**Solid Dynamics**
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**Lecture - 16**
**Multiple Degree of Freedom System (MDOF) - Part 5**

Hello, today we will continue our discussion on Multiple Degree of Freedom System. We can call it also as Multiple Degrees of Freedom System.

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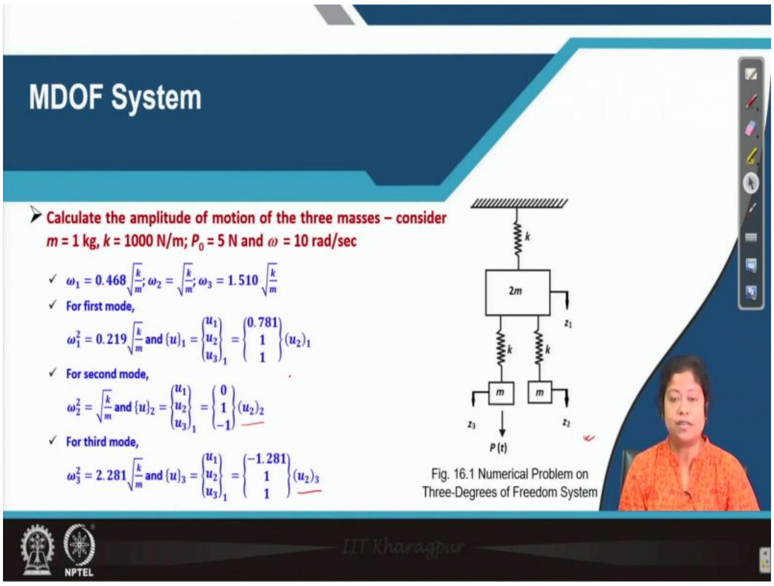


Last class two classes, we have discussed on these three degrees of freedom system where we have calculated the three undamped natural frequencies, which are omega 1, omega 2 and omega 3 for this system. Then, we have determined that u vector, u vector for the first mode, u vector for the second mode and u vector for the third mode. So, here you can see u vector means, you u 1, u 2, u 3. These are the three elements for the u vector and this subscript one outside the bracket represent the mode number here.

So, what we have determined is u vector for the three different modes in terms of u 2 1, u 2 2 and u 2 3. U 2 is the element of u vector, second element I can say. And when I am writing u 2 1 that means, the second element of the u vector for the first mode, when I am writing u 2 2 it means second element of the u vector for the second mode of vibration and u 2 3 means, second element of the u vector for the third mode of vibration. So, with this now, we will proceed our discussion.

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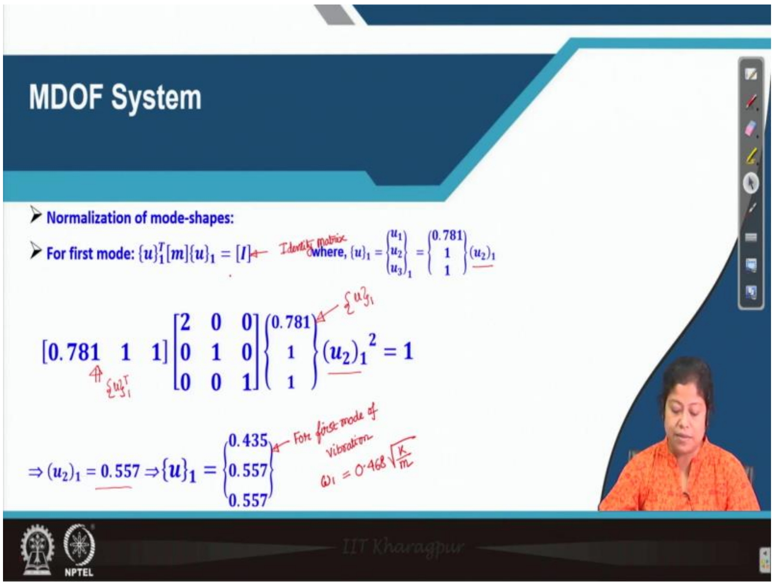


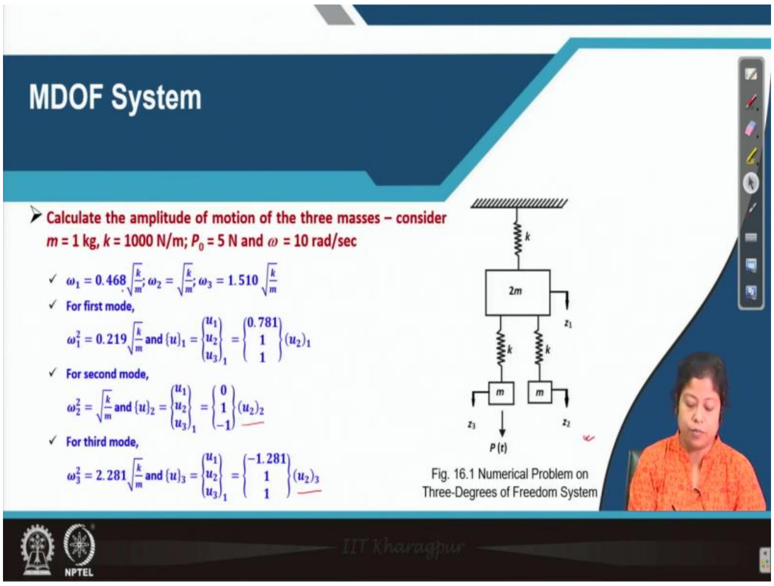


Now, the first step is normalization of the mode shape. For that what we will do? We will pre multiply u transpose to m times u vector and obviously, the right hand side will be equal to identity vector that we have already seen identity matrix. So, here we already have seen the three equations of motion for the system which we are currently choosing for our discussion. I am not repeating those parts.

