Soil Dynamics
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Lecture 14
Multiple Degree of Freedom System (MDOF) - Part 2

Hello friends, today we will continue our discussion on Multiple Degrees of Freedom System. So, far we have studied the definition of multiple degrees of freedom system, then how to get the Eigen values and eigenvectors for the multiple degrees of freedom system. For this purpose in last class we have taken one example of two degrees of freedom system which was under free vibrating condition.

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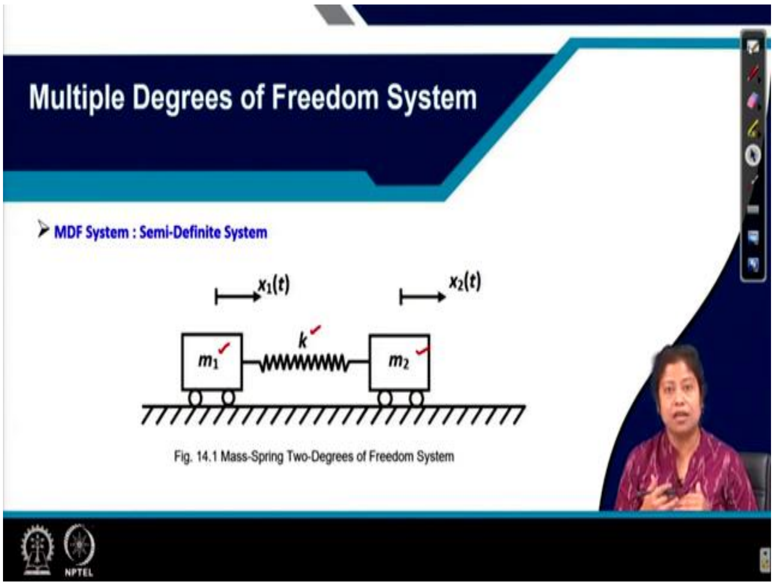

So, today we will extend the, our earlier discussion for two degrees of freedom system. So, in this case what we can see there are two masses, one is M 1 the other 1 is M 2, so M1 and M 2 are attached by a spring K, mass M 1 is subjected to are displacement x 1 in the horizontal direction, likewise, mass M 2 is also subjected to our displacement x 2 in horizontal direction, both x 1 and x 2 are time dependent.

So, these kind of system is called as semi definite system, what is the specialty of this kind of system that we will now see. So, in this case what we can see, first we can in order to know the Eigen values that means the natural frequencies for this type of multiple degree of freedom system, first what we need to do, we need to draw the free body diagram for M 1 and M 2. And then we will form the equation of motions for M 1 and M 2 respectively.

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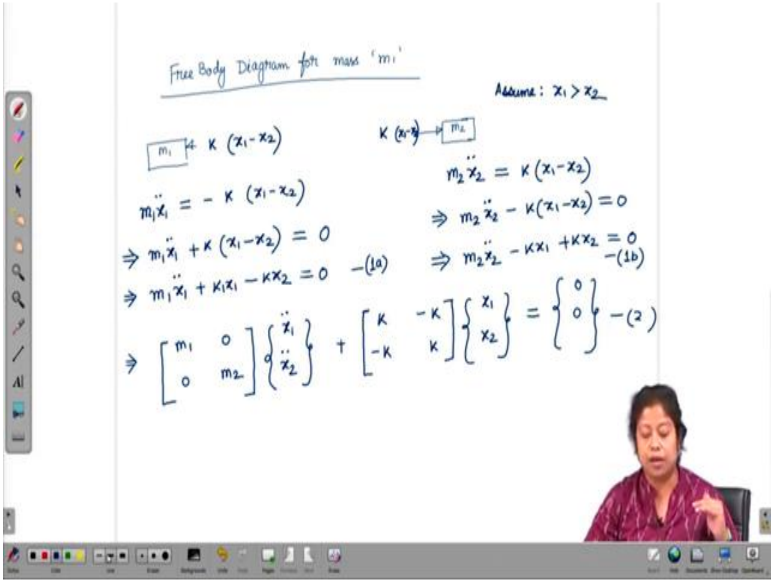

