Soil Dynamics
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Lecture 21
Determination of Dynamic Properties of Soils (laboratory Tests-Part 1)

Hello friends, today we will discuss our new topic which is determination of dynamic properties of soils. So, there are two different ways to determine that dynamic properties of the soil one set of tests can be carried out in the laboratory, other sets of tests can be carried out in the field. So, this week we will study we will see how to do the laboratory tests to determine the dynamic properties of soils.

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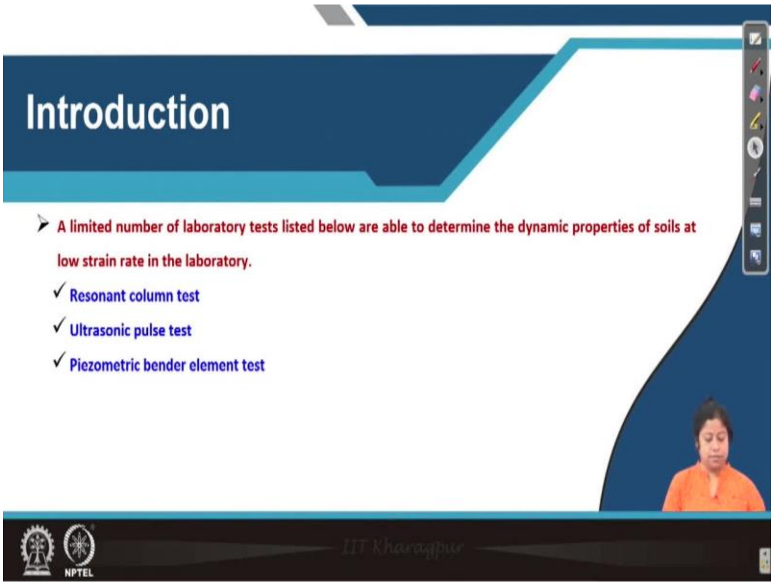

There is on a limited number of tests, which can find out the dynamic properties of soils at low strain in the laboratory. These tests are resonant column test, ultrasonic pulse test, and Piezometric bender element test and all these three tests are for the low strain level, which is 10 to the power minus 4 or below that.

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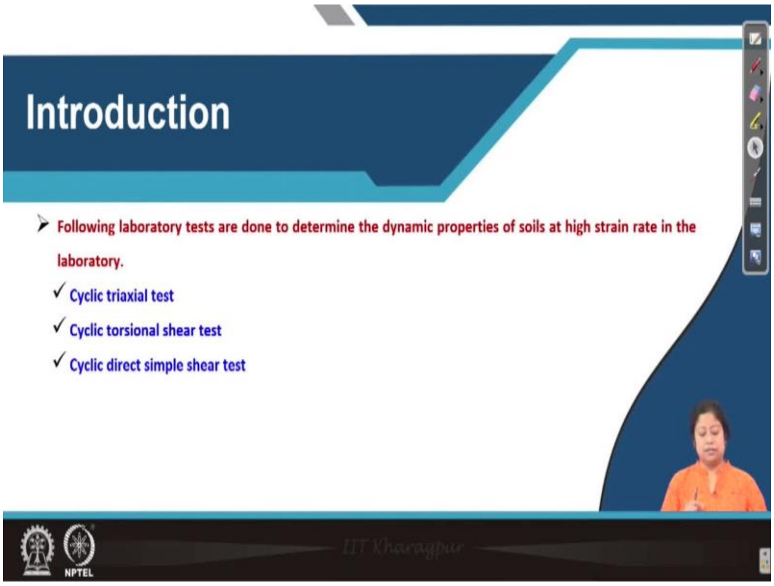

Now, there are a few test or good number of test I can say which we can do in our laboratory to determine the dynamic properties of the soil at high strain rate. These tests are cyclic triaxial test, cyclic torsional shear test, cyclic direct simple shear test and there are few other tests also.