So, here now, we will write u vector for the first mode in terms of the second element of u vector u 2. Then we will we can write this is actually your u transpose for the first mode, this is your u vector for the first mode and these term is coming because u vector for the first mode is represented in terms of u 2 that is the reason.

So, now, if we will do the matrix multiplication here, what we will get? We will get an expression in terms of u 2 1 square, and if we will take the square root of u 2 1, then we will get u 2 1 is equal to 0.557. Now, since we know what is the value of u 2 1 from that we can calculate u vector for the first mode.

So, this is for the first mode and that time omega 1 is equal to we can see the previous two slides, omega 1 is equal to 0.468 times square root of k by m so, 0.468 times square root of k by small a. Now, we can repeat the same exercise for the other two modes that means, for the second mode and third mode.

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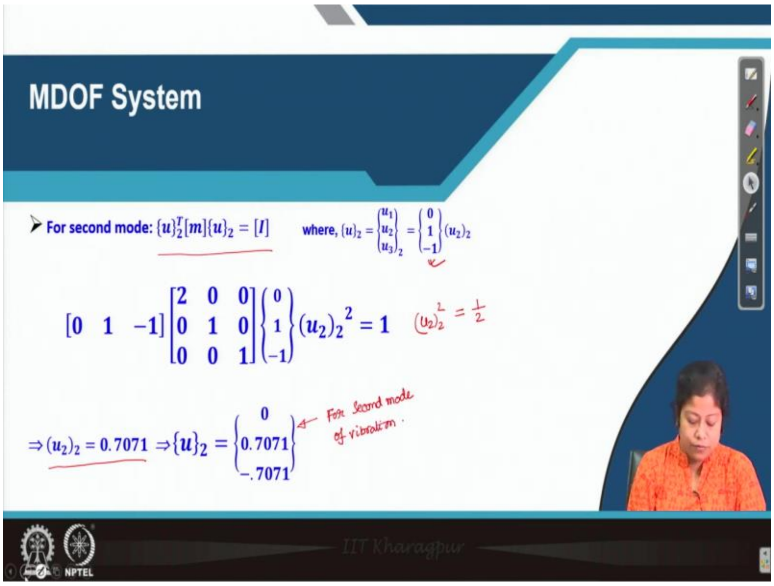


So, for the second board we will we can write this equation because the u vector for the second mode can be expressed in terms of u 2 2 like this way, where the first element is 0, second element is u 2 2 and third element is minus u 2 2. So, if we will expand this relation, we can write it this way.

From these, we will finally write u 2 2 actually if we will write here u 2 2 square we will get one by 2. Now, if we will take the square root on both sides, we will get u 2 2 is 0.7071 and from this we can write our u vector for the second mode. So, this is for the second mode of vibration.

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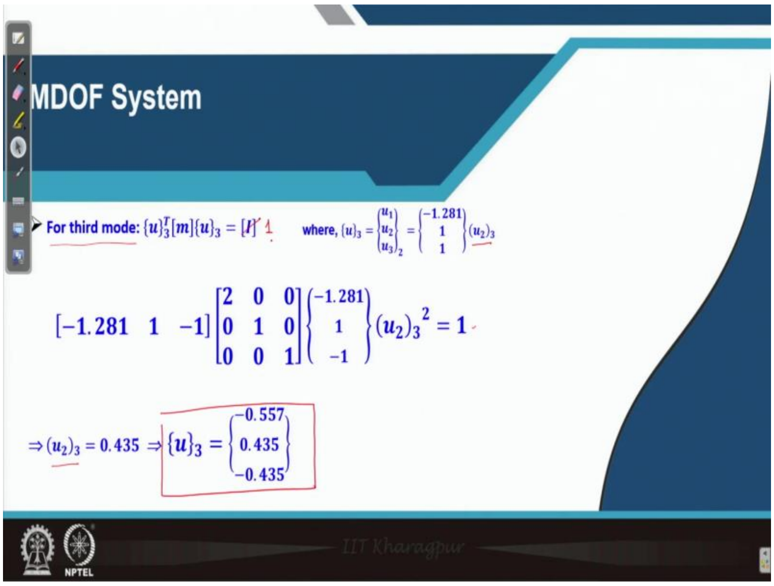


