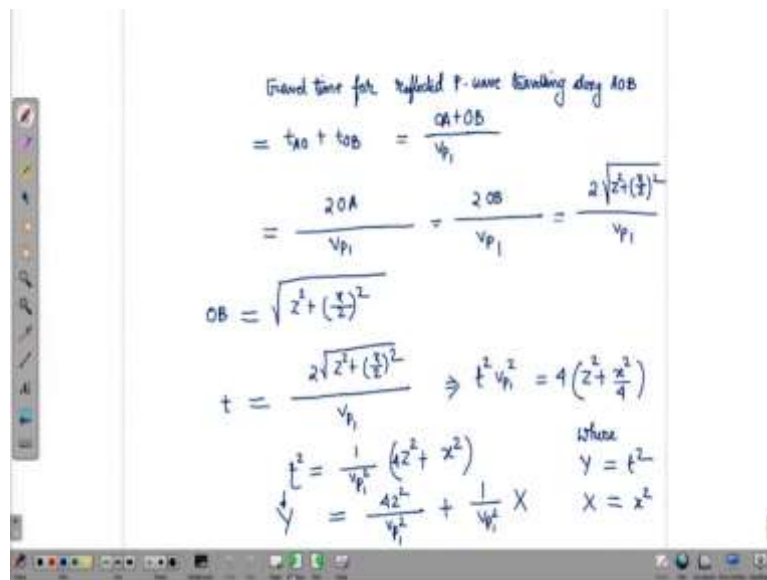
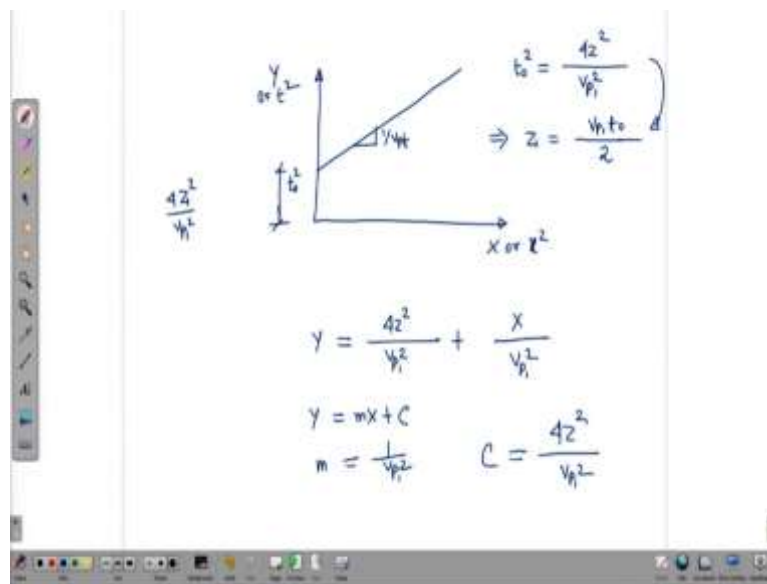
So, I can write it here reflected wave and this is for direct wave. So, what we can see here, the reflected wave initially takes more time to travel some distance  $x$ , which direct wave take less time to travel, but after some point you can see the distance is very close to each other, the difference is very close to each other and slope of both the curve is that time give you this velocity of P wave, slope actually, slope are same, that we can see from these figure b.

So, now, in equation 1 which is the equation for the reflected wave, represents it is this equation. So, if we extend these reflect curve for the reflected wave back, what we can see, at distance  $x$  is equal to 0 that means that A itself we can get the intercept of the curve to the time axis and that  $t_0$  we can calculate in a way we will put  $x$  is equal to 0 in equation 1.

Now, another way we can present the result. So, another way means we have total travel time, which is equal to  $2 \sqrt{z^2 + \frac{x^2}{4}}$  divided by  $VP_1$ . Now, if I will square both sides, what I will get  $t^2 \times VP_1^2$  is equal to  $4z^2 + x^2$ . Or I can also write it as  $t^2$  is equal to  $\frac{4z^2 + x^2}{VP_1^2}$ . So, from this if I represent  $t$  by  $y$  or I can take some other parameter instead of writing it  $y$  I am writing capital  $Y$ .

So,  $t^2$  is capital  $Y$ . And let us take  $x^2$  as capital  $X$ . So, then what we have that I am trying to write here, plus where  $Y$  means  $t^2$  and capital  $X$  means small  $x^2$ .

