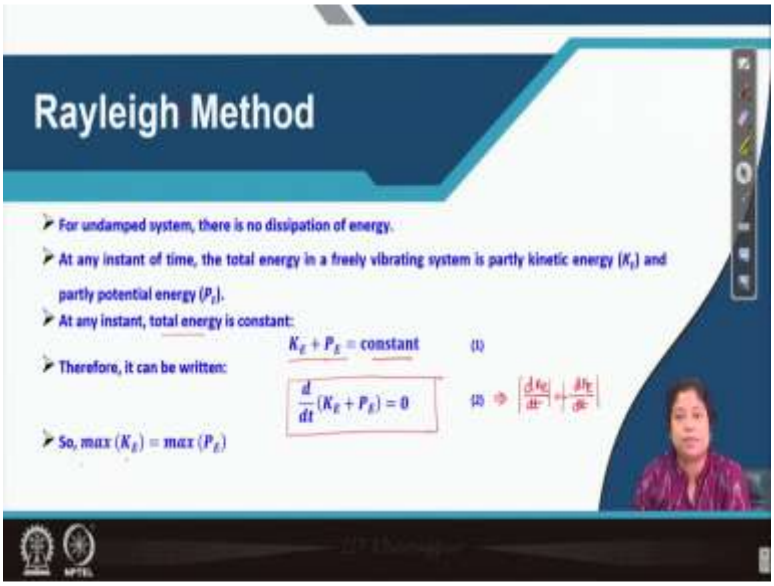
Soil Dynamics
Professor. Paramita Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture - 10
Single Degree of Freedom System (SDOF) - Part 8

(Refer Slide Time: 00:31)

Hello friends, today we will discuss one new thing. So far, we have studied one method to calculate the natural frequency or I can say undamped natural frequency of single degree of freedom system.

(Refer Slide Time: 00:55)

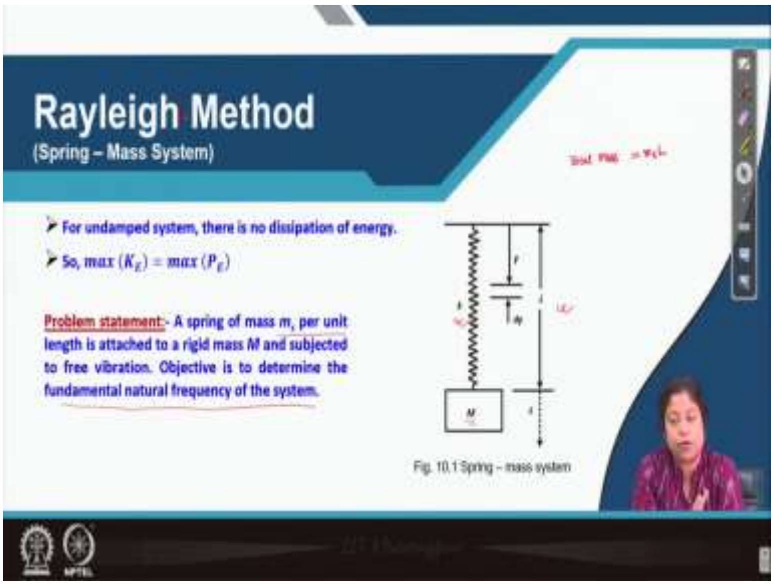


So, today we will say another method which is called Rayleigh method, I can write it here as Rayleigh's that sounds good. So, in this method basically it is said that there is no dissipation of energy from the system. Then, if there is no dissipation of the energy, the total energy should remain constant. Now, in this method, it is also said that at any instant of time the total energy in a freely vibrating system is partly kinetic energy and partly potential energy. That means what? That means, total energy is the sum of kinetic energy and potential energy.

So, then what we can say, from these two statements, we can say total kinetic energy and potential energy is constant or we can say total energy which is sum of kinetic energy and potential energy is constant. Let us give it as a number 1. Then if we differentiate this equation 1 with respect to time, then that will be 0. That is the reason in equation 2 what we can say d of dt of this term KE plus PE which is the total energy that is 0 or I can say in other words, differentiation of total energy that is the sum of kinetic energy and potential energy with respect to time is 0, obviously, because, you can see here total energy is constant.

Now, when we have written equation to from that we can conclude that maximum kinetic energy is equal to maximum potential energy, because what equation 2 says; differentiation of KE with respect to t is equal if I consider the magnitude here. Now, if you will take the magnitude, we can write it like this way. So, from this we can say maximum kinetic energy is equal to maximum potential energy.

