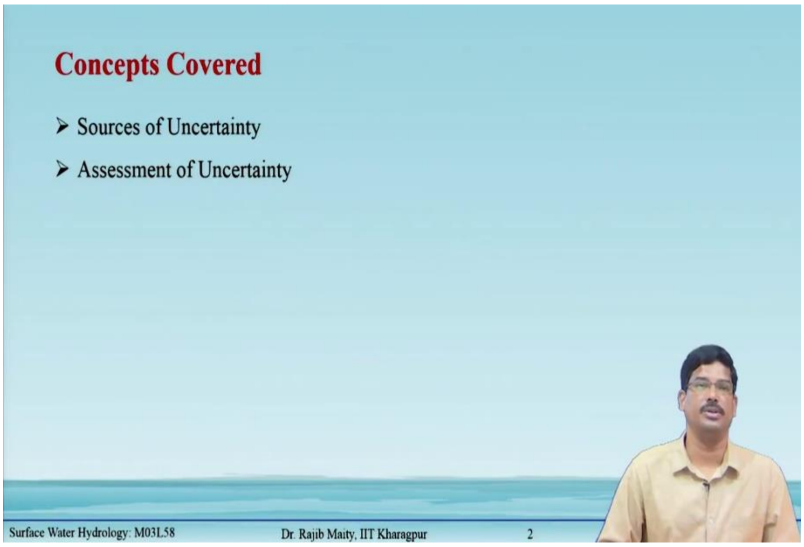
Surface Water Hydrology
Professor Rajib Maity
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 58
Uncertainty in Hydrologic Analysis

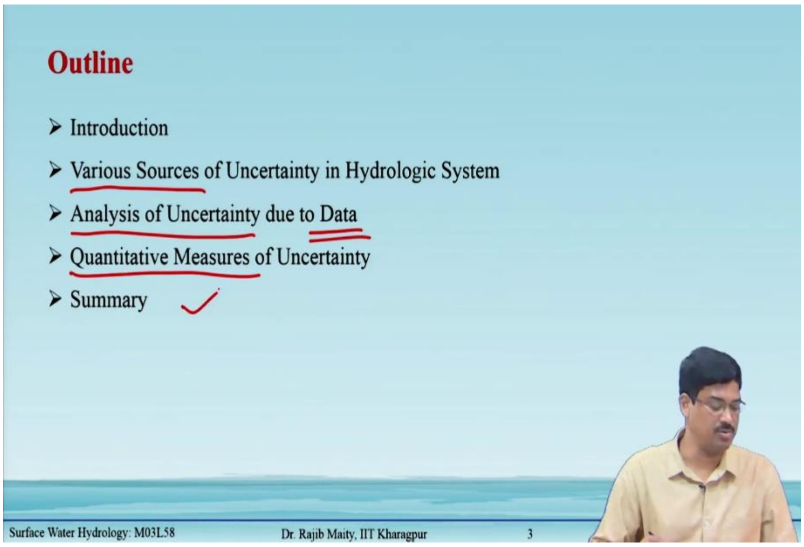
In this very last lecture, we will discuss one very important topic, which is the Source of uncertainty and the assessment of uncertainty.

(Refer Slide Time: 00:49)



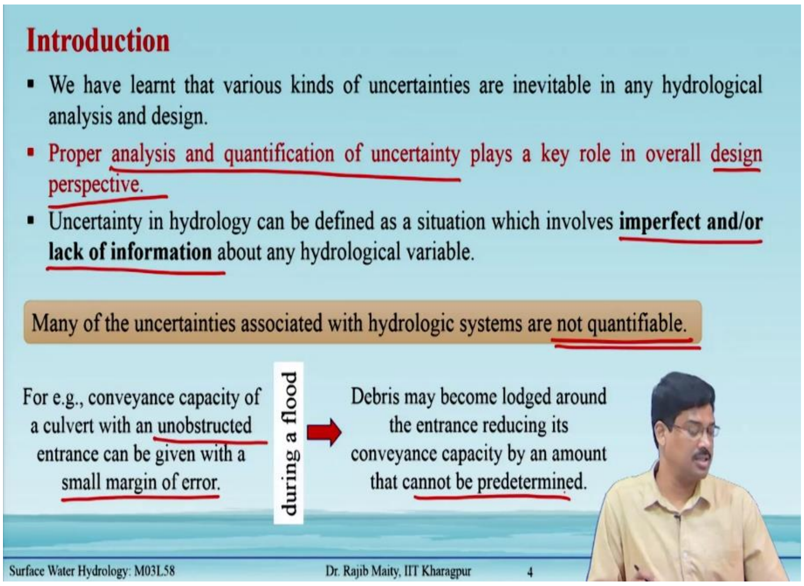
Under this concept cover, we will be covering the different sources of uncertainty and some assessments of uncertainty and their representation also.