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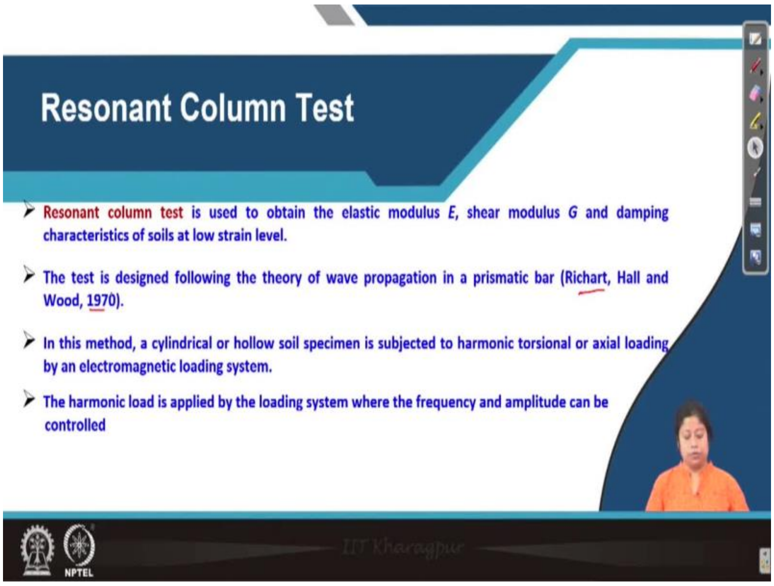

So, today we will see what is resonant column test? How we can get the dynamic properties of the soil from these tests? So, the resonant column test is used to obtain the elastic modulus E, shear modulus G, and damping characteristics of soil at low strain level. Here, when I am writing elastic modulus E it means elastic it is dynamic elastic modulus.

Likewise, when I am talking about shear modulus, it also means that we are focusing on dynamic shear modulus of the soil. Now, the resonant column test is designed following the theory of wave propagation in a prismatic bar that we have already discussed. And that is designed by Richart, Hall and Wood in the year of 1970.

Now, in resonant column test the soil specimen either be cylindrical or be spherical and the specimen is generally subjected to harmonic torsional or axial loading by an electromagnetic loading system. The harmonic load is applied by the loading system where the frequency and amplitude can be controlled.

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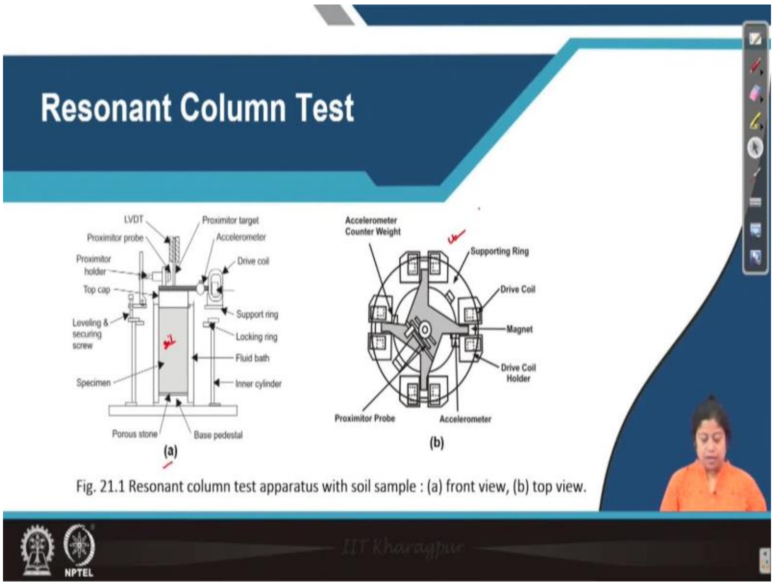

So, let us see the diagram of the resonant column test apparatus. So, here this one is the front view or elevation view of the resonant column test apparatus with the soil sample. So, you can see, this is our soil specimen. There are different components of the instrument and from that top if you will see it, you will get these few, so, the second one is the top view of the resonant column test apparatus.

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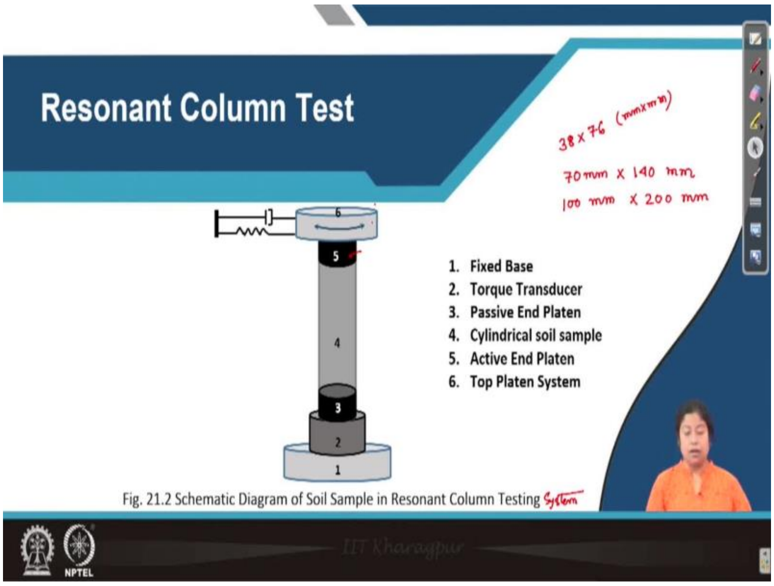