(Refer Slide Time: 04:20)



Now, let us take an example. It is an interesting problem, because so, far we have calculated the undamped natural frequency of spring-mass system. What do we have considered? We have considered that the spring is massless. Now, we will consider that the spring which you can say here which is attached to a rigid mass capital M. So, this spring is not massless anymore for this problem, we have considered the mass of the spring which is a ms per unit length.

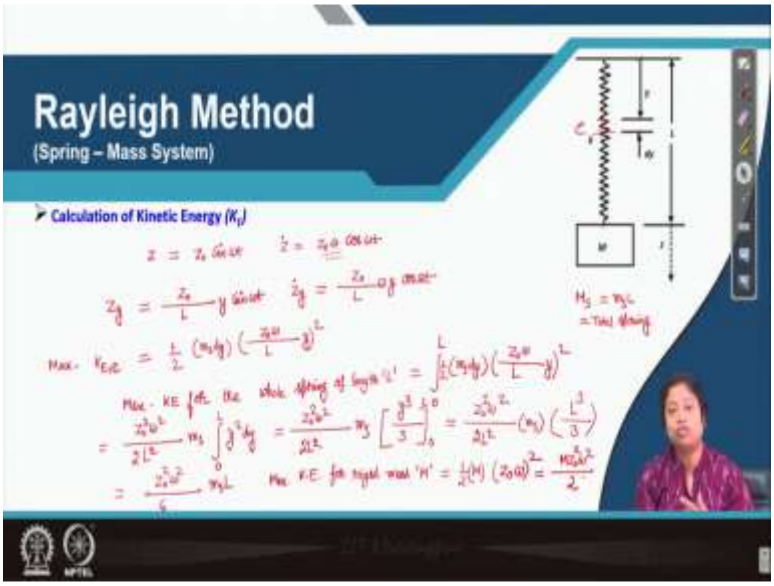
Now, for undamped system, what Rayleigh's method says? It said that there is no dissipation of energy. So, maximum kinetic energy should be equal to maximum potential energy. So, basically when kinetic energy is maximum potential energy is 0 when potential energy is maximum kinetic energy is 0.

Now, with this we will try to find out the fundamental natural frequency of the system, which you can say in figure 10.1. I am just reading the problem statement, what is said here is spring of mass ms per unit length. So, the total length of the spring is capital L that means, the total mass of the spring is how much it is small ms times capital L. So, this spring is attached to a rigid mass capital M and subjected to free vibration. Now, the objective is to determine the fundamental natural frequency of the system.

So, I want to say something about Rayleigh's method before finding out the fundamental natural frequency for this particular system. The, using the Rayleigh's method that means, writing maximum kinetic energy in the system is equal to the maximum potential energy we can get the natural frequency of the undamped system subjected to free vibration, but that natural frequency we, is the fundamental natural frequency.

That means, a system may have an in finite number of natural frequencies if we consider the distributed load or mass for the system, however, fundamental natural frequency is the lowest natural frequency. So, using Ryleigh's method, we will find out the lowest natural frequency for the system.

(Refer Slide Time: 08:12)



So, let us start it. First, we need to calculate the kinetic energy for the system. So, for this I need this figure. So, here what we can do first we will write the displacement z. So, the displacement z at the tip of the spring that means, here is how much it is, it can be written as z0 times sine omega t. Then, what will be the velocity? Velocity will be z dot is equal to z0 omega times, it will be cosine omega not sine times cosine omega t.

Now, for this if we consider one small element at a length y from the starting point of the spring then and the thickness of the element is dy, then what will be the displacement? At this point, it will be zy, which is equal to z0 divided by L times y times sine of omega t. Likewise, the velocity at that point will be z0 divided by L omega times y times cosine omega t. So, now we have velocity at this point.

Then, what is next objective? Next objective is to find out the kinetic energy for the element that means, for this portion. So, how what will be the kinetic energy for that element, I can give it a name small e that will be equal to half of mass, its length is dy. So, ms times dy is that total mass for this element spring element I can call here times velocity square. So, z0 omega by L times y whole square this is the maximum kinetic energy for the element dy.