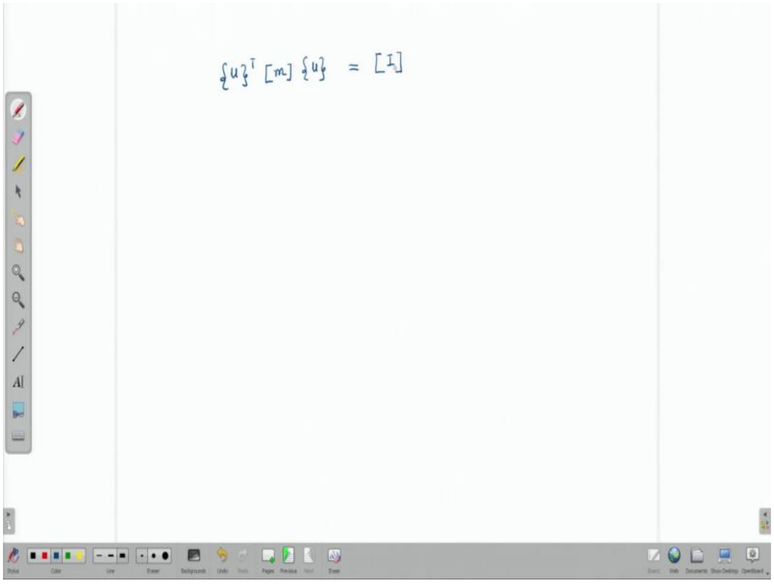


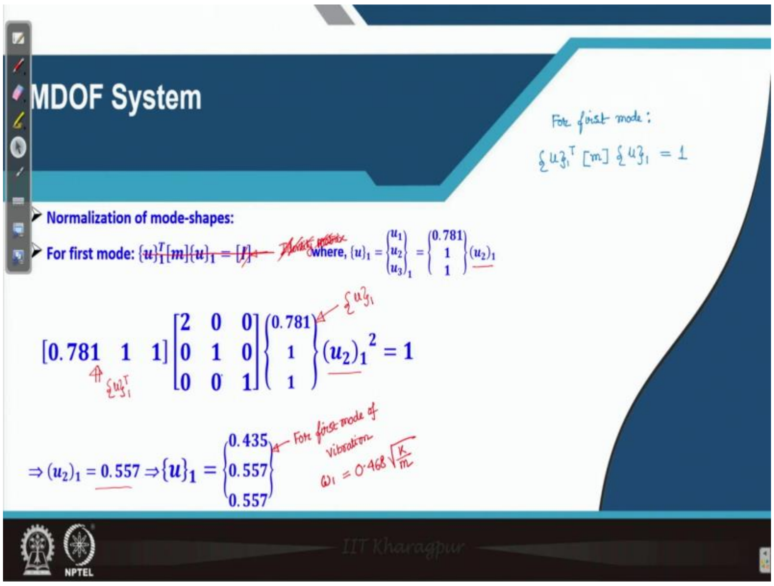

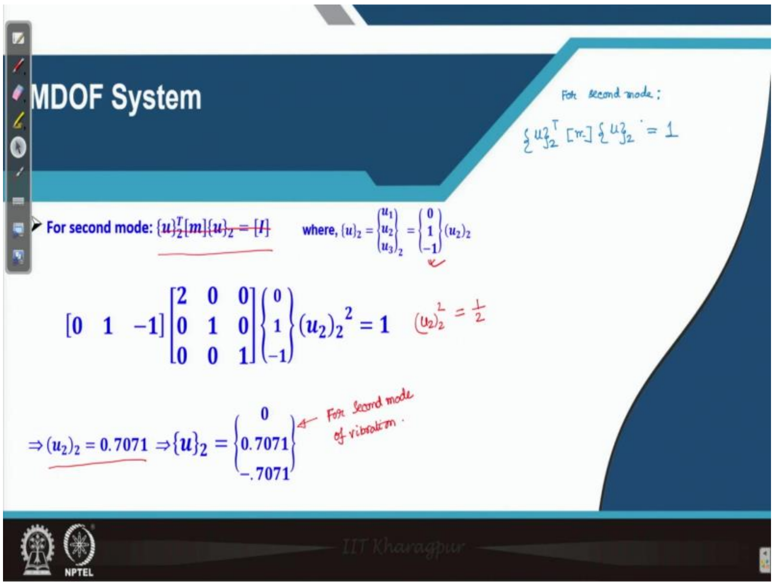


Likewise, we can write the expression for the third mode and from that we can get the first u 2 3 and from that we will find out u vector for the third mode of vibration. Here you can see that u vector for the third mode of vibration. One thing in this explanation, I actually here I made a mistake. Actually, when we are thinking about third mode, that time, if I will write the left hand side of this expression this way, the right hand side should be equal to not the identity matrix, but this will be equal to 1.

That is the reason you can see here also. And so, how I am getting it actually, when we will do this normalization. What is the expression? I am just writing the expression that is u transpose here you can see u transpose times mass matrix times u that is equal to the identity matrix. So, that is the reason now when we are thinking just for the third mode, what will be the expression? u 3 transpose times m times u 3 vector is equal to 1.