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### Seismic Reflection Survey

➤ From Equation (1) we can write:

$$t^2 = \frac{4 \left[ z^2 + \left( \frac{x}{2} \right)^2 \right]}{v_1^2} \quad \dots (4)$$

➤ In Equation (4), at  $x = 0$   $t = t_0$ , which can be expressed as:

$$t_0 = \frac{2z}{v_1} \quad \dots (5)$$

➤ Then,  $z = \frac{v_1 t_0}{2}$

Fig. 26.1(c)

*[A small video inset of a woman is visible in the bottom right corner of the slide.]*

Now, if I will plot these capital X and capital Y like this, then what I, first let me write the equation. So, basically this equation now I am writing it here Y is equal to  $4z^2$  divided by VP 1 square plus capital X divided by VP 1 square, you can compare this equation as Y is equal to  $mX + C$ . So, what is m, m gives the slope of the straight-line C says the intercept of the straight line to the Y axis. So, similarly here we can get the slope and the intercept. So, the line may be like this, where you can say, you can see m is equal to  $1$  divided by VP 1 square.

So, from these you can find out the slope m. And C is what? C is equal to  $4z^2$  divided by VP 1 square. So, this is  $4z^2$  divided by VP1 square. So, now, instead of capital X, I can also write X or t is what I have taken earlier capital X means small x square, and t square means capital Y. So, this is small x square, and this is or t square. So, this is nothing but then  $t_0^2$ . So, I can write then  $t_0^2$  is equal to  $4z^2$  divided by VP 1 square from this drawing.

Then, we can find out z is equal to VP 1 times  $t_0$  divided by 2, from these. So, let us go back to the figure. So, here the same thing is stated. First, we have drawn the straight line here, we know the equation of the straight line, equation of the straight line is t square is equal to 4 times z square plus x by 2 whole square divided by VP 1 this is equation of straight line, and you can get the slope also here, here also I can just complete this, sorry it is square.

So, after that if in this equation itself I will write in place of x squared if I will write x square is equal to 0 or at x is equal to 0 t is  $t_0$ . So, from that we can get  $t_0$  is equal to  $2z$  divided by VP 1 or z is equal to VP 1  $t_0$  divided by 2, that we have already seen.

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### Seismic Reflection Test in Soil (Inclined Layering)

In Fig. 26.2,  $AO+OB = A_1OB$

$A_1OB = \sqrt{(A_1A_2)^2 + (A_2B)^2}$

$DA = DA_1$   
 $OA = OA_1$

Fig. 26.2 Reflection survey in soil with inclined layering

### Seismic Reflection Test in Soil (Inclined Layering)

In Fig. 26.2,  $AO+OB = A_1OB$

$A_1OB = \sqrt{(A_1A_2)^2 + (A_2B)^2}$

$A_1B = \sqrt{4x^2 + x_B^2 + 4x'x_B \sin \theta} = AO+OB$

Travel time of the reflected P-wave along path AOB is:

$$t_B = \frac{\sqrt{4x^2 + x_B^2 + 4x'x_B \sin \theta}}{v_{p1}} \quad \dots (7)$$

Travel time of the reflected P-wave along path AO'C is:

$$t_C = \frac{\sqrt{4x^2 + x_C^2 + 4x'x_C \sin \theta}}{v_{p1}} \quad \dots (8)$$

Fig. 26.2 Reflection survey in soil with inclined layering



For  $\Delta A_1 A_2 B$

$$A_1 O B = A_1 B = \sqrt{A_1 A_2^2 + A_2 B^2}$$

$$A_2 B = A_2 A + AB$$

$$A_1 A_2 = A_1 \sin \theta = 2z' \sin \theta$$

$$AB = x_B$$

$$A_2 B = (2z' \sin \theta + x_B)$$

$$A_1 A_2 = A_1 \cos \theta = 2z' \cos \theta$$

$$A_1 B = \sqrt{(2z' \cos \theta)^2 + (2z' \sin \theta + x_B)^2}$$

$$= \sqrt{4z'^2 + 4z'x_B \sin \theta + x_B^2}$$

So, in previous slides, we have seen how to get the thickness of the layer one when horizontal layering is happened. Now, what will be happen when the interface of the when this layering is inclined that means, here you can see the interface of these two layer is in is one inclined plane, in that case how I will get the, how I will get that information about the layer 1 that we will now see.