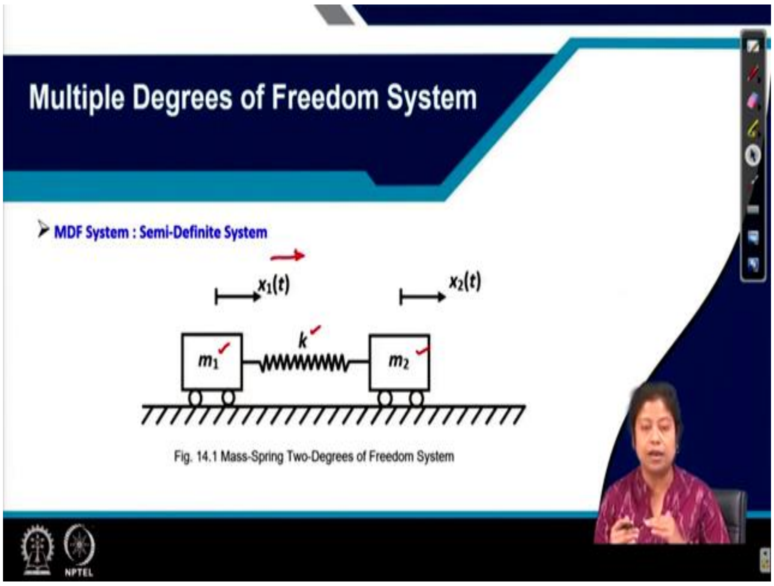

So, let us draw the free body diagram for M 1 and M 2. So, first I am drawing give me 1 minute, first I am drawing free body diagram for mass M 1, this is the first job for us. So, what we have seen in the given problem M 1 is subject to our displacement x 1 which is a function of time t.

Then what will be happened if M 1 is moving in positive d x direction, then it will push the spring k and as a reaction spring will also exert force to M 1, so that spring force on M 1 will be how much, it will be M 1 sorry it will be K 1 times x 1 minus x 2. In this case what I have assumed? I have assumed x 1 is greater than x 2.

If now I will draw the free body diagram for the mass M 2, then what will be happened here, in this case spring K will exert a force on M 2 in positive x direction. So, we can write here K 1 times x 1 minus x 2, x 1 minus x 2 is the relative displacement of the spring, or yes spring. Now, we will form the equation under equilibrium condition for M 1 and M 2 what we can write for M 1 we can write inertia force should be equal to the unbalanced force here.

So, in this case for mass M 1 inertia forces M 1 times x 1 two dot and for your system for mass M 1 what is the unbalanced force unbalanced force is K 1 times x 1 minus x 2. But its direction is opposite to the positive direction. So, I will write here minus K1 times x 1 minus x 2. Then what we can write here M 1 times x 1 two dot plus K 1 or I can I have taken only K, so I will write here in in place of K 1 only K I will write, so, these will be only K.

So, now come to the equilibrium equation for mass M 1, this will be equal to this will be M 1 times x 1 two dot plus K times x 1 minus x 2 is equal to 0. So, we can write it also as M 1 times x 1 two dot plus K 1 x 1 minus K times x 2 is equal to 0, likewise, for the mass M 2 we can write inertia force is equal to the unbalanced force.

So, in this case for mass M 2 inertia force is M 2 times x 2 two dot that should be equal to the unbalanced force which is acting in this case in positive x direction. So, I can write here as unbalanced force is equal to K times x 1 minus x 2. So, from these we can write the next line as these and finally the equation of motion for mass M 2 is M 2 times x 2 double dot minus K x 1 plus K times x 2 is equal to 0.

So, now we have two equation, one a and one b, so we can write these two equation together as M 1 0 0 M 2 this is our mass matrix times acceleration vector plus what we can write now stiffness matrix, so in this case stiffness matrix will be K minus K next row minus K and plus K times displacement vector, so x 1 and x 2 and right hand side there is a null vector. So, this is our equation of motion for the mass M 1 and mass M 2.