I think I made the probably the same mistake for other two modes. Just let me check yeah this is for the first mode. So, here also you please make this correction. This is not identity matrix. It is I can write it first delete me strike of this expression, what is the exact expression that I am writing here. So, for first mode we can write u 1 transpose, so, transpose of u 1 vector times mass matrix times u 1 vector that is equal to 1 and this is for the first mode.

Likewise, for the second mode, let me check yes, again here also I made the mistake, so, please strike off this expression, what I can write here for second mode I can write transpose of u 2 vector times mass matrix times u 2 vector is equal to 1. This 2 stands for the mode number here. So, when I am talking u 2 vector that means, u vector for the second mode, so, please make this correction. Now, we can go back to the third mode.

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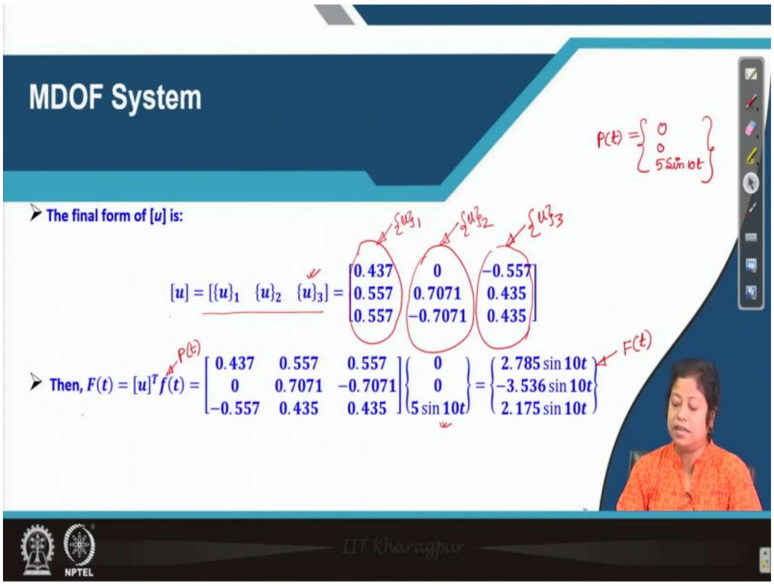


Then the final form of the u vector will be we can write this way the final form of the u here I have written it in terms of matrix. So, you can see u matrix which can be written like this way. Now, we know u 1, we know u vector for second mode, we also know that u vector for the third mode. So, basically if I will write this is our u vector for the first mode, this one is our u vector for the second mode and this one is our u vector for the third mode of vibration.

After this we will try to find out the right hand side of our equation of motion. So, that is nothing but the u transpose times f t. In our case, f t is already you can see here actually it is we can call it also as we can call it as P t not f t, so, if you recall P t for our case for the first mass, which is to m on that no external force.

So, the first element is 0, for the second mass which was on the right hand side there also no external force, for the third mass there was an external force and that can be represented by these. So, here it is P t. So, we can write then this expression this way and finally, what we are getting, I am just erasing this thing and writing. So, finally, we are getting f t so, this is our capital F t, F t vector, you can see here that it is a function of time.

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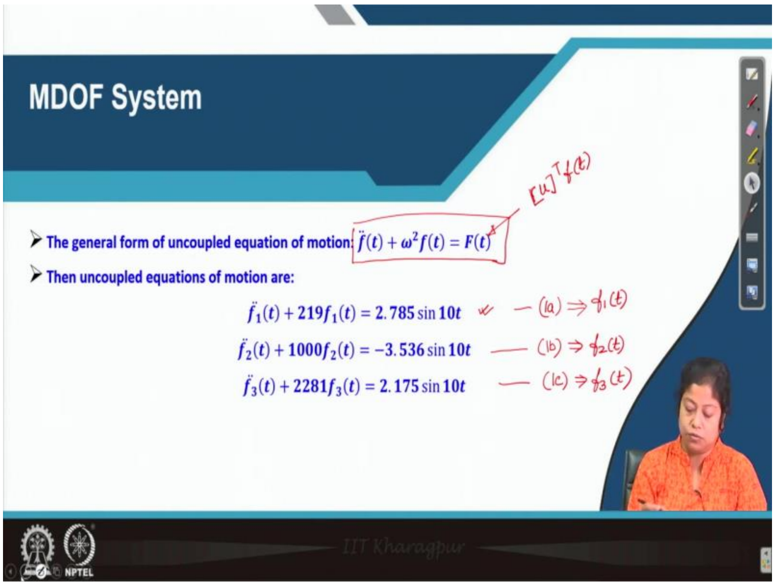


In the next step, what we need to do? We need to form the general equation for uncoupled motion or we can call it as uncoupled equation of motion. So, what is the general form of uncoupled equation of motion? Here we are the general form of uncoupled equation of motion is f two dot plus omega square if is equal to capital F, so, what is capital F t here, that is nothing but we have already seen u transpose small f t.

