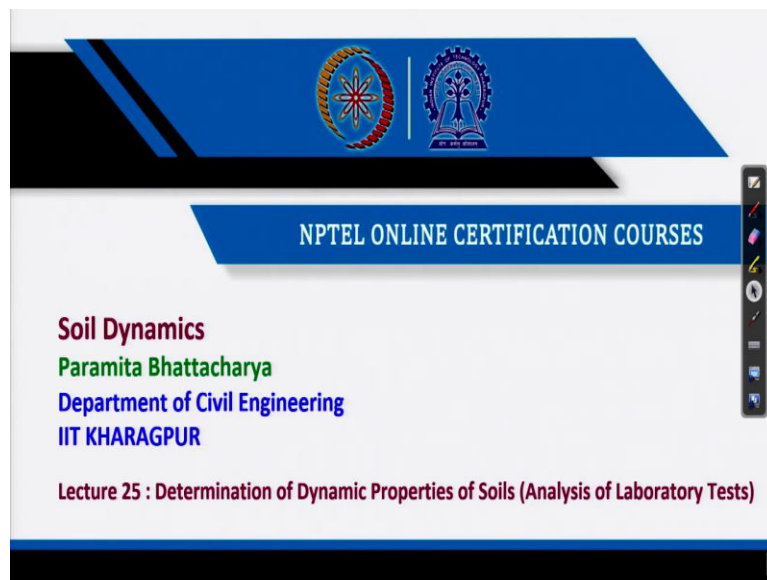


**Soil Dynamics**  
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**Department of Civil Engineering**  
**Indian Institute of Technology Kharagpur**  
**Lecture 25**

**Determination of Dynamic Properties of Soils (Analysis of Laboratory Tests)**

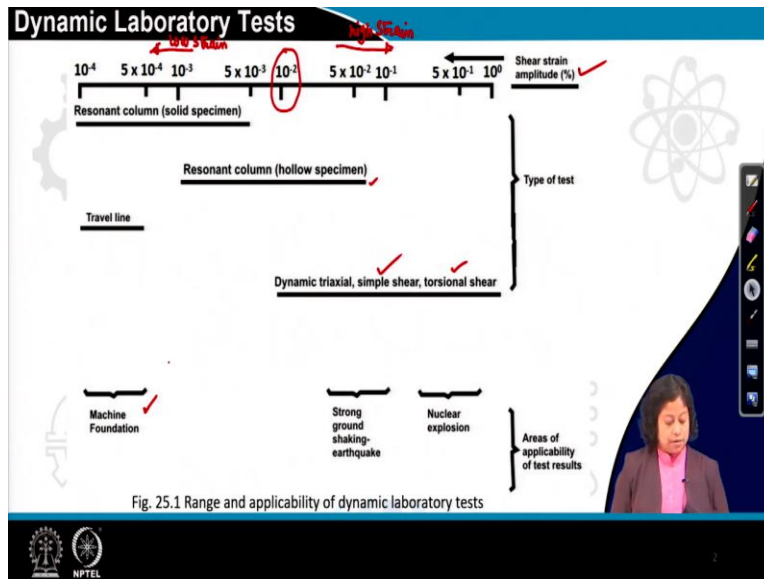
Hello friends, in last four classes, we have studied different laboratory tests to determine the dynamic properties of soils.

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Those tests are resonant column test, cyclic triaxial test, cyclic torsional , shear test, etcetera. Now, today we will see what information's we generally get by conducting laboratory test and how we will interpret the results.

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So, first we will, here we can see the range and applicability of different dynamic laboratory tests. First thing which we need to remember is the boundary between low strain test and high strain test. The boundary shear strain amplitude is 10 to the power minus 4 or I can say 10 to the power minus 2 in percentage. So, here you can see this is the boundary for low strain and high strain laboratory test. So, this side is mainly for the high strain and the site mainly for low strain.

Now, here you can see resonant column test with solid specimen can be done only at low strain level, which is 10 to the power minus 2 or below 10 to the power minus 2 percent. Whereas, if we do the resonant column test with hollow specimen that can be possible to carry on at high strain level as well, here you can see it can be done also up to 5 times 10 to the power minus 2 percent strain level, here you can see.

On the other hand, cyclic triaxial test or you can call it as Dynamic triaxial test, then Torsional shear test or simple shear test, these tests can be done only at high strain level. High strain level means the amplitude of cyclic shear strain more than 10 to the power minus 2 percent. Now, where these tests, the results from these tests can be used, the low strain test, is the result from the low strain test is generally used for the design of machine foundation.

Whereas, the results obtained from the high strain test is useful for strong ground shaking earthquake analysis. It is useful for the analysis of nuclear explosion etcetera. So, that means, if

you are interested for the design of machine foundation that time you will if you need to do any kind of laboratory test for determination of dynamic properties of the soil, then you will do resonant column test. Whereas, if you are interested to do analysis related to earthquake, then you will do cyclic triaxial test or cyclic torsional shear test.