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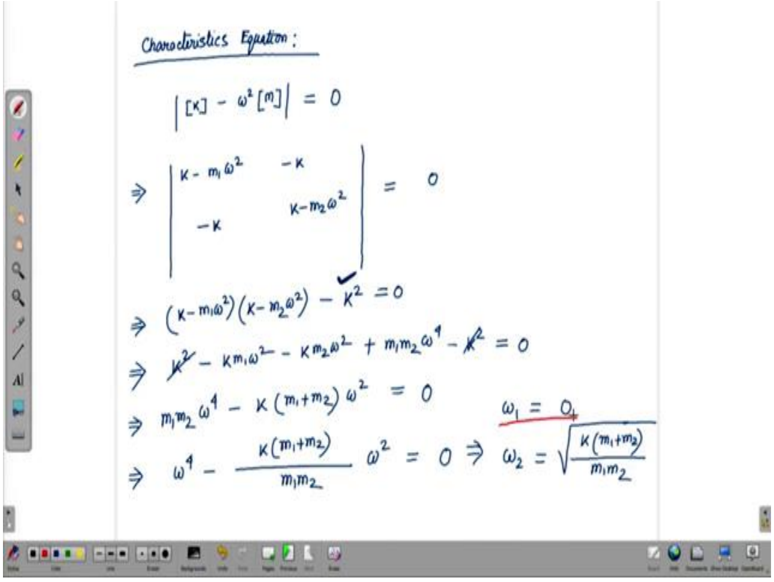

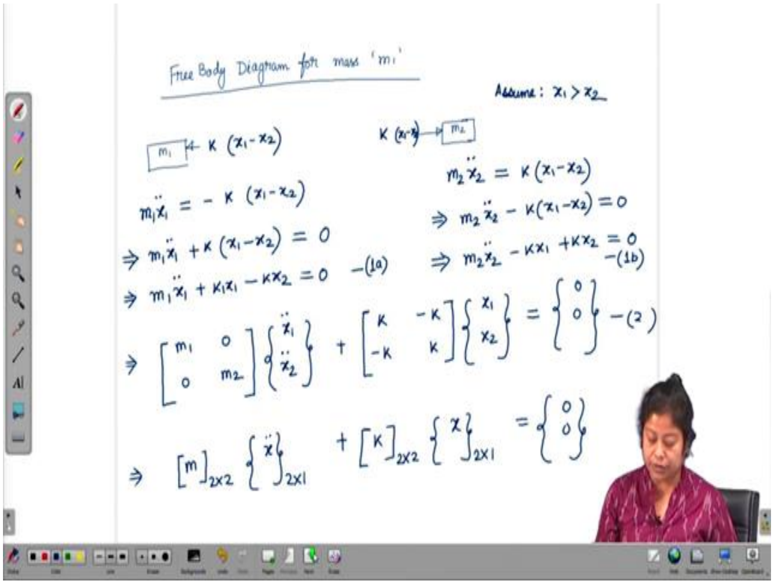

Now, already we are familiar to the characteristics equation, so for non-trivial solution we have learned that the characteristics equation will be 0. So, write the characteristics equation for this equation I am just going back to the previous page. So, in equation 2 what we get that we can also write as small m matrix which is mass matrix in this case times acceleration vector plus stiffness matrix which we another 2 by 2 times displacement vector that is equal to null vector, we can write it this way also.

So, now we will write the characteristics equation for equation 2. So, characteristics equation will be then stiffness matrix minus omega square times mass matrix determinant of this should be equal to 0, this is true for non-trivial solution for equation 2. Then we can again expand it and can write it as K minus M1 omega square minus K minus K in the second row first element and then K minus M 2 omega square which is the second row second element and that should be equal to 0.

So, from these what we can get K minus M 1 omega square times K minus M 2 omega square minus K square is equal to 0. Now, we can write it also as K square minus here I will get k M 1 omega square minus K M 2 omega square plus M 1, M 2 omega to the power 4 is equal to 0, there is another term minus K square which we can see here, so I need to write this minus K square here itself and then if left hand side is equal to 0 on the right hand side.