Now, since we are interested to find out the dynamic properties, so, I will focus on how to get the dynamic properties of the soil conducting resonant test column, a resonant column test. Since, we are interested to determine the dynamic properties of the soil, so, I like to concentrate on the soil sample which is subjected to either torsional vibration or longitudinal vibration. So, let us focus on this schematic diagram of soil sample in resonant column testing system here system should be there.

Now, you can see this is our soil sample nowadays, we can use different soil sample of the size. So, generally for we are familiar with this size of soil sample, where 38 millimeter is the diameter of the soil sample and 76 millimeter is the height of the soil sample. Other than that, we can use another size which is 70 millimeter by 140 millimeter, we can also use 100 millimeter by 200 millimeter height of soil sample.

So, all these three dimensions which I have written here for all the three. First one first number represents the diameter of the soil sample and the second number represent the height of the soil sample. So, here what you can see, it is a fixed base on which a torque transducer is placed and then you can see the passive end platen on which soil sample is resting. And at the top one active end platen is provided this one and then top platen system is attached.

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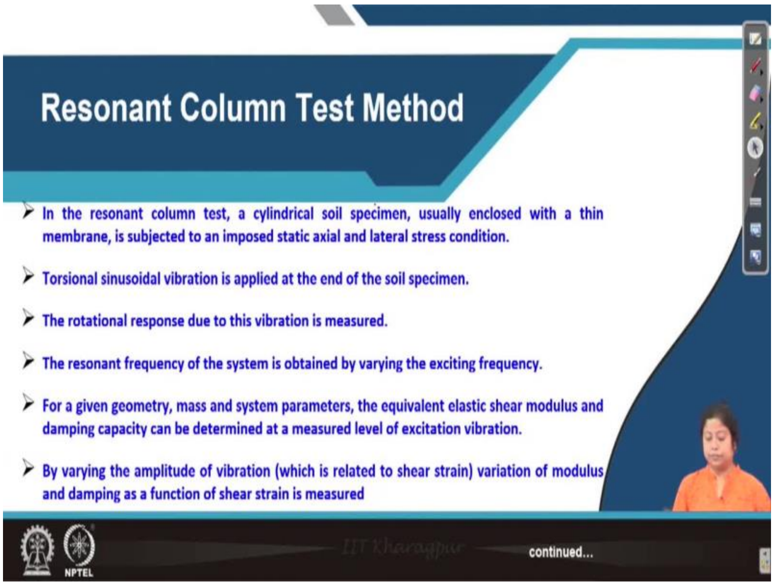

Now, in this system, when we apply the torque what is happening it is just like the case of propagation of torsional waves through our prismatic bar case with fixed free end condition. So, let us see first the different steps of the resonant column test method. So, the first step is in the resonant column test is cylindrical soil specimen which you have already seen in the previously figure is usually enclosed with a thin membrane and is subjected to an imposed static axial and lateral stress condition. What does it mean? That means, apply confining pressure light triaxial test.

The next step ask to apply that torsional sinusoidal vibration at the end of the soil specimen the torsional or it causes rotation so, the rotational response due to these vibration is needed to measure. The resonant frequency of the system is obtained by varying the exciting frequency up to the resonant. For a given geometry, mass and system parameters the equivalent elastic shear modulus and damping capacity can be determined at a measured level of excitation vibration.

By varying the amplitude of vibration which is related to the shear strain variation of modulus of variation of modulus that means, shear modulus in these case and damping as a function of shear strain can be measured.