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**Dynamic Laboratory Tests**

**Table 25.1 Relative Quality of Laboratory Techniques for Measuring Dynamic Soil Properties (after Silver, 1981)**

	Relative quality of Test Results				
	Shear modulus	Young's modulus	Material damping	Effect of number of cycles	Attenuation
Resonant column	Good ✓	Good ✓	Good ✓	Good	-
Ultrasonic pulse	Fair	Fair	-	-	Poor
Cyclic triaxial	-	Good	Good	Good	-
Cyclic simple shear	Good	-	Good	Good	-
Cyclic torsional shear	Good	-	Good	Good	-

Now, in this table you can see relative quality of the different laboratory tests that are used for the measurement of dynamic soil properties, here we can see from resonant column test, we can get dynamic shear modulus of good quality, we can get Young's modulus, dynamic Young's modulus of good quality and we can also get the information about material damping and the quality of the result is good. Whereas, ultrasonic pulse test which is another low strain laboratory test can provide shear modulus and Young's modulus of fair quality.

Now, cyclic triaxial test can only give the dynamic Young's modulus and material damping of good quality, it cannot provide any information of shear modulus or dynamic shear modulus. Sometime we convert Young's modulus to shear modulus, but directly we cannot get any information of shear modulus from the cyclic triaxial test. Whereas, from cyclic torsional shear test and cyclic simple shear test, we can get information of dynamic shear modulus of good quality we cannot get any information of dynamic Young's modulus from both the tests.

We can also get the information of material damping of good quality from cyclic simple shear test and cyclic torsional shear test. All these tests, which I have mentioned can provide good quality result related to the effect of number of cycles on the parameters like shear modulus or Young's modulus.

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**Table 25.2 Parameters Measured in Dynamic or Cyclic Laboratory Tests**

Parameters	Resonant column	Cyclic triaxial	Cyclic simple shear	Torsional shear
<b>Load</b>	Resonant frequency	Axial force	Horizontal force	Torque
<b>Deformation</b>				
<b>Axial</b>	Vertical displacement	Vertical displacement	Vertical displacement	Vertical displacement
<b>Shear</b>	Acceleration	Not measured	Horizontal displacement	Rotation
<b>Lateral</b>	Not usually measured	Not usually measured	Often controlled	Not usually measured
<b>Volumetric</b>	None for undrained tests Volume of fluid moving into or out of the sample for drained tests			
<b>Pore water pressure</b>	Not usually measured	Measured at boundary	Measured at boundary	Measured at boundary

Now, in this table you can see what are the different parameters that we actually calculate or measured from different dynamic tests in the laboratory. So, first one is resonant column test. From resonant column test in terms of load actually, what we get is resonant frequency, we can also measure vertical displacement, we can, generally cannot measure the pore water pressure from resonant column test, generally we do not.

Now, what are the parameters, main parameters we can measure from cyclic triaxial test, we can measure the axial force, we can measure the vertical displacement, generally we do not measure the lateral displacement we cannot get any information about the shear strain. But from these tests we can get information of the pore water pressure at the boundary of the soil specimen.

Similarly, from torsional shear test, we can measure the torque applied to the sample. We can measure the vertical displacement we can also measure the rotation of the specimen and we can measure the pore water pressure at the boundary of the specimen. From cyclic simple shear test, we can measure the horizontal force acting to the sample, we can measure the vertical

displacement, we can measure the horizontal displacement as well, also we can measure the pore water pressure at the boundary of the specimen. So, these are the different parameters, which we can get or measure by conducting different laboratory tests.

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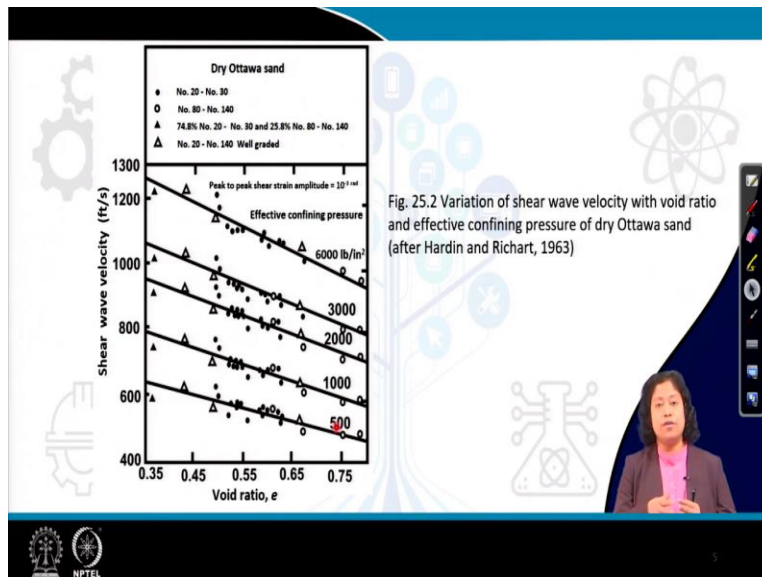