So, now what we can see here, the final expression will be then M 1 times M 2 times omega to the power 4 minus K times M 1 plus M 2 times omega square is equal to 0, or we can also write it as omega to the power 4 minus K times M 1 plus M 2 divided by M 1 M 2 whole thing is multiplying by omega square is equal to 0.

Then now what we can see there should be 4 there are 4 roots of the omega, 2 roots are negative and 2 roots are positive, we are here interested only for the positive roots. So, what are those positive 2 roots? One will be omega 1 which is equal to 0 and the other one is omega 2 which is equal to square root of K times M 1 plus M 2 divided by M 1 M 2. So, these are the two non-negative roots for this characteristics equation or omega.

So, what we can call these two omegas? These are the natural frequencies. Now, the interesting thing which we can note here is omega 1 is equal to 0, what does it imply, it implies that system is not oscillating.

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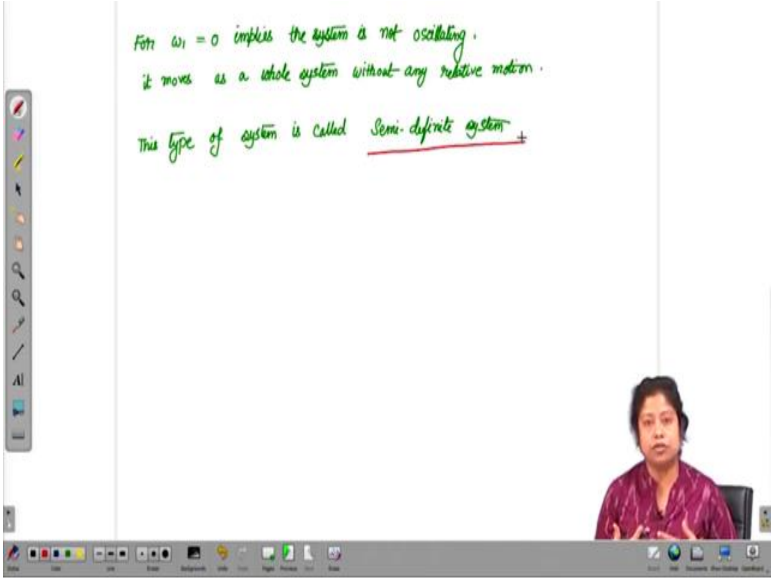

So, for next page we can write then for omega 1 is equal to 0 implies the system is not oscillating and what else it implies, it also implies that the system moves as a whole body without any relative motion that means as a rigid body. So, these type of system I just write the next line it moves as a whole system, so these type of system is called semi definite system, likewise, we can see similar type of semi definite system for three degrees of freedom system as well.

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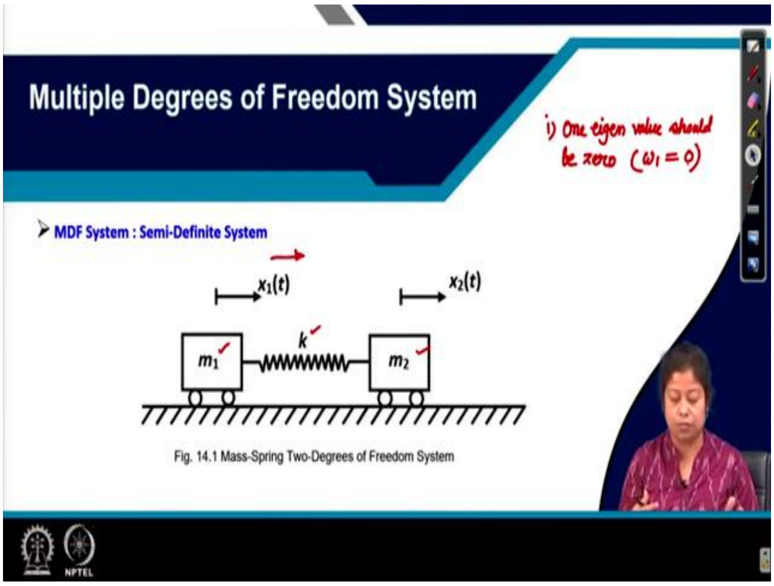

So, here also I can write the characteristics of semi definite system first one is one Eigen value should be zero, one Eigen value should be zero in this case that is omega 1 is equal to

0. Then what will be happen the whole system will move as a whole system without any kind of relative motion between the masses M1 M2.