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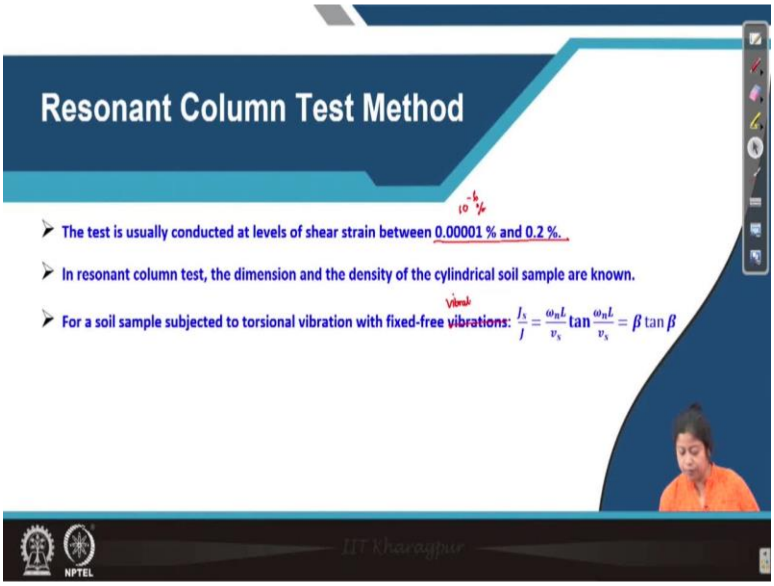

In these tests generally we keep the strain level in between these value this is basically 123. So, you can see it is 10 to the power minus 5 or minus 6 to in percentage to 0.2 percentage, but it is the 0.2 percent is the upper limit.

In resonant column test the dimension and the density of the material of the cylindrical specimen or any type of soil specimen if we are using even hollow specimen that is known to us. Now, for a soil subjected to torsional vibration with fixed free condition vibrating condition please correct here it will be vibrating condition.

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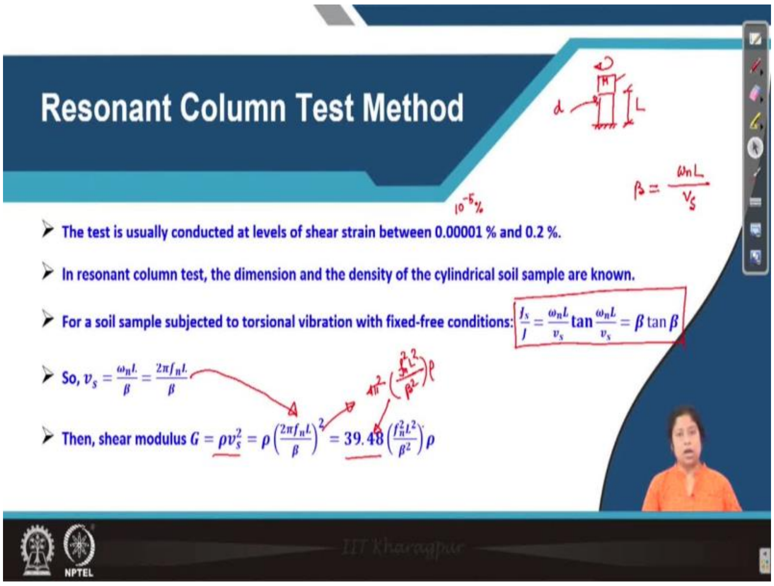

So, the test is usually conducted at levels of shear strain between say it is your 10 to the power minus 5 I can write the number here also 10 to the power minus 5 percentage to 0.2. So, 0.2 percent is the upper limit. In resonant column test the dimension and the density of the cylindrical specimen is these two are known to us.

Now, we have already studied these things what is happened when torsional wave is propagating through a prismatic bar and the end conditions are fixed free end conditions. So, that time we can get this expression that Js is divided by J is equal to omega n times capital M divided by v s times tangent of omega n L divided by v s, What is this actually? So, if you recall I have discussed soil sample having length capital L it has a diameter say d, d is the diameter.

When it is on that at the top of the soil sample a mass of m is placed and it is subjected to torsion what is happened we can get this equation. Here v s is the velocity of the shear wave propagating through this soil sample you can see L is the height of the soil sample what is J s and J, J s is the mass polar moment of inertia of the cylindrical soil sample and capital J is the mass polar moment of inertia of the system.

So, from this we can final, so, for us since, we know that geometry and the density of the soil specimen. So, basically we know the value of J s we can calculate actually J s from this and capital J is provided by the manufacturer of the apparatus. So, from this we can calculate finally, the beta what is beta? Here beta is omega a, omega n L divided sorry beta I can write here beta is omega n L divided by v s.