Fig. 25.2 Variation of shear wave velocity with void ratio and effective confining pressure of dry Ottawa sand (after Hardin and Richart, 1963)

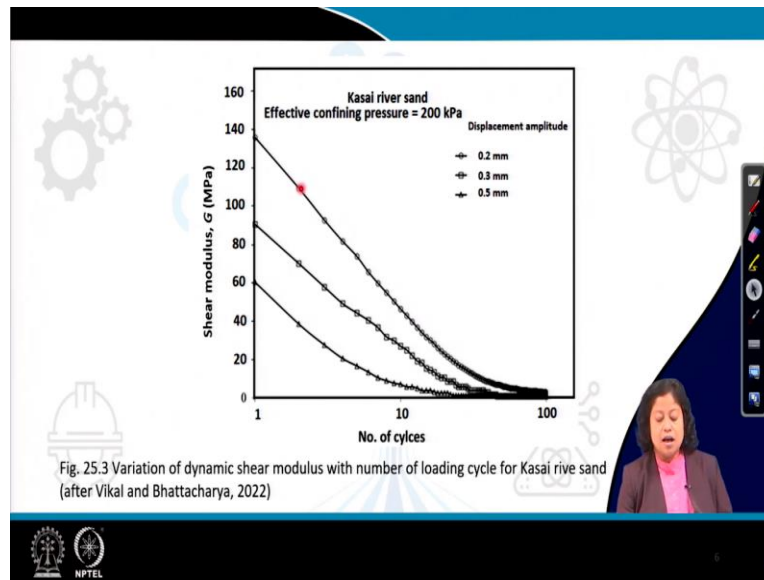
Now, let us see the trends of the results that we get from the different laboratory test. Here you can see the result of resonant column test. In this diagram, the variation of shear wave velocity with void ratio and effective stress is presented. So, what we can see these numbers represent the gradation of the soil sample in this case, Ottawa sand is used and these results is taken from the paper of Hardin and Richard which is published in 1963. So, here, what we can observe is that the shear wave velocity does not depend upon the gradation of the sand, you can see these dot circular marker represents the sand specimen size given, sieve number 22, sieve number 30.

Whereas, these triangular marker without any fill, that represent well graded sand, so, whether it is well graded sand or poorly graded sand, it does not matter the shear wave velocity remains same if the void ratio and the confining pressure remain same. So, it does not depend upon the shear velocity sorry shear velocity does not depend upon the gradation of the sand, but it depends upon the void ratio and the effective confining pressure.

So, in this figure, what we can see is that the shear wave velocity decreases with an increase in void ratio, what does it mean? When void ratio increases, it means we are using loose sand of

low relative density that means, shear wave velocity is less in sand specimen having low relative density or when it is loose sand. Similarly, you can see when effective confining pressure decrease from top to bottom the shear wave velocity also decreases. So, at higher confining effective confining pressure, we can expect higher shear velocity, whereas, at lower confining pressure the shear wave velocity is less it means the strength also accordingly changing.

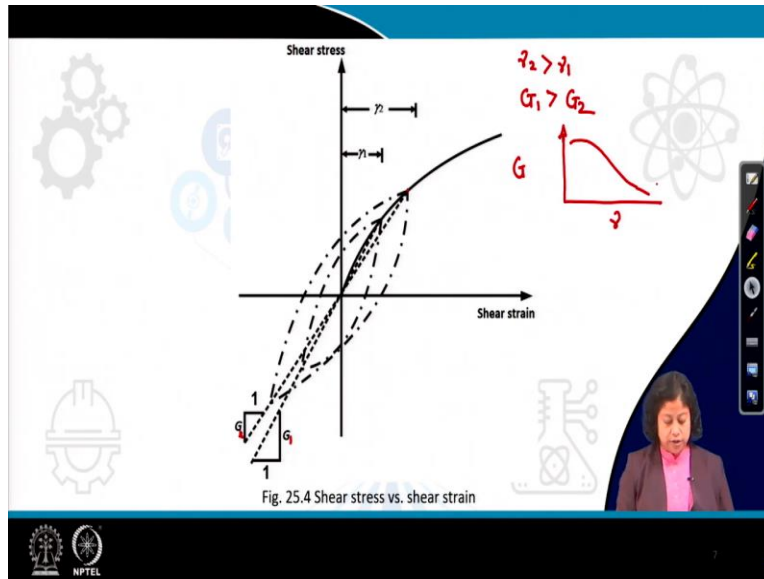
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In this figure you can see the variation of dynamic shear modulus  $G$ , with the number of cycles. This is the result of a strain or displacement controlled cyclic triaxial test where the three, where test were conducted at three different displacement amplitude say 0.2 millimeter, 0.3 millimeter and 0.5 millimeter and these results are for effective confining pressure 200 kPa. So, what we can see from this test, when the number of cycles increases, shear modulus or dynamic shear modulus decreases and if you see the reduction in dynamic shear modulus is very high from 1 to 10 cycles, 10 to 100 if you see the reduction is not that much sharp.