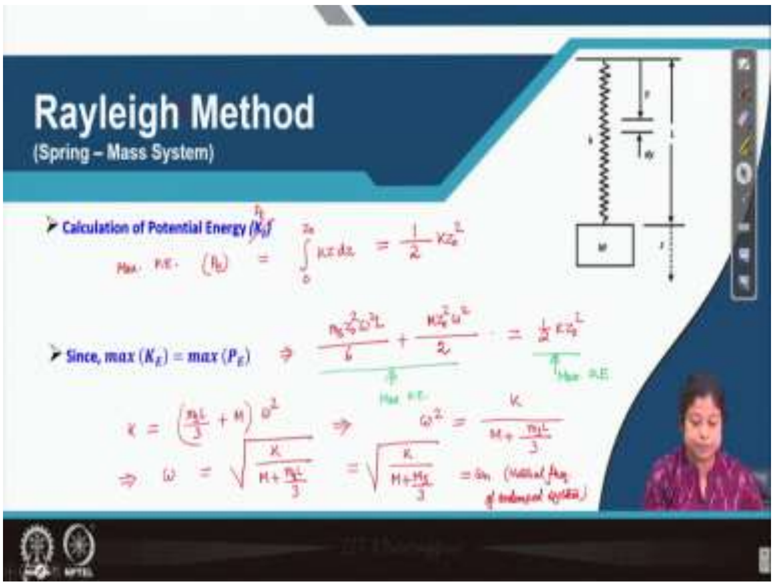
Now, I can get the total maximum kinetic energy for the entire spring for the whole spring of length capital L that will be. How we will get it? If I will integrate this expression. So, ms times dy z0 omega by L times y whole square and this is the limit of this integration from 0 to capital L. So, that can be rewritten as I can take out the constant term outside of this integration so, it will be y squared dy integration of y squared dy with limit 0 to L. So, finally, it keeps. One half is missed here. So, I need to write half also.

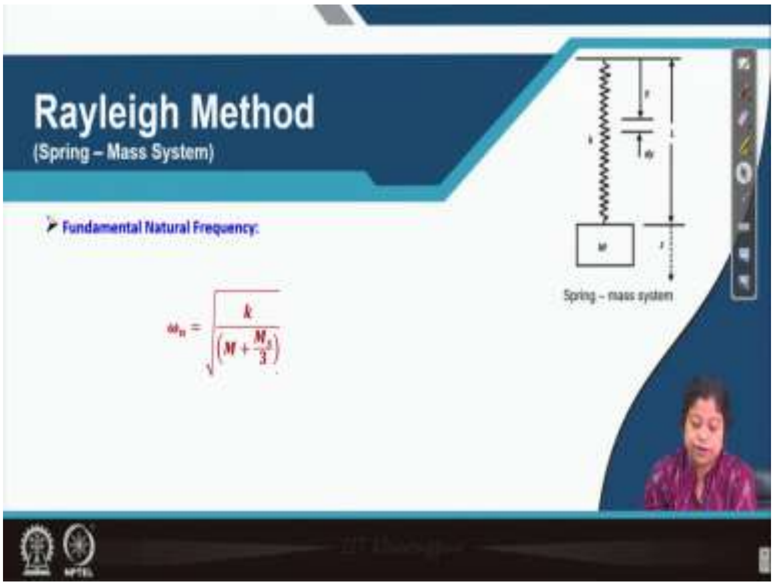
So, now, I can rewrite this expression up to this like this then let us do the integration again limit 0 to L we need to write. So, what we are getting, after putting the limits. So, the final form in this case is z0 square omega square divided by 6 times a ms L, ms L means, if I can say that total mass of the spring is capital MS, then small ms times capital L gives the total mass of this spring. So, this is total mass. Now, the kinetic energy which I have calculated here that is for the spring.

What about the kinetic energy for the rigid mass capital M? For the rigid mass capital M, the kinetic energy for rigid mass capital M is equal to half of capital M times what is the velocity of the spring at t, it was maximum velocity I am talking that is z0 omega, then the maximum kinetic energy when we will calculate the maximum kinetic energy of this rigid mass, that time the velocity is equal to the velocity at the tip of the spring which is z0 omega.

So, in this way, I am getting capital M z0 omega, z0 square omega square divided by 2 this is for the rigid mass capital M. If we will sum these two then we will get the total kinetic, maximum kinetic energy present in the system and that should be equal to the maximum potential energy. So, let us calculate then the maximum potential energy.

(Refer Slide Time: 16:38)





So, maximum potential energy sorry, it should not be KE it will be I think we use P subscript E. So, the potential energy or I can say maximum potential energy that is P subscript E is equal to how much it is kz dz integration of kz dz and the limit 0 to z0 because displacement, the limit for the displacement of spring varies from 0 to z0 which is the amplitude of the displacement that means maximum displacement and it gives us half of k z0 square. So, now, we do maximum potential energy that can be stored in the spring. So, what we can then do here we can now maximum kinetic energy is equal to maximum potential energy.