So, first thing we will put the geophone at A, sorry this we will create the disturbance at A and will vary the geophone, location of the geophone at B and C, B may be any distance C is another one. So, when wave is propagating along AO and then get reflected by the interface of layer one and layer two and then again going back to the layer one or to the ground surface that time if there is a geophone at B that will receive the signal. There is another wave which is propagating from A to O dashed reflecting back at O dashed, when it is reflecting back following the path O dashed C.

So, at this situation what will be how we will get the z dashed, that is our task and also how do we get the information about theta which is the inclination of layering. So, let us see in this figure 26.2, what we can see? The distance AO plus OB is equal to A1 OB. Why so? In this case I have extended the ray OB extended back the ray OB to A1, which intersects the perpendicular at this point. So, let us give it another name say D. So, DA is perpendicular to the interface layer.

So, if I will extend DA also extend back the DA and the OB these two will intersect at A1. If so, then what will be happened? You can see here; this is 90-degree, and this angle will also be 90 degree and what else we can see we can also see these angles are also probably same.

So, finally, from this we can get actually these DA is equal to DA 1. If it is so, then obviously OA is also equal to OA 1. Then, these AO may be replaced by OA 1. So, in place of AO, I can also write it as A1 O plus OB. I can write these A1 O in better way, then that will give us A1 OB, that means this.

So, now, A1 OB means what? A1 B means, if I consider the triangle, A1, A2 B the for the triangle A1, A2 B, I am writing on the board, A1 A2 B. What is A1 OB? A1 OB or I can write it directly as A1 B is equal to A2 B square root of A2 B square plus A1 A2 square. So, I am writing it here, A1 A2 square plus A2B square. The same thing is actually written here also. Now, the question what is A1 A2 and what is A2 B.

I am just erasing this thing. So, what is in this figure what is A2B, A2B is nothing but A A2 plus AB. So, I am writing here A2B is equal to A2A plus AB, let me check A2A plus AB. Now, AB is equal to xB. What is A2A? In place of A2A, what I can write is A A2 times sine theta and sorry A A1 times sine theta is A A2. So, A A2 is equal to once again A A2 that means, this length is equal to A A1 times sine theta. A A1 means what here? A A1 means 2z dashed. So, I can write here 2z dashed times sine theta that is A A2. Likewise in place of AB, I can write xB. So, in this way I can get A2B is equal to 2z dashed sine theta plus xB.

What about A1 A2? A1 A2 that means, this distance is how much that I have to find out. From figure we can see A1 A2 is equal to A A1 times cosine theta. So, A A1 times cosine theta. What is A A1 in this case? 2z dashed. So, I can write it as 2z times cosine theta. Then, what is our A1 B? Our A1 B is equal to you can write 2z dashed cosine theta whole square plus 2z dashed sine theta plus xB whole square. If we will write it expanding these terms under square root what we will get 4z dashed square plus 4z dashed xB times sine theta, sine theta plus xB square.

What I have done, 4z dash square times sine square theta plus 4z dashed square times cos square theta, I have taken together which is equal to 4zdash square. So, in this way, we can get A1 B, A1 B means this length. So, I can write A1 B here as I have got that square root of 4z dash square plus xB square plus 4z dash takes B times sine theta. Now, travel time of the reflected P wave along the path AOB is then, the length of the path divided by VP 1. So, it is divided by VP 1, length of the path already we get here, so, this is VP 1.

Then we can calculate the travel time of the reflected P wave along the path A0 dashed C, that means this one, this one I am talking now. So, the travel time for this wave these reflect

this path is how much if it is tC, then we can write here as in place of xB only xC will be introduced. So, we can write it as this way. So, let us give two numbers for tB and tC seven and eight.

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### Seismic Reflection Test in Soil (Inclined Layering)

➤ From Equations (7) and (8):

$$\sin \theta = \frac{v_p^2 (t_c^2 - t_b^2)}{4z'(x_c - x_b)} - \frac{x_c + x_b}{4z'} \quad \text{-- (9)}$$

➤ Assume:  $\bar{t} = \frac{t_b + t_c}{2}$  and  $\Delta t = t_c - t_b$  then Equation (9) becomes:

$$\sin \theta = \frac{v_p^2 \bar{t} (\Delta t)}{2z'(x_c - x_b)} - \frac{x_c + x_b}{4z'} \quad \text{-- (10)}$$

➤ If  $x_b = 0$  then Equation (10) becomes:  $\sin \theta = \frac{v_p^2 \bar{t} (\Delta t)}{2z'x_c} - \frac{x_c}{4z'}$  -- (11)

➤ For  $\theta = 0$  then from Equation (11),  $\Delta t = \frac{z_c^2}{2v_p^2 \bar{t}}$  -- (12)

➤ If  $\Delta t > \frac{z_c^2}{2v_p^2 \bar{t}}$  then  $\theta$  is positive i.e. reflecting layer is sloping down to in positive x-direction.