However, 1 to 10 the reduction in dynamic shear modulus is very sharp, also, we can note another thing that when the displacement amplitude increases that time what is happened, shear we can if we apply higher displacement amplitude then what will be happened the value of the dynamic shear modulus at any particular loading cycle will be less than the same at higher displays a lower displacement amplitude. So, at top we can see the lower displacement amplitude and at the bottom we can see the higher displacement amplitude.

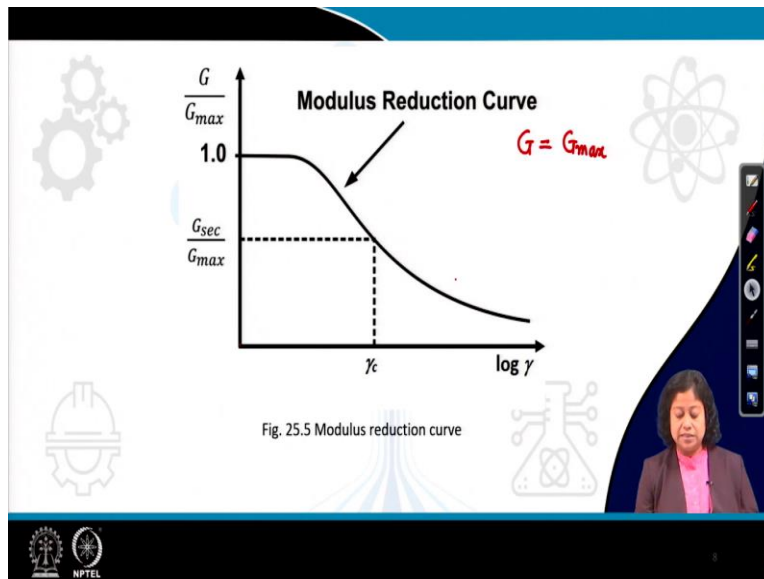
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Now, in this figure, we can see the variation of shear stress versus shear strain. So, what we can see, you can see here two different cyclic shear strain amplitudes are taken, one is gamma 1 and the other one gamma 2. So, gamma 1 is lower amplitude, gamma 2 is higher amplitude from this figure you can say. So, at lower cyclic shear strain amplitude, we can see the  $G_{sec}$  value which is secant shear modulus is high or higher than the secant shear modulus at higher amplitude of cyclic shear strain.

So, you can see here gamma 2 is greater than gamma 1 and if I will write it as this is gamma 2 and I can write instead of  $G$ , I am writing here  $G_2$  and this is  $G_1$  then what we can see here  $G_1$  is, which one is higher  $G_1$  is higher than  $G_2$ , that means, as if now we will plot let us take gamma versus  $G$ , then what will be the pattern? Pattern will be something like this. So, at higher shear strain we will see the lower value of  $G_{sec}$  or secant shear modulus.

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Next is the variation of the ratio of  $G$  to  $G_{max}$ . Here  $G$  means secant shear modulus and  $G_{max}$  is the shear modulus when the cyclic shear strain amplitude is almost 0 that means close to 0 or very, very low. What we can see here, when cyclic shear strain amplitude is 0 that time the modulus ratio which is  $G$  by  $G_{max}$  is equal to 1 that means, as I said we can get the  $G_{max}$ . That time when cyclic shear strain amplitude is 0, 0 means very close to 0 that time, we are noting that  $G$  is equal to  $G_{max}$ . Now, after this value of cyclic shear strain amplitude increases up to some level, we can note a sharp decrease in the modulus ratio.