(Refer Slide Time: 00:59)



The outline of today's lecture goes like this. First, we will give some basic introduction, and then we will discuss various sources of uncertainty that may come to the system, then we will see some analysis of uncertainty due to data, and then the quantitative measure of uncertainty, how we can measure them and represent them before we go to the summary of this lecture.

(Refer Slide Time: 01:30)



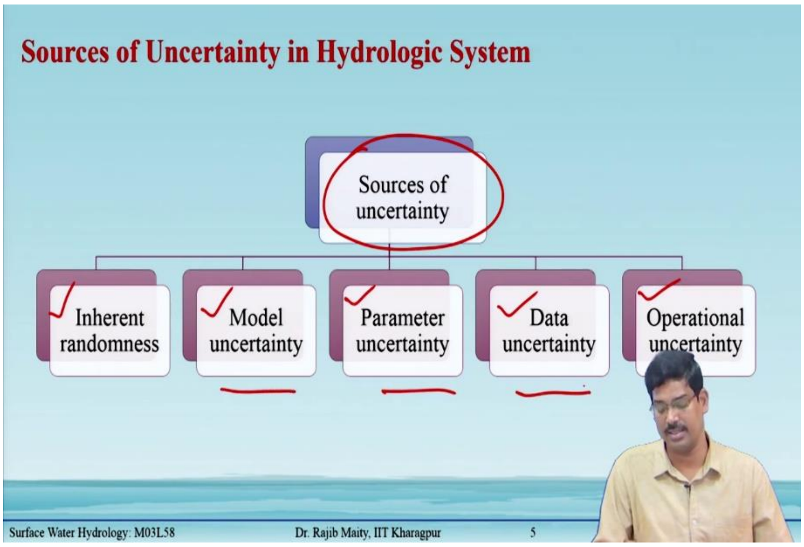
Introduction

The uncertainties are inevitable in different analyses and in the design proper analysis and quantification of uncertainty that plays a key role in the design perspective also. So, that is why the uncertainty in hydrologic can be defined as a situation that involves, imperfect under a lack of information about some of the hydrologic variables or information.

Many of the uncertainties associated with the hydrologic system are not quantifiable. But at the same time, there are few sources out there that are quantifiable we can do some analysis on that one, those are the things basically will mostly focus on in this lecture. As an example of that, no quantifiable associated uncertainties. For example, the conveyance capacity of a culvert with an un-obstructed entrance can be given with some small margin of error.

During the heavy flood time and when there is a lot of debris coming along with the flow of the water that may be logged around the entrance and that may reduce the conveyance capacity to some extent and that cannot be predetermined. So, this is just one of the typical situations. So, what kind of debris what is the position of logging and also those are some of the things we may not even quantify at all.

(Refer Slide Time: 03:15)



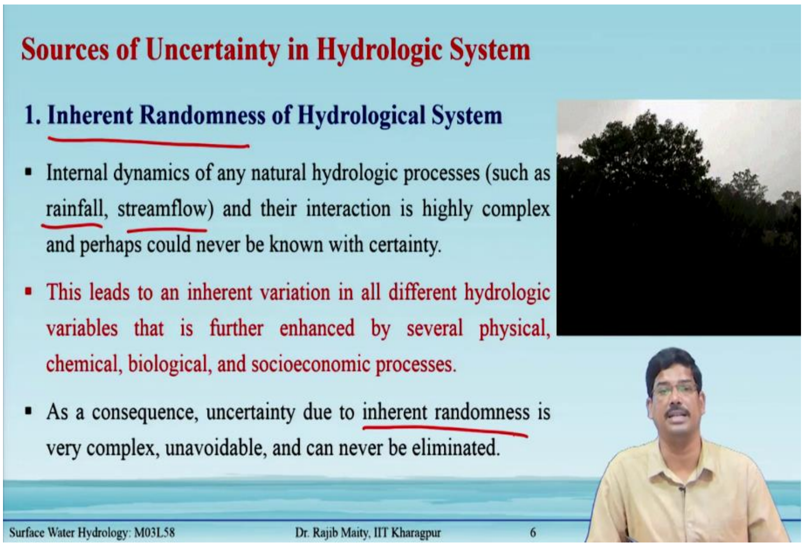
Sources of Uncertainty in Hydrologic System