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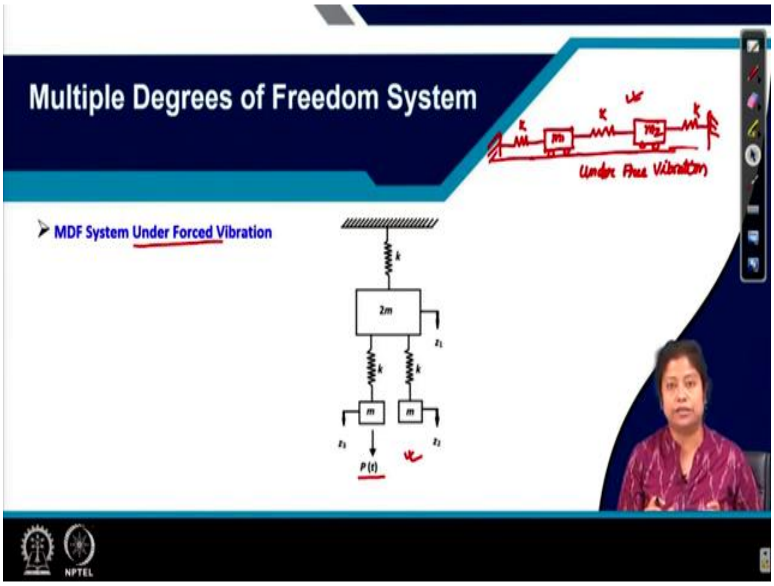