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Table 25.3 Effect of Different Factors on Modulus Ratio of Normally Consolidated and Moderately Overconsolidated Soils (after Dobry and Vucetic, 1987)

Increasing Factor	$G/G_{max}$
Effective confining pressure, $\sigma'_m$	Increases with $\sigma'_m$ ; effect decreases with increasing PI
Void ratio, $e$	Increases with $e$
Geological age, $t_g$	May increase with $t_g$
Cementation, $c$	May increase with $c$
Overconsolidation ratio, OCR	Not affected
Plasticity index, PI	Increases with PI
Cyclic strain, $\gamma_c$	Decreases with $\gamma_c$
Strain rate, $\dot{\gamma}$	$G$ increases with $\dot{\gamma}$ , but $G/G_{max}$ probably not affected if $G$ and $G_{max}$ are measured at same $\dot{\gamma}$
Number of loading cycles, $N$	Decreases after $N$ cycles of large $\gamma_c$ ( $G_{max}$ measured before $N$ cycles) for clays; for sands, can increase (under drained conditions) or decrease (under undrained conditions)

Here you can see the effect of different factors like, Effective confining pressure, Void ratio, then Over consolidation ratio, Plasticity of plasticity index of soil, Cyclic strain etcetera be on the magnitude of modulus ratio that means,  $G$  by  $G_{max}$ . So, what we can see from this table the value of  $G$  by  $G_{max}$  depends upon several factors which are mentioned on the left hand side column like, Effective confining stress, Void ratio.

Then the plasticity index over consolidation ratio then cyclic shear strain which you have already seen and strain rate and all the cases  $G$  by  $G_{max}$  what we have seen is that all the except cyclic strain for rest of the cases  $G$  by  $G_{max}$  value increases with an increase in effective confining pressure, with an increase in void ratio, increase in plasticity index and plasticity index. Now, it does not actually depend upon OCR, which is over consolidation ratio. However, the modulus ratio you can see here decreases with an increase in cyclic strain that we have already seen also.

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## Dynamic Laboratory Tests

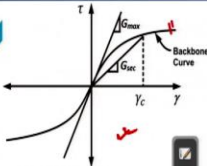



Fig. 25.6 Backbone curve

- It is seen from the laboratory tests that the soil stiffness depends upon the cyclic strain amplitude, void ratio, mean principal effective stress, plasticity index, overconsolidation ratio and number of loading cycles.
- Secant shear modulus (or simply call it as shear modulus) of soil decreases with an increase in cyclic shear strain amplitude.
- The locus of the points corresponding the tips of hysteresis loops at various cyclic shear strain amplitudes is called a **Backbone curve**.

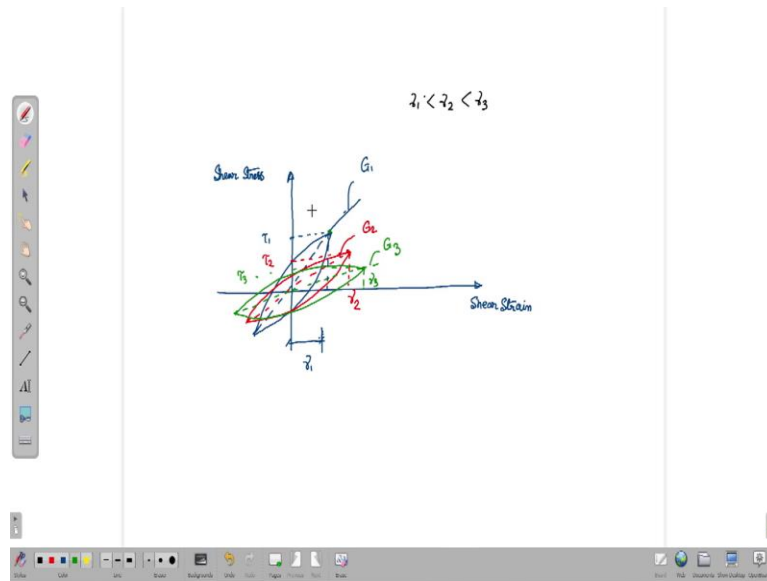


Now, there is, now dynamic from the dynamic laboratory tests what we have seen, we have seen that the soil stiffness depends upon the cyclic shear strain or I can say cyclic shear strain amplitude or cyclic axial strain amplitude in case of cyclic triaxial test, then it depends upon void ratio, over consolidation ratio, mean principal effective stress, plasticity index and number of loading cycles. Secant shear modulus we generally call it as dynamic shear modulus or simply shear modulus also of soil decreases with an increase in cyclic shear strain amplitude.

Now, from cyclic triaxial test we get hysteresis damping. So, the locus of the points corresponding the tips of the hysteresis loops at different cyclic shear strain amplitudes is called the backbone curve. Here you can see the backbone curve I am reading once again the definition of the backbone curve. So, backbone curve is the locus of all the points corresponding the tips of the hysteresis loops obtained from various obtain for various cyclic shear strain. So, let us take for one test this is 1 hysteresis loop.

Now, if we will do another test like this or at lower strain, then it may be like so, the tip the points first we need to select which is corresponding the tip of the hysteresis loop. Now, if we will try to find that locus. So, what the way I have shown actually this drawing is not correct, let me draw it on the board.