So, from that we will write the value or the three different elements for these capital F vector. So, let us do that exercise. Since we have three modes, so, we will get three uncoupled equations of motion here. So, this is for the, this is related to the first this is our first equation I can write it as 1 a. This is our second equation, I can write it as 1 b and this is our third equation which I can write as 1 c.

So, now, we have three uncoupled equations of motion. Now, from these from the first one we will find out f 1 t, from the second one we will find out f 2 t and from the third one we will find out f 3 t. So, how we will find it out? We know if you recall what we have learnt in single degree of freedom system there also we have solved the uncoupled equations. So, similarly, here also we can solve that uncoupled equation.

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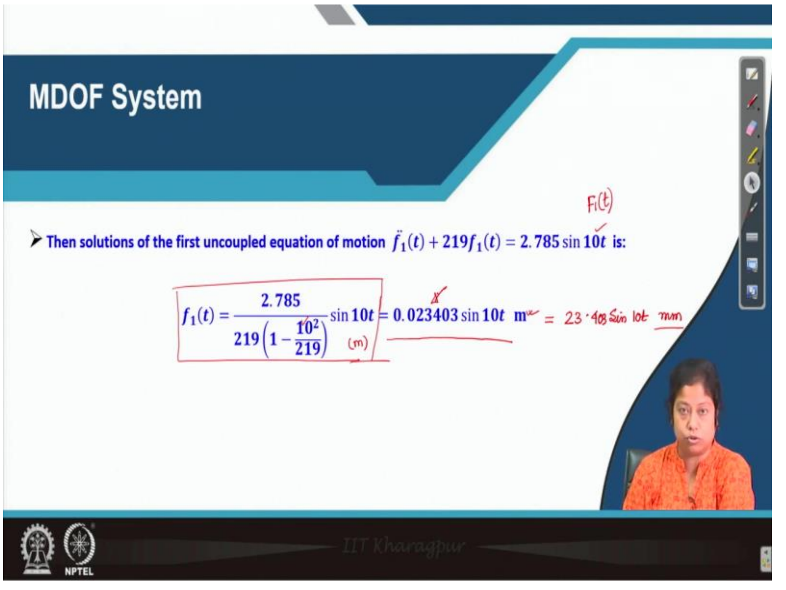


So, for the first uncoupled equation, this one we will get f 1 t. So, f 1 t can be expressed by this form. So, this 10 is related to the sorry it is not P 1, it is now F 1 t, capital F 1 t. So, from this we can find out f 1 t and this f 1 t has a unit which is in meter. So, final expression for f 1 t if I will do the calculation, I will get the 0.02, I have taken here up to 6 decimal place, the reason is that it is in meter. Now, if I will express it in millimeter, how much it will be?

It will be 1, 2, 3, so, I can get 23.4 sin 10 t in millimeter, that is the reason I have taken or I can write it like this. Generally, if you recall I said that we will take up to two decimal place or three decimal place and n generally for the displacement we for this type of problem we use, we get the magnitude of displacement in millimeter, there is a reason when I will experience it in millimeter that time I will multiply it by 1000.

So, automatically now the number of decimal place will be reduced and I due to that reason I have taken when I am expressing these in meter up to 6 decimal place so that when I will express it in millimeter I will be able to finally write the answer up to 3 decimal place, it is just a matter of choice also I can say.

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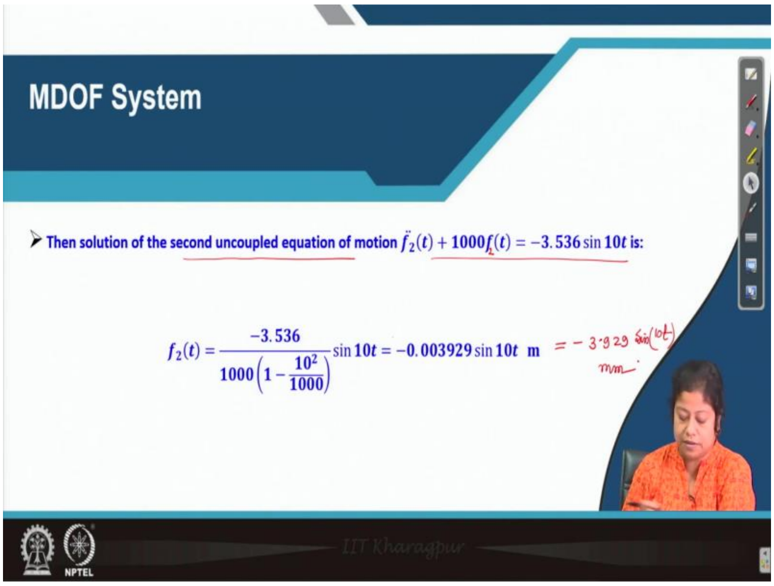


Now for the second uncoupled equation of motion, you can see what will be the final solution? It will be f 2 t is here 2 is missing please correct it, it is there is one f 2 instead of f here, so, the solution is if 2 t is equal to this, so, when I am writing it in millimeter, it will be minus 3.929 times sin, I do not need to write it in. So, what I will do? I will write it as sin of 10 t. So, this is in millimeter.