What was maximum kinetic energy? Maximum kinetic energy was for the spring it was ms times z0 square let me check once ms times z0 square times omega square times L divided by 6, ms z0 square times omega square divide times L divided by 6 plus maximum kinetic energy stored in the rigid mass capital M and that is how much that is capital M z0 square omega square divided by 2 that, so, this what I have written now is nothing but maximum kinetic energy.

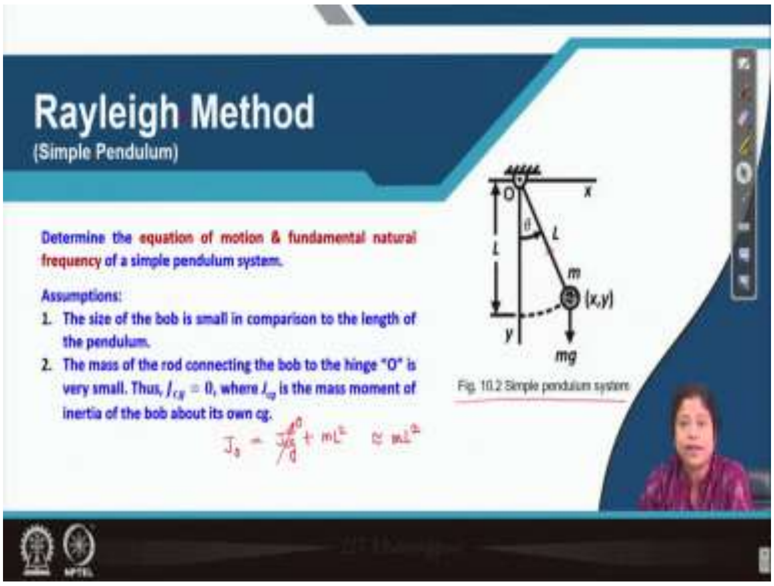
Now, that should be equal to maximum potential energy and maximum potential energy is how much half of k z0 square so, this is our maximum potential energy. So, from these what we can write finally? We can write here then k z0. Now, z0 cannot be equal to 0 and it is present in on both side.

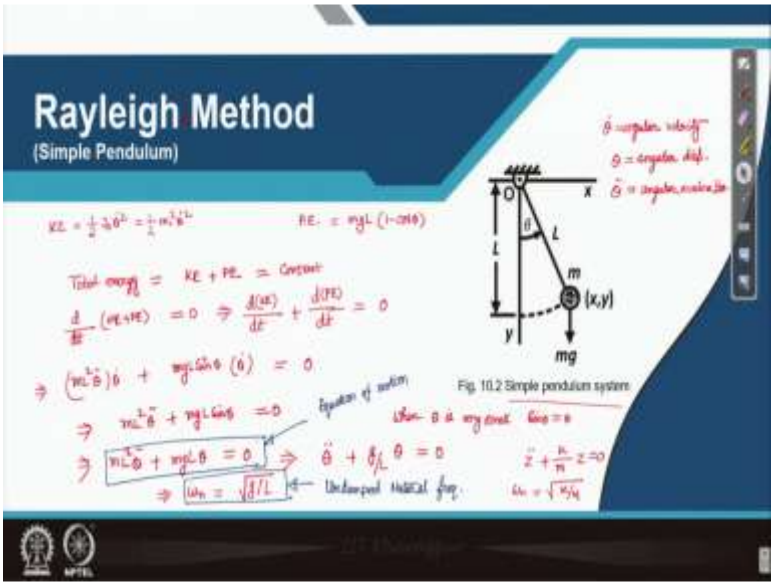
So, instead of writing z0, we can divide both sides by z0 and can write these expression as K is equal to small ms L divided by 3 plus capital M times omega square and from this we can write omega square is equal to k divided by capital M plus ms divided ms times L divided by 3, then omega should be equal to square root of K divided by capital M plus ms times L divided by 3 ms that means, small ms times capital L is nothing but the total mass of the spring.

So, I can rewrite this expression as K divided by capital M plus capital MS by 3 and these omega is called the natural frequency of the undamped system, undamped spring mass system. So, what we can see here, when we are considering the mass of the spring that time what is happening the natural frequency actually decreases by an amount and in the denominator of the expression for natural frequency one third of the mass of the spring is added which was not present when we have not considered the mass of the spring.