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So, here first we will draw a hysteresis loop for different values of shear strain amplitudes, here horizontal axis represents shear strain, vertical axis represents shear stress. Now, at low shear strain what we can see the hysteresis loops look like this one so, the  $G_{sec}$  is this one and this is the tip of the hysteresis loop in this case. Now, at higher shear strain so, let us take another one. So, this one is at higher strain so, what we can see here this is the tip and  $G_{sec}$  is this one. So, I can write then here this one is suppose,  $\gamma_1$ .

Let us take this one is  $\gamma_1$ , so, this will give you the  $G_{sec}$  value the slope of this line and that you can write  $G_1$  or  $G_{sec 1}$ , likewise for the second one you may get  $G_2$  or  $G_{sec 2}$  and here it is  $\gamma_2$ . Now, in the next step what we can see for, so, one important thing we need to remember here  $\gamma_1$  is less than  $\gamma_2$  now, if we will consider  $\gamma_3$  also what we will get, so, there we will get the third curve. So, that may be like this one. So, this is  $G_3$ , I can write  $G$  properly  $G_3$  and so, the corresponding so, the tip is this one and the corresponding shear strain here is  $\gamma_3$ .

So, now, what we have obtained here for different values of  $\gamma_1$ ,  $\gamma_2$ ,  $\gamma_3$ , we can get the tip of the hysteresis loop and that represents different magnitude. So, here for the first case it is we can write  $\tau_1$  and this is  $\gamma_1$ , likewise for the second one we can write  $\tau_2$  and shear strain is  $\gamma_2$ , for the third one it is  $\tau_3$  and  $\gamma_3$ , so, this maximum or tip

value of the hysteresis loops, three different loops or there maybe the number of loops may be more that can be used to plot the backbone curves.

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## Dynamic Laboratory Tests

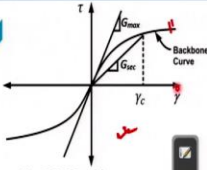



Fig. 25.6 Backbone curve

- It is seen from the laboratory tests that the soil stiffness depends upon the cyclic strain amplitude, void ratio, mean principal effective stress, plasticity index, overconsolidation ratio and number of loading cycles.
- Secant shear modulus (or simply call it as shear modulus) of soil decreases with an increase in cyclic shear strain amplitude.
- The locus of the points corresponding the tips of hysteresis loops at various cyclic shear strain amplitudes is called a **Backbone curve**.
- The slope of this curve is maximum at the origin i.e. at zero cyclic shear strain amplitude.
- At greater cyclic shear strain amplitude, the modulus ratio becomes less than 1.



So, here is the backbone curve you can see, so, this is. Now, the slope of this backbone curve at origin that means, when shear strain is or I can say cyclic shear strain amplitude is 0 that time we get the  $G_{max}$ , at greater cyclic shear strain amplitude that means, when now we are shifting towards this direction the modulus ratio becomes less than 1, these we have already seen in the previous slide.

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**Maximum Dynamic Shear Modulus**

$$\sigma'_m = (\sigma'_1 + \sigma'_2 + \sigma'_3) / 3$$

- In order to characterize the soil stiffness information on both  $G_{max}$  and variation of the modulus ratio with cyclic shear strain amplitude are subjects of interest.
- From the laboratory tests like resonant column test with cylindrical specimen, bender element test data  $G_{max}$  can be determined.
- Using other laboratory test data  $G_{max}$  can be calculated using following empirical relationships:

$$G_{max} = 625F(e)(OCR)^k P_a^{1-n} (\sigma'_m)^n \quad \dots (1)$$

Where  $F(e)$  is a function of void ratio,  $k$  is an overconsolidation ratio exponent,  $P_a$  is atmospheric pressure,  $\sigma'_m$  is mean principal effective stress,  $n$  is stress exponent.

NPTEL

Now, how to get maximum dynamic shear modulus? So, first of all we have to understand one thing that in order to characterize the soil stiffness we need the information of both  $G_{max}$  and the variation of the modulus ratio with cyclic shear strain amplitude. Now, there are different tests to find out  $G_{max}$  like resonant column test, or we can do bender element test for this purpose. However, if we have no such facility then how to get  $G_{max}$ , using some laboratory test data also we can calculate the  $G_{max}$  value.

Here you can see if we know the over consolidation ratio of the soil if we know the void ratio and also the effective confining pressure we can calculate  $G_{max}$ . So,  $G_{max}$  is equal to 625 times a function  $F$  times over consolidation ratio to the power  $k$  times  $P_a$  to the power  $1 - n$  times  $\sigma'_m$  to the power  $n$ ,  $n$  is an,  $n$  is a stress exponent, whereas,  $F(e)$  is a function of void ratio,  $K$  is a ratio exponent for over consolidation ratio,  $P_a$  is atmospheric pressure,  $\sigma'_m$  is mean effective, mean principal effective stress that means, the if we know that  $\sigma'_1$ ,  $\sigma'_2$  and  $\sigma'_3$ .