➤ If  $\Delta t < \frac{z_c^2}{2v_p^2 \bar{t}}$  then  $\theta$  is negative i.e. reflecting layer is sloping down to in negative x-direction.

$$t_B = \frac{\sqrt{4z'^2 + 4z'x_B \sin \theta + x_B^2}}{v_{p1}}$$

$$t_C = \frac{\sqrt{4z'^2 + 4z'x_C \sin \theta + x_C^2}}{v_{p1}}$$

$$t_C^2 - t_B^2 = \frac{v_{p1}^2 (x_C^2 - x_B^2) + 4z'(x_C - x_B) \sin \theta}{v_{p1}^2}$$

$$\Rightarrow \sin \theta = \frac{v_{p1}^2 (t_C^2 - t_B^2) - (x_C^2 - x_B^2)}{4z'(x_C - x_B)}$$

$$= \frac{v_{p1}^2 (t_C^2 - t_B^2)}{4z'(x_C - x_B)} - \frac{x_C + x_B}{4z'}$$

Now, from seven and eight, if I am just writing a few steps to understand that thing. So, we get tB which is square root of 4z dash square plus four z dash xB sine theta plus xB square divided by VP 1, likewise tC is equal to square root of 4z dash square plus 4z dashed xC sine theta plus xC square divided by VP 1. Now, if I will subtract tC square, I subtract tB square from tC square, then what is left that I am trying to write here. So, then we will get x square minus xB square, then plus 4z dashed xC minus xB sine theta. This is the expression.

So, here you can see the same thing is written but before that some more calculation is done. So, what they have done I am just showing it for you all. So, sine theta from these I can write as these  $VP_1^2$  will be multiplied to  $t_C^2 - t_B^2$ , if I want to take it other side and then that will be from that we will subtract  $x_C^2 - x_B^2$  and the entire thing will be divided by  $4z(x_C - x_B)$ . So, from this now we can write  $VP_1^2(t_C^2 - t_B^2) = 4z(x_C - x_B)(x_C + x_B)$ . I can write here  $x_C + x_B$  divided by  $4z$  dash. You can see the same thing is written here also.

Only thing, you need to do some calculation one or two steps here, giving it a number equation 9. So, in equation 9, if we consider the difference between  $t_C$  and  $t_B$  as  $\Delta t$  it is already written. So, I should not write it once again here. So, if the difference between  $t_C$  and  $t_B$  is  $\Delta t$  and the average of  $t_C$  and  $t_B$  is considered as is written as  $\bar{t}$ , then the equation 9 can be written as  $\sin \theta = \frac{VP_1^2 \bar{t} \Delta t}{2z(x_C - x_B) - (x_C + x_B)}$ , this is one part and  $VP_1^2 \bar{t} \Delta t$  divided by  $4z$  dash, this is the other part.

So, here if  $x_B$  is 0, what will we happen? Sine theta will become this. So, it is our equation 11. Now, in equation 11, if theta is 0 then what will be happen, sine theta will be 0 also. So, from that we can get  $\Delta t$  is equal to  $\frac{x_C^2}{2VP_1^2 \bar{t}}$ . Simple way we will make it 0 and then we will get this. So, this is the condition when theta is 0, that means horizontal layering. Now, if this  $\Delta t$  is greater than the expression written on the right-hand side, then what does it mean? It means that the reflecting layer is sloping down to positive x direction that means, I can show it on drawing like this, this is our positive x direction.

So, this is our reflecting layer, but if these  $\Delta t$  is less than the expression written on the right-hand side, then what does it mean? The reflecting layer is sloping down in a negative x direction. So, that time it will be like this. So, this direction is negative x this is positive x. So, here you can see this is theta, in this case this is theta. So, seeing the expression for  $\Delta t$ , so, first we will calculate  $\Delta t$  to understand whether the inclination or whether that reflecting layer is sloping down to positive x direction or negative x direction.