So, if we know beta from beta, we can calculate v s which is the velocity of the shear wave propagating through the soil sample here. Now, omega n which is the circular natural frequency that can be written as 2 pi f n. Now, if we write 2 pi f n in place of omega n, then what we can write from this we know what is v s? v s is equal to square root of G by rho or we can write G which is the dynamic shear modulus of soil is equal to rho times v s square v s is the velocity of the shear wave.

So, from this, from here we can write the value of v s. So, what we are getting is for one step I can also write here that we are getting 4 pi square times f n squared times L square divided by beta square times rho. So, 4 pi square is equal to 39.48 I have approximated the value up to two decimal places. So, from this we are getting this number 4 times f n squared times L squared divided by beta squared times rho. So, here already we have calculated beta from the J ratio J s by J. So, we can get in this way the dynamic shear modulus of the soil.

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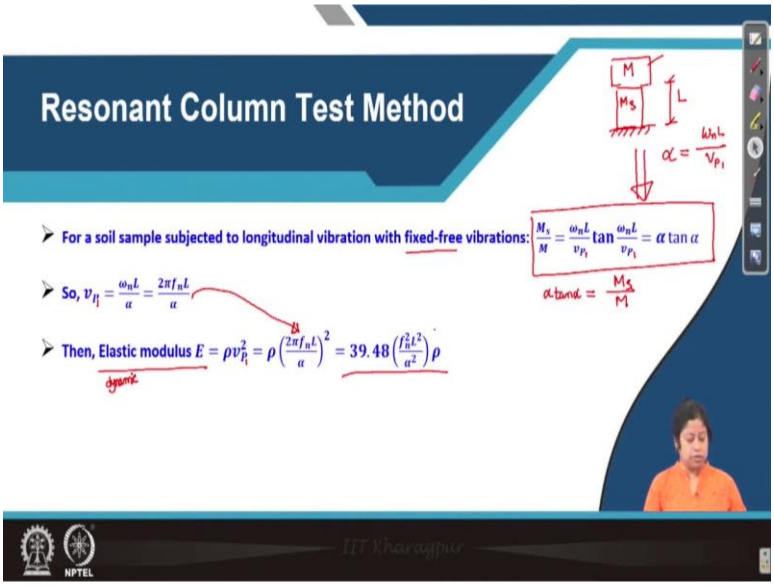

We can also determine that elastic dynamic elastic modulus of the soil. For that we need to apply, we need to create longitudinal vibration to the soil system, soil sample. So, for a soil samples subjected to longitudinal vibration, we can measure the P velocity of the P wave using this equation where the boundary conditions imposed is fixed at bottom and free at top. So, here it is fully fixed at the top as mass or M is placed.

These soil sample has a length L. It has mass capital Ms. So, for these type of cases we can write this equation. Here what is alpha then? Alpha is omega n L divided by v p where v p is the velocity of the P wave. Here one thing you please note I have written v p actually when we have started the propagation of the longitudinal wave through prismatic bar that time we have used the term v p 1 to denote one dimensional motion.

So, if you wish you can call it as v p 1 also there is no harm or you can call it v p also, since we are familiar with v p 1, so, I am just writing here v p 1. So, then v p 1 is equal to omega n L divided by alpha and that alpha is determined from this equation alpha times tangent of alpha is equal to a Ms divided by M, what is Ms here? Mass of the soil sample capital M is the mass which is placed at the top of the soil sample.

So, omega n which is the circular natural frequency can be replaced by 2 pi f n then this we can get elastic modulus or I can call it as dynamic elastic modulus as well it is E which is equal to rho times v p 1 square that is equal to rho times in place of v p 1 square now, we can write this expression here. So, finally, we are getting it as equal to 39.48 times f n square times capital L square divided by alpha square whole thing is multiplied by the density of the material which is rho here.