Then we can calculate  $\sigma'_m$  as  $\sigma'_1 + \sigma'_2 + \sigma'_3$  divided by 3, in this empirical equation one thing we need to remember is that the unit of  $P_a$  which is atmospheric pressure,  $\sigma'_m$ , which is mean principal effective stress and  $G$

max should be same. So, if the, if we choose the unit for  $\sigma_{vm}$ , is kPa then raised 2 should have the same unit that means in kPa.

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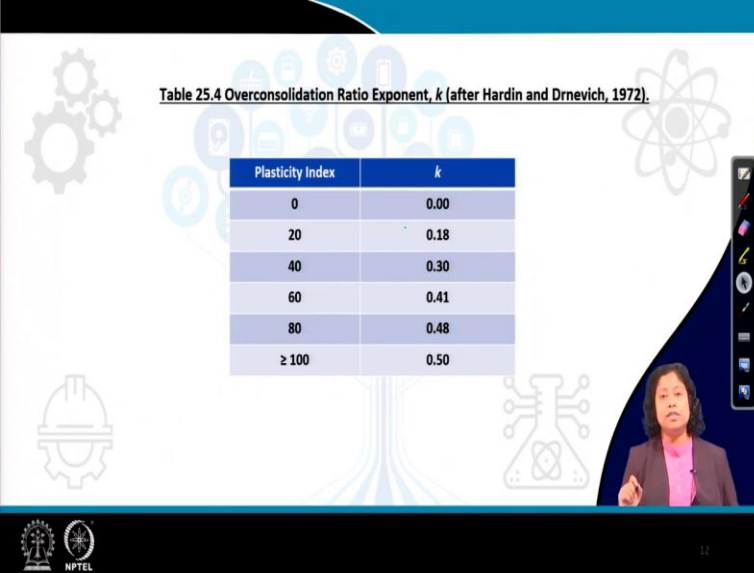


Table 25.4 Overconsolidation Ratio Exponent,  $k$  (after Hardin and Drnevich, 1972).

Plasticity Index	$k$
0	0.00
20	0.18
40	0.30
60	0.41
80	0.48
$\geq 100$	0.50

Now, how to get  $k$  which is over consolidation ratio exponent? So,  $k$  depends upon the plasticity index we all know what is plasticity index, it is the range over which soil exhibits plasticity. So, how we get it? By we if we subtract plastic limit from the liquid limit value then we can get the plasticity index of soil. Now, the OCR exponent which is  $k$  depends upon  $P_i$  and it increases with the plasticity index, the maximum value of  $k$  is 0.5 and its minimum value is 0. So, here you can see as plasticity index increases,  $k$  value also increases. For  $n$  I think I have already mentioned generally we take  $n$  is equal to 0.5 which is stress exponent.

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**Maximum Dynamic Shear Modulus**

- Hardin (1978) suggested following relationship:  
$$F(e) = \frac{1}{0.3 + 0.7e^2} \quad \dots (2)$$
- Jamiolkowski et al. (1991) provided following relationship:  
$$F(e) = \frac{1}{e^3}$$

NPTEL

Now, what about  $F(e)$  which is the function of void ratio. So, Hardin 1978 suggested a relationship which is  $F(e)$  is equal to 1 divided by 0.3 plus 0.7  $e$  square, where  $e$  is the void ratio. We can also use another empirical relationship which is provided by Jamiolkowski and others in 1991 which says  $F(e)$  is equal to 1 divided by  $e$  cube, where  $e$  is the same void ratio.

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**Maximum Dynamic Shear Modulus**

- For sand, the maximum shear modulus can be estimated by using the following empirical relationship:  
$$G_{max} = 1000 K_{2,max} (\sigma'_m)^{0.5} \quad \dots (4)$$
- Here  $\sigma'_m$  is  $n$  lb/ft<sup>2</sup> (Seed and Idriss, 1970) and  $K_{2,max}$  is determined from the void ratio or relative density.

NPTEL

Now, for sand what will be the  $G_{max}$  value or how do we find out the  $G_{max}$  value. For sand generally we use we may use this equation  $G_{max}$  is equal to 1000 times  $K_{2,max}$  times  $\sigma'_m$

dash m to the power 0.5. Here  $K_2$  max depends upon the void ratio or the relative density, if we know the relative density of the sand, we can get the  $K_2$  max if we know the void ratio instead of the relative density then also, we can get the value of  $K_2$  max. So, in this empirical equation the important thing is that the effective, mean effective stress  $\sigma$  dash m, is expressed in unit pound per square inch.