Sources of uncertainty

- Inherent randomness

- Model uncertainty
- Parameter uncertainty
- Data uncertainty
- Operational uncertainty

(Refer Slide Time: 03:57)

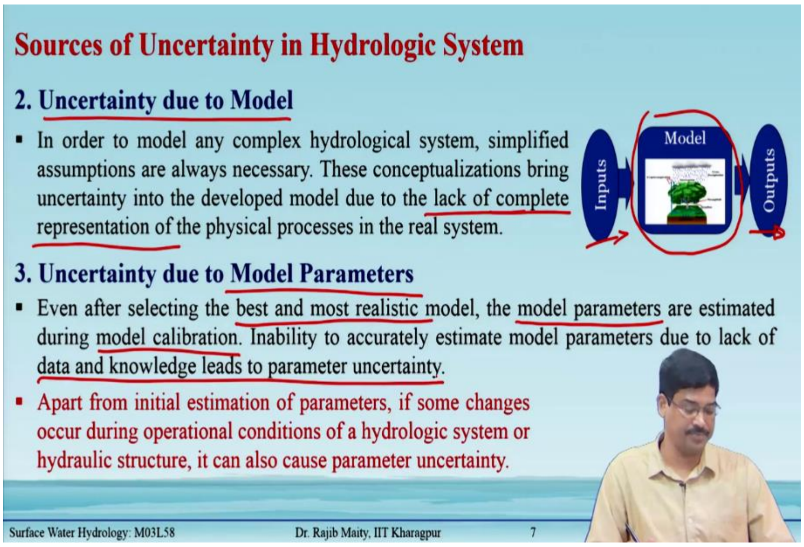


1. Inherent Randomness of Hydrological System

Internal dynamics of any natural hydrological process such as the rainfall and streamflow, interaction is highly complex and perhaps could never be known with certainty to whatever the spatial scale is, and this leads to an inherent variation in all different hydrologic variables.

These primarily originate from the variation or the uncertainty, and that will be further enhanced by several other processes also like physical, chemical or biological, or socioeconomic processes are involved as they are going through different processes of the hydrologic cycle. So, as a consequence, the uncertainty due to the inherent randomness is very complex, unavoidable, and can never be eliminated to the possible extent that we desire to be.

(Refer Slide Time: 05:15)



2. Uncertainty due to Model

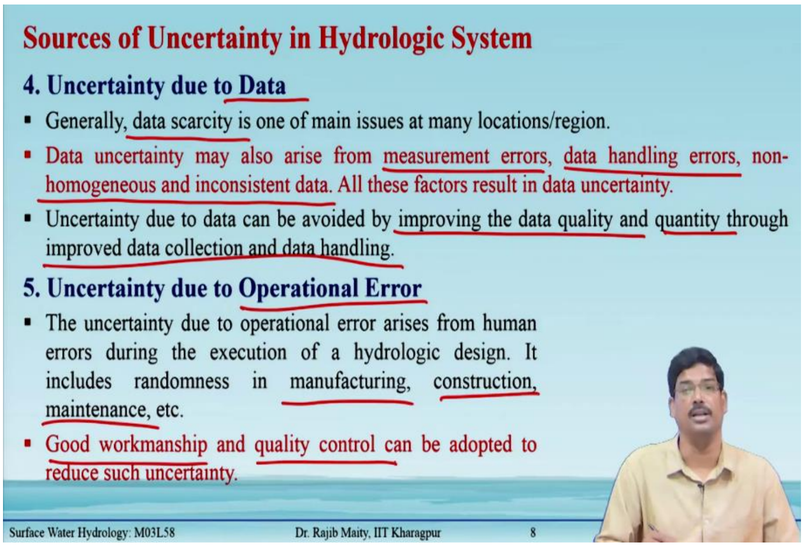
To model any complex hydrological system, simplified assumptions are always necessary. These conceptualizations bring uncertainty into the developed model due to the lack of complete representation of the physical processes in the real system.

3. Uncertainty due to Model Parameters

Even after selecting the best and most realistic model, the model parameters are estimated during model calibration. Inability to accurately estimate model parameters due to lack of data and knowledge leads to parameter uncertainty.

Apart from the initial estimation of parameters, if some changes occur during operational conditions of a hydrologic system or hydraulic structure, it can also cause parameter uncertainty.