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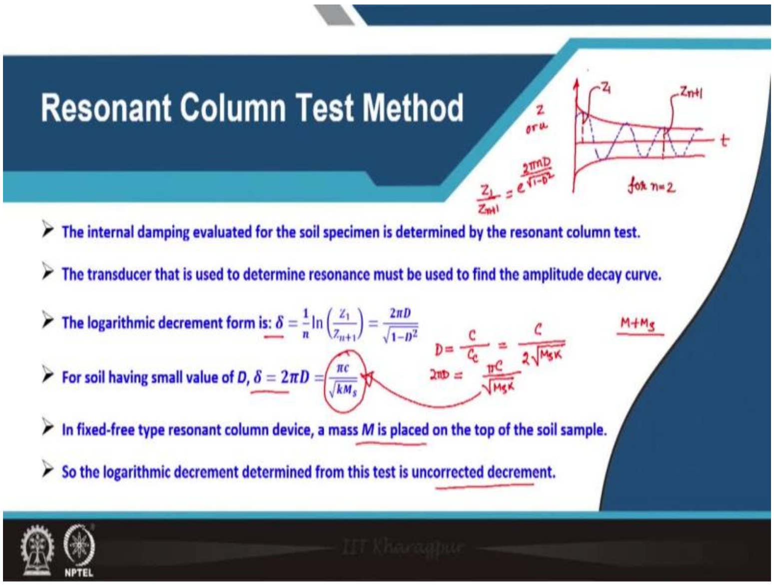

Now, the internal damping evaluated for the soil specimen is determined by the resonant column test. So, other than dynamic shear modulus and dynamic elastic modulus or dynamic Young's modulus, we can also find out the internal damping by conducting resonant column test.

The transducer that is used to determine the resonance must be used to find the amplitude decay curve. I hope you all remember what is amplitude decay curve let me draw it we have discussed it when we discussed the topic under damped system there so, amplitude give me one minute time to erase this line so, the amplitude decay curve if I will draw, it is our we can take it a time horizontal axis and vertical axis may be z which represents the displacement or you can represent it by u also where u is displacement.

So, how does this amplitude decay curve look, let me draw it something like this where if I draw the thing in the sin curve then probably look like. So, for the first cycle it looks like this, now, for the second cycle this is the curve for the third cycle. So, like this way, now, in this curve what we can find out? We can find out the ratio of Z a divided by sorry z 1 divided by z n plus 1. So, what is z 1 here let us take this is our z 1, when n is equal to 2 that time, this is our z n plus 1 for n is equal to 2.

Now, what we have studied, we studied the z 1 divided by z n plus 1. So, this ratio we know and that ratio basically can be represented by e to the power 2 pi n capital D divided by square root of 1 minus D square. Now, here D is the damping ratio. So, from this what we can write? From this we can write logarithmic decrements delta, which is equal to 1 divided by n times log of z 1 divided by z n plus 1.

In this figure n is equal to 1 but n can be any integer and this logarithmic decrement delta is equal to 2 pi D divided by square root of 1 minus D square. Now, when D is very small that time, what we can write we can write delta is equal to 2 pi D. So, for soil having small values of damping ratio capital D, we can write delta is equal to 2 pi D, which is equal to pi c divided by square root of k times Ms.

Now, how the expression pi c divided by square root of k times Ms appears here. We know what is damping ratio. So, I am writing here, D is damping ratio, which is equal to c divided by c c, where c is the coefficient of viscous damping and c c is critical damping, how we can write c c, we can for c c we can write 2 times square root of a Ms times k. So, now, in this expansion of delta that means, logarithmic decrement if we use the value of D is equal to c divided by 2 times square root of Ms times k then what we get 2 pi D is equal to pi c divided by square root of Ms times k. So, that is written here.

Now, in resonant column device if we consider the boundary condition for the soil sample it is fixed free type boundary conditions where at the top of the soil a mass M is placed. So, for that what is happened? Now, the total mass acting on the soil is increased to capital M plus capital Ms, where capital M is the additional mass placed on the top of the soil and capital Ms is the mass of the soil sample, which we can determine at the beginning of the test.

Then, what we get actually from the resonant column test, the logarithmic decrement based on these mass. So, we can call that logarithmic decrement as uncorrected decrement. Thank you (()) (28:05).