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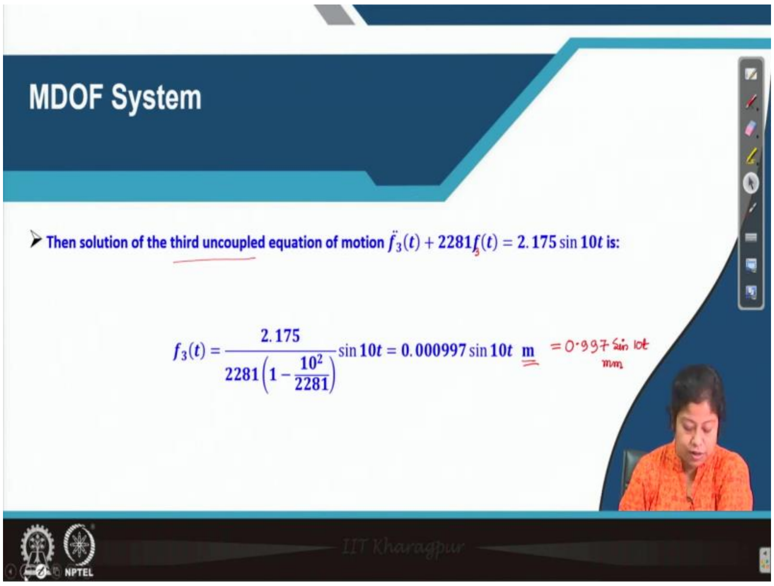


Likewise, for the third mode of vibration, we can get third uncoupled equation of motion here also it is f 3 t then what will be the final solution for f 3? The final solution for f 3 t is this. This is written in meters so, I can express it in millimeter. In millimeter, it is 0.997 times sin of 10 t it is in millimeter. So, in this way, we can solve the three uncoupled equations of motion which is shown here.

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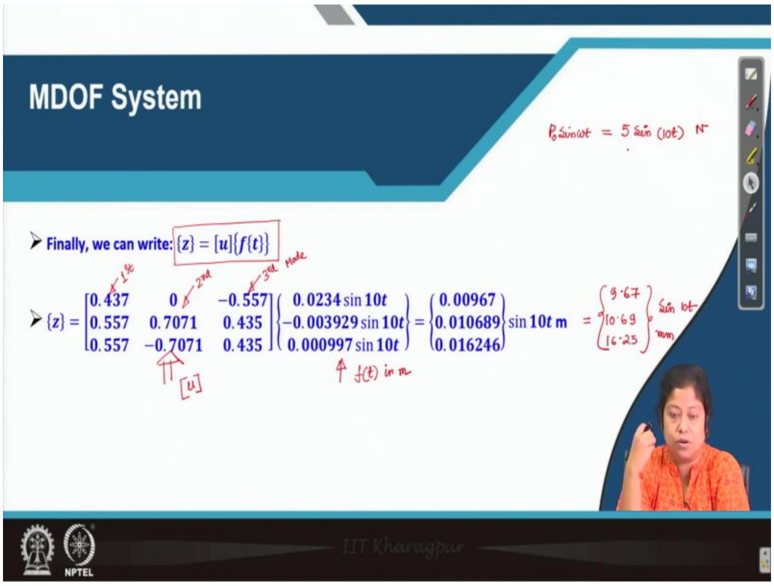


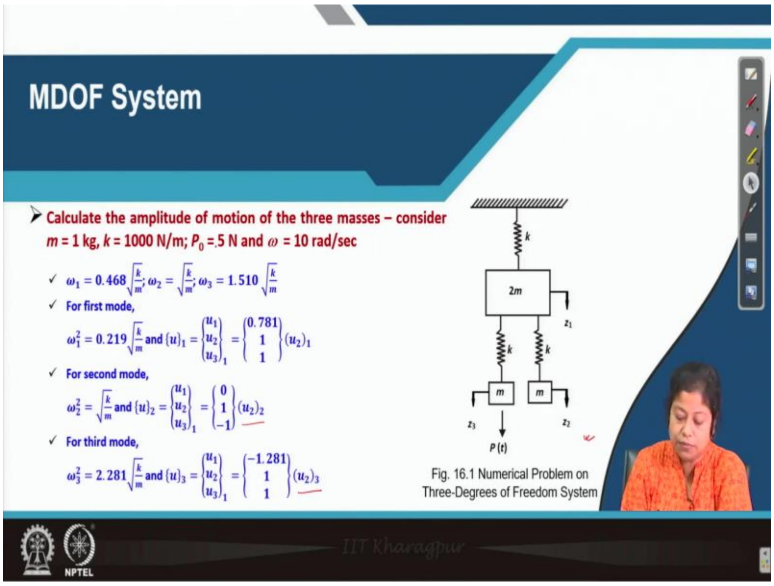


And then finally, our objective is to find out the response for the multiple degrees of freedom system subjected to force vibration. So, because of the force vibration, what will be the final solution or response that we can get from this relation if you recall first class or second class we have assumed actually this and then we have done a lot of exercise also.