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**Table 25.5 Values of  $K_{2,max}$**

$e$	$K_{2,max}$	$D_r(\%)$	$K_{2,max}$
0.4	70	30	34
0.5	60	40	40
0.6	51	45	43
0.7	44	60	52
0.8	39	75	59
0.9	34	90	70

Source: After Seed and Idriss (1970).

And you can see the value of  $K_2$  max it increases with the increase in relative density of the sand that means, for denser sand  $K_2$  max value is high, for loose sand the value of  $K_2$  max is low or in other words, I can see the value of  $K_2$  max you can see for low  $e$  value that means, for lower void ratio  $K_2$  max value is high, for high void ratio that means, for loose sand  $K_2$  max value is low. This is low and this is high, obviously, this is for loose sorry, this is for dense, so, this is dense and this is loose soil or sand I can say.



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Increasing Factor	Damping Ratio, $D$
Confining pressure, $\sigma'_m$	Decreases with $\sigma'_m$ ; effect decreases with increasing PI
Void ratio, $e$	Decreases with $e$
Geological age, $t_g$	Decreases with $t_g$
Cementation, $c$	May decrease with $c$
Overconsolidation ratio, OCR	Not affected
Plasticity index, PI	Decreases with PI
Cyclic strain, $\gamma_c$	Increases with $\gamma_c$
Strain rate, $\dot{\gamma}$	Stays constant or may increase with $\dot{\gamma}$
Number of loading cycles, $N$	Not significant for moderate $\gamma_c$ and $N$

Now, another important parameter is damping ratio, which we can measure from the dynamic laboratory tests. So, here we can see that the damping ratio also depends upon the effective confining pressure, it depends upon the void ratio, it depends upon the over consolidation ratio and plasticity index for fines, it depends upon the cyclic shear strain and strain rate. Interestingly, damping ratio decreases with increasing effective confining pressure void ratio, plasticity index whereas, damping ratio increases, with an increasing cyclic shear strain, damping ratio is the measurement of the loss of energy which increases with increasing cyclic strain.

So, if suppose I am interested to see just the curve for damping ratio versus cyclic shear strain, then how this curve will look, it will look like this. Another thing the strain rate, the effect of this strain rate is on generally seen in damping ratio, but you can see it the damping ratio remains constant, with an increase in strain rate or it may slightly increase with an increase in strain rate. So, the effect of the strain rate is very small on the damping ratio.

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**SUMMARY**

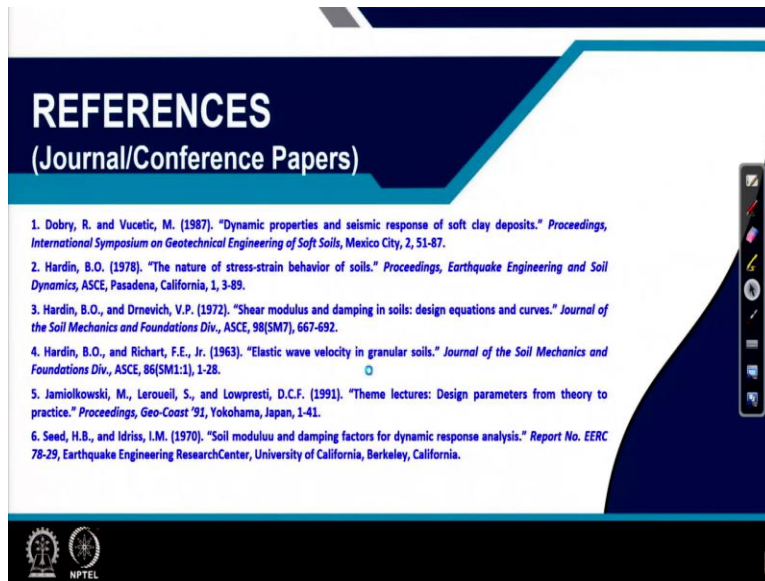
In this lecture the following topics related to dynamic properties of soils are discussed:

- Range and applicability of different laboratory dynamic tests
- The different parameters on which dynamic properties of soils depend
- Modulus reduction curve and backbone curve
- Estimation of Maximum shear modulus
- Parameters controlling the magnitude of modulus reduction and damping ratio

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So, come to the summary of today's lecture, we have discussed about the range and applicability of different laboratory dynamic test, then we have discussed the different parameters on which dynamic properties of soils depend, then we have studied the modulus reduction curve and backbone curve, then we have the studied how to determine the maximum shear modulus, we can do it by conducting laboratory test or we can do it by conducting some dynamic test in the lab also. Then, we have studied the parameters that controlling the magnitude of modulus reduction and damping ratio. So, with this I am stopping today's class.

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These are the references which are shown in today's lecture. Thank you.