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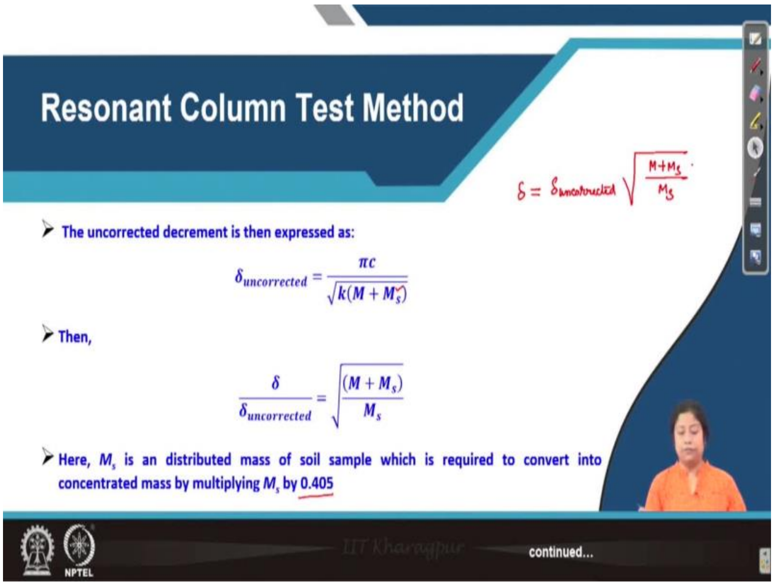

So, here uncorrected logarithmic decrement can be written by this equation delta uncorrected is equal 2 pi times c divided by square root of k, which is the stiffness coefficient times total mass that is capital M plus capital Ms that means, the mass additional mass kept at the top of the soil sample plus the mass of the soil sample which is capital Ms.

Now, we can try to get our ratio which is called delta divided by delta uncorrected which is equal to square root of capital M plus capital Ms divided by capital Ms. So, in this equation, what parameters are known to us, we know what is the value of capital M that means the additional mass kept at the top of the soil sample, we also know what is the total mass of the soil sample that is capital Ms.

Also we can measure delta uncorrected from the resonant column test. So, three parameters are known here only unknown is delta that can be determined by writing delta is equal to delta uncorrected times square root of M plus Ms divided by Ms. Now, in this case, we need to note one interesting thing, when I am seeing a Ms capital Ms is the total mass of the soil sample, in soil sample this mass is distributed.

Whereas, when we have studied the single degree of freedom system that time the mass was concentrated at some point. So, we need to convert the distributed mass Ms of the soil samples to the concentrated mass by multiplying it by a factor 0.405 here.

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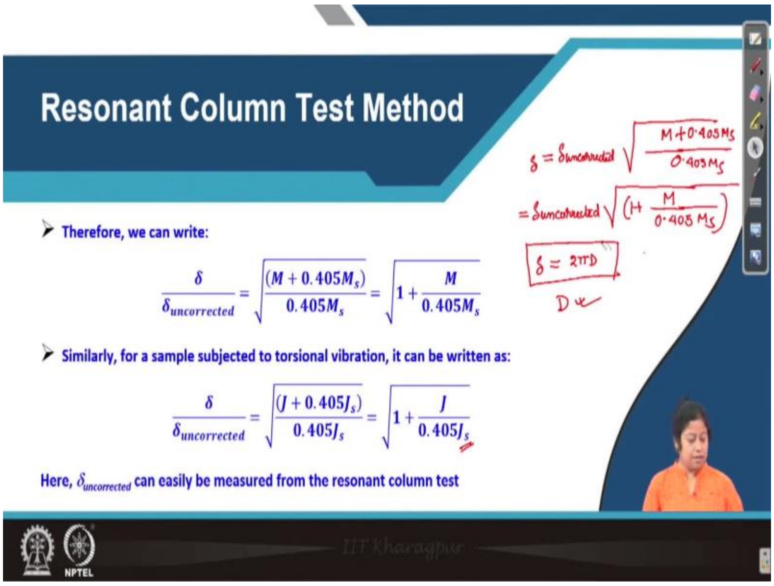

So, in place of capital Ms now we can write 0.405 times capital Ms so, that we have done here. So, from this finally, we can calculate the delta is equal to delta uncorrected times square root of M plus 0.405 Ms by 0.405 Ms or the way I have written I can write that way delta uncorrected times square root of 1 plus M divided by 0.405 Ms. So, in this way we can calculate the logarithmic decrement.

Now, if we know that logarithmic decrement then already we know delta is equal to 2 pi D. So, using this equation finally, we can find out capital D which is the damping ratio of the soil.

Similarly, when the soil sample is subjected to torsional vibration, that time this relationship can be written as delta divided by delta uncorrected is equal to square root of 1 plus J divided by 0.405 J s because what is J here? J is mass polar moment of inertia of the system that means, because of the additional mass whatever is happened to the system that is and J s is the mass polar moment of inertia of the soil sample itself.