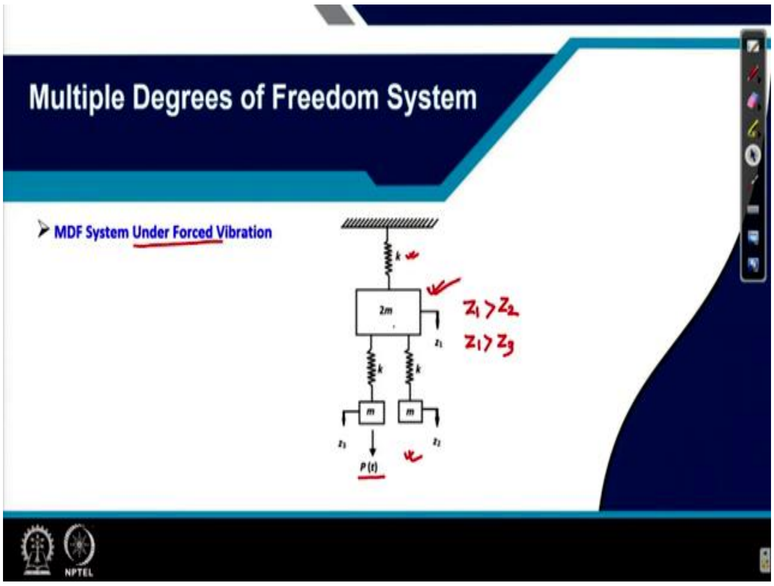

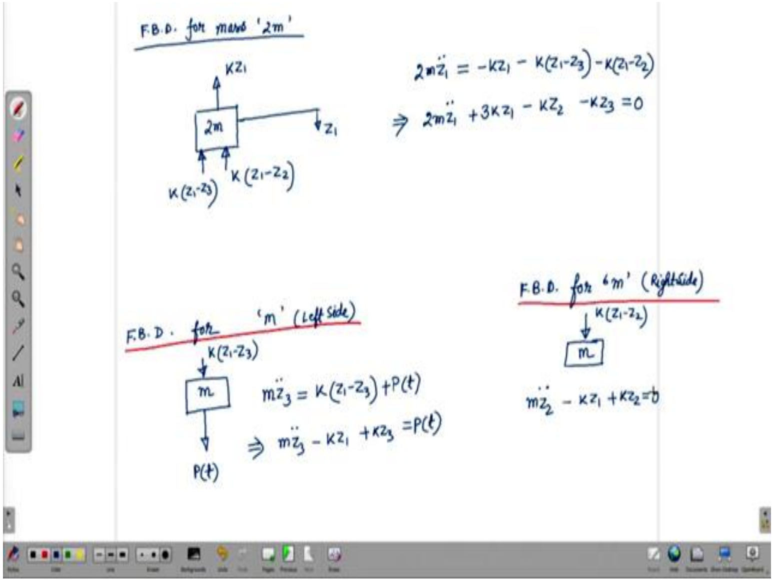

Now, now next is a three degrees of freedom system, now last time also we have studied multiple degrees of freedom system, just now we have seen one, two degree of freedom system. So, where is the difference between that or this system or if you recall last class we have solved a system like this where 2 masses were connected by springs something like this, this type of system we have solved in last class may be something like this.

So, now where is the difference between our this system which we saw last class and the system which you can see right now, the difference is that current system is subjected to an external force or vibration which is p t function of time and last class what we have solved that was under free vibration. So, this was under free vibration and this problem is under forced vibration, so this is the difference, I can erase this figure.

So, for these type of problem how we will find out the response that is our current concern. So, for that first we will solve the characteristics equation to find out the Eigen values for this system. So, let us start to solve the to find out the Eigen values for this system. So, first step will be to draw the free body diagram for mass 2m, then for mass m and then finally there are two masses of magnitude m.

So we will draw free body diagram for the three masses shown here. So, let us start with mass 2m that means first for this mass, this is our mass m 1 or I can write it as 2m also so 2m and displacement for these is z 1 from the equilibrium position. So, I can show it other way also, so it from equilibrium position it is subjected to displacement z 1.

So, in this case we are considering z 1 is greater than z 2 and z 1 is also greater than z 3, then what will be happened on mass 2m, the spring K this one will exert a force on this mass. Similarly, the other two springs connected at the bottom of these mass will also exert force and the magnitudes of these spring forces depend upon the relative displacement of the springs.

So, let us take first one, so here I will draw the free body diagram for this mass force is in this case for mass 2m if you see the spring K which is connected at the top is subjected to a displacement z 1 only. So, here sorry it is not K 1 only k spring force is K z 1, now what about the bottoms 2 springs those 2 Springs will exert also forces on these mass and that force is K times z 1 minus K times z 1 minus z 3 and this one will be K times z 1 minus z 2.

So, this is the free body diagram for mass 2m. Likewise, we can draw free body diagram for mass m left side and also the FBD for mass m on right hand side. So, let us do that, so in this case it is p t and these force is K times z 1 minus z 3, this is mass m, likewise, for other mass it will be m I should draw it on that side. Now, we can write then the equation of motion for these three masses.

So, for mass 1 it will be or first one the equation of motion will be inertia force is equal to minus of k z 1 minus K times z 1 minus z 3, sorry minus K z yes I am writing it correctly minus K times z 1 minus z 2. So, I can rearrange entire thing on the and take everything on the left hand side, then it will be plus 3 k z 1 minus k z 2 minus k z 3 and that is equal to 0, same way I can write the equation of motion for left hand side mass of magnitude m.

Then it will be m times z 3 two dot is equal to K times z 1 minus z 3 plus p t because this mass is subjected to an external vibrating force. So, we can write it as also m z 3 two dot minus k z 1 plus k z 3 is equal to p t. Similarly, I can write the equation of motion for the mass in on the right hand side and I am directly this time writing the equation of motion for this mass it will be m times z two dot minus K times z 1 plus K times z 2 is equal to 0 on the hand side.

So, these are the three equations of motions which we need to solve to get the Eigen values for this system. We will continue this lecture in next class and that time we will find out first the Eigen value, then the eigenvectors for this problem. Thank you for today.