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In Equation (10) put  $x_c = -x_b = x$  and  

$$\sin \theta = \frac{v_1^2 \bar{t}(\Delta t)}{4z'x} \quad \dots (13)$$

$$AA_1 = 2z' = (A_1B + A_1C) / 2 \quad \dots (14)$$

Divide Equation (14) by  $v_1$  we get,  

$$\frac{2z'}{v_1} = \frac{1}{2} \left( \frac{A_1B}{v_1} + \frac{A_1C}{v_1} \right) = \frac{1}{2} (t_B + t_C) = \bar{t} \quad \dots (15)$$

Using Equation (15) into Equation (13) we get,  

$$\sin \theta = \frac{v_1 \bar{t}(\Delta t)}{2x} \quad \dots (16)$$

$$z' = \frac{\bar{t} v_1}{2} \text{ in Eq. (15)}$$

$$t_B = \frac{\sqrt{4z'^2 + 4z'x_B \sin \theta + x_B^2}}{v_1}$$

$$t_C = \frac{\sqrt{4z'^2 + 4z'x_C \sin \theta + x_C^2}}{v_1}$$

$$t_C^2 - t_B^2 = \frac{v_1^2 (x_C^2 - x_B^2) + 4z'(x_C - x_B) \sin \theta}{v_1^2}$$

$$\Rightarrow \sin \theta = \frac{v_1^2 (t_C^2 - t_B^2) - (x_C^2 - x_B^2)}{4z'(x_C - x_B)}$$

$$= \frac{v_1^2 (t_C^2 - t_B^2)}{4z'(x_C - x_B)} - \frac{x_C + x_B}{4z'}$$

$x_C = -x_B = x \Rightarrow \sin \theta = \frac{v_1^2 (\bar{t}) \Delta t}{2z'(x+x)} = \frac{v_1^2 \bar{t}(\Delta t)}{4z'}$

Then, how to get z dashed, for getting z dashed what we will do? We will create the disturbance at A and we will put two geophones on two sides of A, one at C and one at B. Now, as far as this figure, what we can see we can see x in this case  $x_B$  and  $x_C$  are same, which is taken as x. So, in the equation 10, whatever we get in so, actually equation 10 was something like this. So, in this equation, if I will put  $x_C$  is equal to minus  $x_B$  b is equal to x. Then what will be the sine theta? The sine theta then will be this part will be 0, this will be okay.

Actually, there is one more step. So, I am trying to write first that, times 2 z dashed now x plus x. So, it is becoming  $v_1^2 \bar{t} \Delta t$  times 4z dashed, let us see. Again, from this figure, we can find out  $AA_1$  which is  $2z$  dashed, you can see her. So  $2z$

dashed is equal to  $A_1 B$  plus  $A_1 C$  divided by 2. So, now, if I divide right hand side and left-hand side by  $VP_1$ , what I will get? I will get  $2z$  dashed divided by  $VP_1$  is equal to this expression. What is  $A_1 B$  by  $VP_1$ ?

$A_1 B$  by  $VP_1$  represents that time, travel time let us give a number  $O$  dashed. So,  $A_1 C$  divided by  $VP_1$ , gives the travel time of the reflected P wave following the path  $AO$  dashed  $C$ , which is  $t_C$ . So, whatever written in the bracket on the right-hand side in place of that we can write for these it is  $t_B$  and for these it is  $t_C$ . So, that is written here. And already we have assumed that average of  $t_B$  and  $t_C$  is  $\Delta t$ . So, that is written here. Then finally, what we are getting in place of  $t$  bar we can write  $2z$  dashed divided by  $VP_1$  in equation 30. Then, it becomes like this or actually in not in place of  $t$  bar basically in place of  $z$  dashed I have written in place of  $z$  dashed I have written  $t$  bar  $VP_1$  divided by 2 in equation 13 and then get this expression which is equation 16 here. So, in this way we can calculate sine theta and the  $z$  dashed also.

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Come to the summary of today's lecture. Today we have studied the seismic reflection test in soil with horizontal layering and also how to get the velocity  $V_P$  1 from the graph that also we have discussed. Then we have studied the seismic reflection test in soil with inclined layering. With this, I am stopping today's class. These are the references which I have used for today's lecture. Thank you.