Now, we are considering for single degree of freedom system when we derive the expression for delta we have considered lumped mass system and here mass is distributed as a reason this correction factor 0.405 is multiplied to J s. So, J s is the mass polar moment of inertia of the soil sample. Now, from this since, delta uncorrected is measured by the resonant column test, so, we can get delta this.

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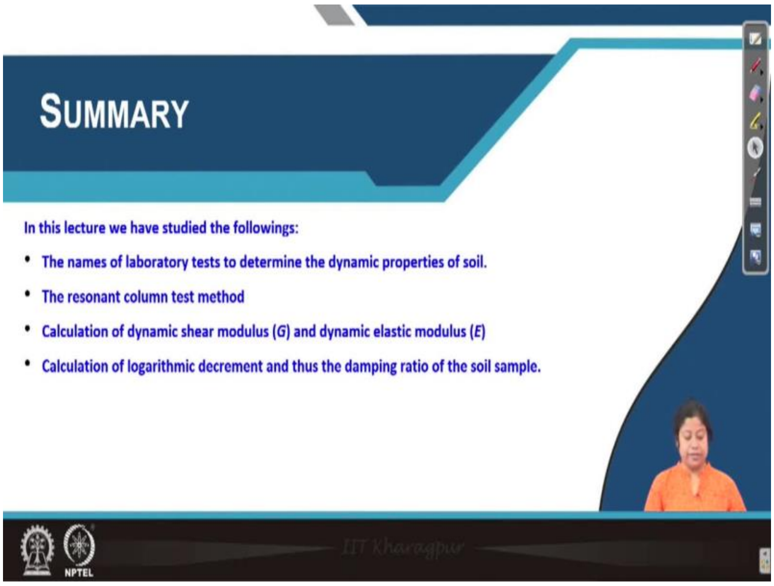

Come to the summary of today’s lecture. So, the today’s lecture, we have learnt the following things which are listed here. First we have learned the names of different laboratory tests that are used to determine the dynamic properties of soil those names are resonant column test, then piezometric bender element test, then density triaxial test then we see the name of next one was cyclic torsional shear test then cyclic direct simple shear test etc.

So, we became familiar with different names of laboratory test. Then, we the we saw, what is the difference between resonant column test and cyclic shear test in terms of the strain rate? One is low strain of course, resonant column test is the low strain test where a cyclic triaxial test is the high strain test.

Then, we have gone through the testing method for resonant column test that followed the ASTM code, D4015 then, we have learned how to calculate that dynamic shear modulus, capital G and the dynamic elastic modulus E for the soil. Then we have studied how to calculate the logarithmic decrement and thus the damping ratio of the soil sample.

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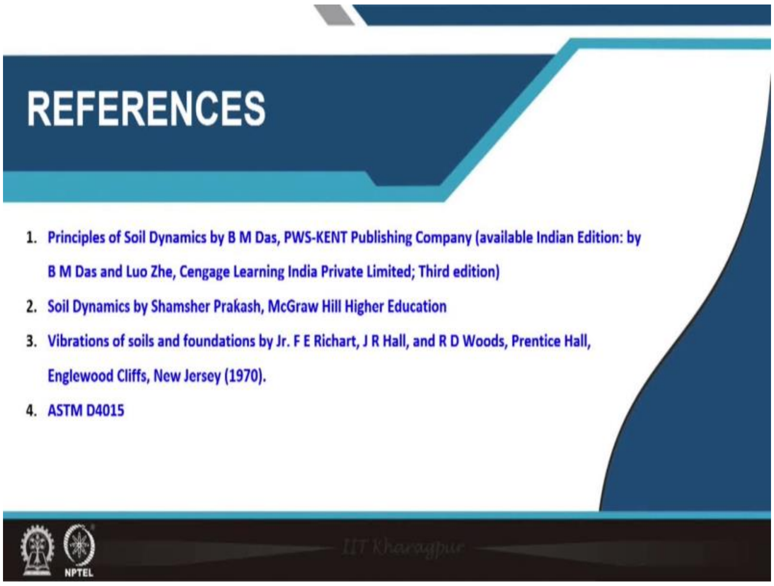

So, here is the list of references, which I have used for today's class. First three are the reference of textbooks and ASTM D4015 is used to describe the test methodology for resonant column test with this I am concluding today's class. Thank you.