So, that relationship will be used here. So, we can write first one is this is our u matrix. So, this is related to the first mode. This is related to the second mode and this is related to the third mode. This is our f t vector. Just now we have calculated it. So, here I have expressed all the elements in millimeter sorry in meter.

So, the final solution I get in meter. You can express it also in millimeter. So, if we will express it in millimeter, what it will be, 9.67. This element will be 10.69. And this one will be 16.25 times sin of 10 t and it is in millimeter. Here I have in final answer I have taken I have expressed the value up to two decimal places.

So, this is that this is all about the three degrees of freedom system subjected to forced vibration and we need to follow the steps which are shown here to solve the to get the response for such kind of multiple degrees of freedom system. Now, in our most of the discussion, we have taken for forced vibration basically, we have the expressed the force as a harmonic function.

Even in this case, if you recall the external force which was acting on the mass m on the left hand side that was P 0 sin omega t or I can write the magnitude of P 0. Let me see one thing, what was the unit, P was in Newton, you can see here also, so, this is a Newton. So, this is a harmonic motion what will be happened, sinusoidal force this is of course, so, what will be happened if the external force is not sinusoidal if it is a kind of impulse loading, let us see that.

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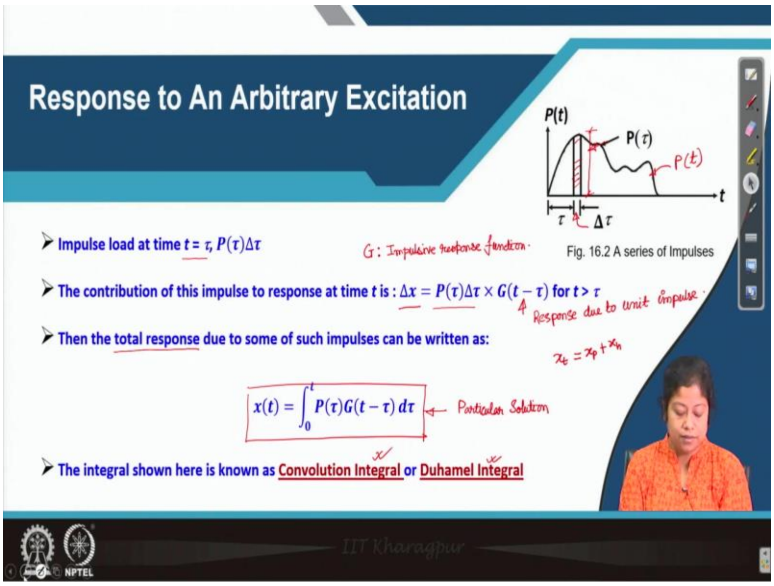

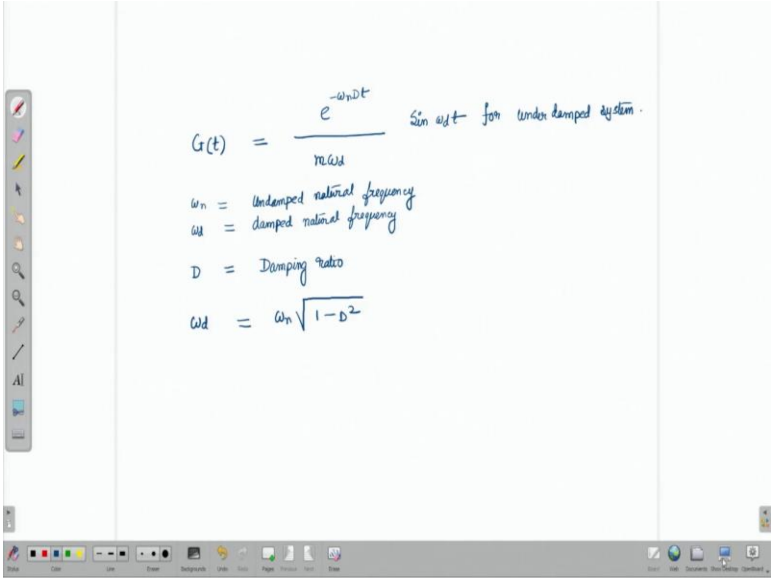


So, in this figure what we can see? This function P t, this is function actually P t and this is P tau just let me correct this is P tau and this is our function P t. So, what we can see here? Here we can see at different time for an example, at time t is equal to tau there is an impulse of magnitude t which is a function of tau here. In such cases, what will be the response? Now, this is for one impulse.

There is a possibility of a series of impulse like this. So, in such case what will be the total response that we will see now. So, impulse load at time t is equal to tau that means, this is the impulse load, how much it will be? It will be if the magnitude is P, if the magnitude that means, this actually is P tau, I am just correcting this figure.

So, this is P tau then the impulse load at time t is equal to tau we can write P tau times this delta tau, delta tau is this width of the strip. Then what will be the response only for this much impulse? The response is delta x which is equal to P times tau times delta t that means, the impulse load at time t times G times t minus tau, what is this? This is the response due to unit impulse. So, G t minus tau is the response due to unit impulse.