So, this is the difference in case of considering some finite mass for the spring. So, using Rayleigh's method in this way, we can find out the natural frequency of the spring mass system considering the mass of the spring as well. Now, we can say another example of this Rayleigh's method. What we have got that is written in the slide.

(Refer Slide Time: 23:51)





So, now, take another example of Rayleigh's method. Here in this figure 10.2, you can see a simple pendulum system in this case, small m is the mass of the bob which is connected to the hinge by a rod having length capital L. Theta is the angular displacement at any time t for this bob. What else we can see? We need to assume some important things before proceeding for the determination of the equation of motion and the fundamental natural frequency for this simple pendulum system.

What are those assumptions? First assumption is that this size of the bob is very small in comparison to the length of the pendulum, this is the first assumption. Second assumption is that the mass of the rod that means mass of this portion connecting the bob to the hinge is very small. So, we can neglect the mass of this rod. If we neglect the mass of these rod, then what we can do in the next step, we can calculate actually not here, better I should write here itself, if we calculate J0.

What is J0? J0 is mass moment of inertia of these bob about O, that is equal to Jcg that means, mass moment of inertia of the bob about its own cg plus small m times capital L square that is the distance between the cg of the bob to the hinge. So, we can write it this way. Now, when we are saying that the size of the board is very small and mass of the rod is neglecting, so, size of the bob is small so, we can take Jcg is almost equal to 0, the J0 is equal almost approximately equal to small m times L square.

So, now we will calculate the kinetic energy for this simple pendulum system. So, kinetic energy will be equal to half of J0 times theta dot square where theta dot is angular velocity, theta is angular displacement and theta two dot will get this expression also that is angular acceleration. Then, potential energy for the system is how much this is m times g times L times 1 minus cosine theta. Now, in Rayleigh's method what we have learned is that, total energy of a system will remain same and the total energy is partly kinetic energy and partly potential energy. Then what we can say that summation of kinetic energy and potential energy is constant.

And once again repeating total kinetic energy is constant as per Rayleigh's method. So, and total energy means total kinetic energy plus total potential energy. So, total energy is equal to I can write here as kinetic energy plus potential energy and that is constant. So, if I will take the derivative of total energy with respect to time, then what I can write 0, right hand side should be 0.

Or I can write the same expression as summation of derivatives of kinetic energy plus summation of derivatives of potential energy. Derivatives means division with respect to time is 0. Now, we have already calculated the kinetic energy for this simple pendulum system in terms of J0 and theta dot, it is said that kinetic energy is equal to half of J0 theta dot square. In case of J0 I can write mL square times theta dot square.

Then, if we take the first derivative of kinetic energy with respect to time. What we will get? We will get plus and this should be equal to 0 and theta dot which is the angular velocity is not 0 here. So, what we can write mL square theta two dot plus mgL sine theta that should be equal to 0. Now, when theta is very small, we can say sine theta is approximately equal, is equal to theta.

So, here in case of simple pendulum since theta is very small, we can write sine theta as theta and this expression will then become equals to mL square theta 2 dot plus mgL theta is equal to 0. Now, we can simplify it as theta 2 dot. So, theta 2 dot plus g by capital L times theta is equal to 0.

Now, if you say this expression is similar to the expression like mz 2 dot plus k or I can write it in other way basically not exactly this one, but I can write it as z 2 dot last k by m times z is equal to 0. This is the equation for free undamped system and for free undamped system, then omega n naught omega d, omega n which is the natural frequency of the undamped system is expressed like square root of k by m.

Now, same way for simple pendulum what we can then write we can write omega n which is the natural undamped frequency or undamped natural frequency of the pendulum simple pendulum system is equal to square root of g by L. So, in this way we can find out the natural frequency and the equation of motion we can write this one as also equation of motion or the bottom one. For small angle theta basically, we will consider the bottom one not this top one, so that this is our equation of motion and next expression gives us the undamped natural frequency.

So, in this lecture, we have studied how to use Rayleigh's method to construct the or to derive the equation of motion for simple pendulum and then how to get the natural frequency for the system. In this case, since we are using the Rayleigh's method, the natural frequency is the fundamental frequency of the system that means the lowest frequency of course, for this kind of system, where degree of freedom is one we have considered, so, we will get only one natural frequency.

So, I am stopping here. In next class, we will study some new thing, new thing means we will study the coulomb damping and the corresponding equation of motion, the characteristics of coulomb damping, etcetera. Thank you.