Now, we have impulse of this much so, the response because of this much impulses P tau times delta tau times response due to unit impulse which is G. We can call it also as for G we can call it also as impulsive response function. So, this is for only this much impulse which you can see here, then what will be the total contribution of this total impulse? So, for that we need to do the total impulse also.

Then what we can do if we will integrate these small small contribution due to this small small impulse or individual impulse, then we can get the total impulse sorry total response and that is integration of this tau which you can see here. So, we are integrating this to get the total response. So, in this case the in this integration is called actually the Convolution integral or Duhamel integral.

Here one thing I should mention that the response which we are noting is because of the external load only. Now, for any such kind of motion actually, total response has two components, one component is called complementary function for the homogeneous differential equation and the second part is called the particular solution. So, this response is part of a particular solution of the equation of motion, this is related to the particular solution.

So, I can write here also as so, if we are asked to find out the total response, so, total response will be actually particular solution plus solution of homogeneous equation. So, I am writing it as x h. So, in this way, we can calculate that total response due to some impulses which is P t here. One important thing here how we can write this G function?

So, depending upon the system, we can define the impulsive response function, if we are working with let us I can write here, if we are working with the under damped system, then these function will be e to the power minus omega n D t divided by mass times omega D times sin. Here I can write it as a function of time only I am writing impulsive response function as a function of time only.

So, it is e to the power minus omega n D t divided by omega D times mass times sin of omega D t. So, we all know what is omega D? Omega D is damped natural frequency, omega n is undamped natural frequency and capital D is damping ratio or damping factor and if we know omega n and capital D then omega D which is damped natural frequency is equal to omega n times square root of 1 minus b square.

All these things we have studied already so, I am just writing here for your understanding. So, here G t is for under damped system. So, I conclude today’s class.

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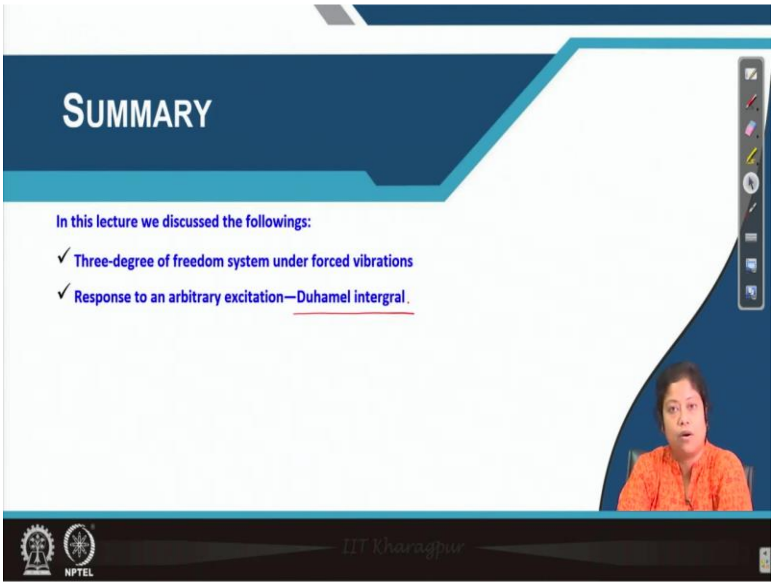


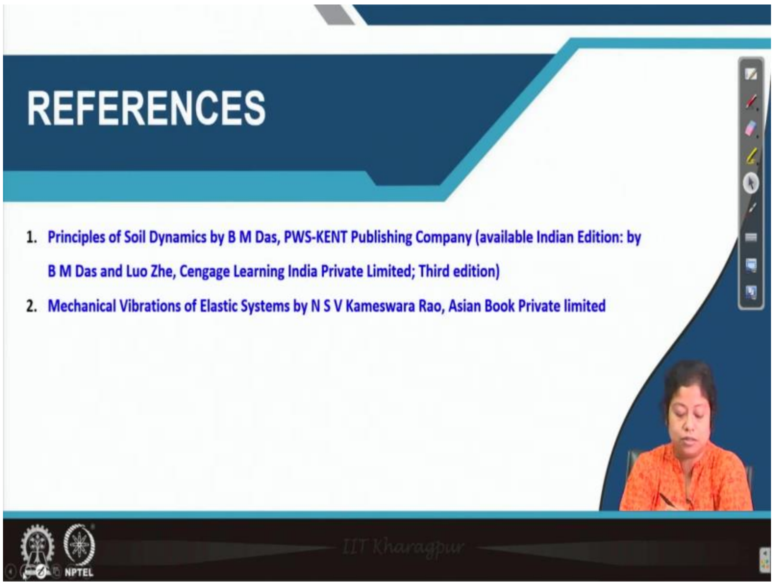


Today first we have the completed our discussion on three degree of freedom system under forced vibration. We have studied the response to an arbitrary excitation using Duhamel integral. For today's discussion, I have used these two references. Thank you for attending today's class.