(Refer Slide Time: 08:13)



4. Uncertainty due to Data

Then comes uncertainty due to data. Generally, the first and obvious thing particularly for developing countries like us, is that data scarcity is one of the main problems. So, the quality of the data or quantity of the data that we need for a, for a location for a project location, that may not be available always or to the extent possible whatever we need, that may not be there and that is true for many regions and locations.

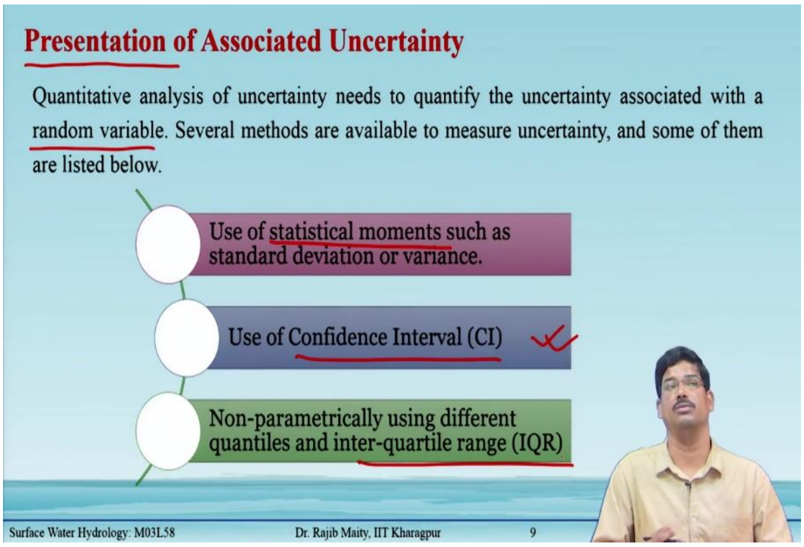
Data uncertainty may also arise from the measurement errors, the data handling error and non-homogeneous and inconsistent data, all these factors, result in leads to that data uncertainty. Uncertainty due to data can be avoided by there are some ways that are improving the data quality, quantity through the improved data collection and data handling methods.

5. Uncertainty due to Operational Error

Then comes uncertainty due to operational error, the uncertainty due to operational error arises from the human error during the execution of a hydrological design, it includes say randomness in manufacturing, construction, maintenance, so those are the things that are there during the process when we implemented on the actual site, that time there could be some barriers and what is what was adopted during the design process and what is implemented in the on the ground. So, that gives another source of uncertainty which is called this uncertainty due to operational error.

To remedy this source of uncertainties the good workmanship and quality control during the implementation, so, will help to reduce this uncertainty.

(Refer Slide Time: 10:10)

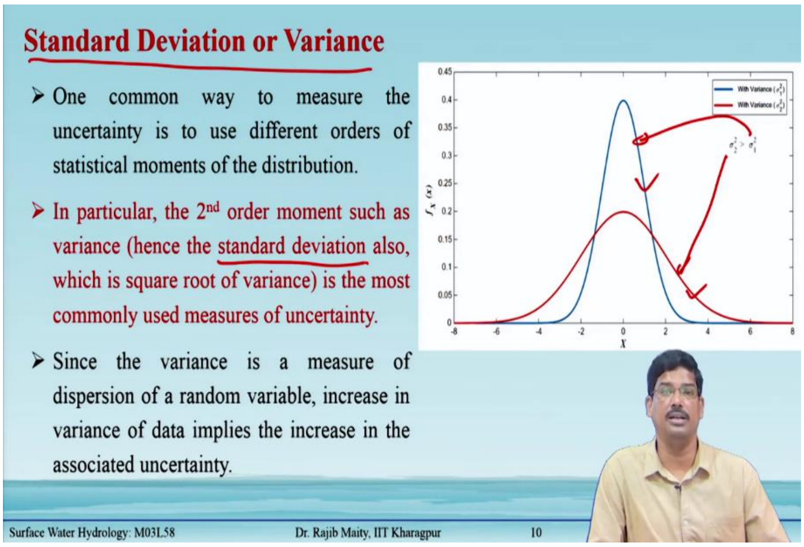


Presentation of Associated Uncertainty

Quantitative analysis of uncertainty needs to quantify the uncertainty associated with a random variable. Several methods are available to measure uncertainty, and some of them are listed below.

- Use of statistical moments such as standard deviation or variance.
- Use of Confidence Interval (CI)
- Non-parametrically using different quantiles and inter-quartile range (IQR)

(Refer Slide Time: 12:00)

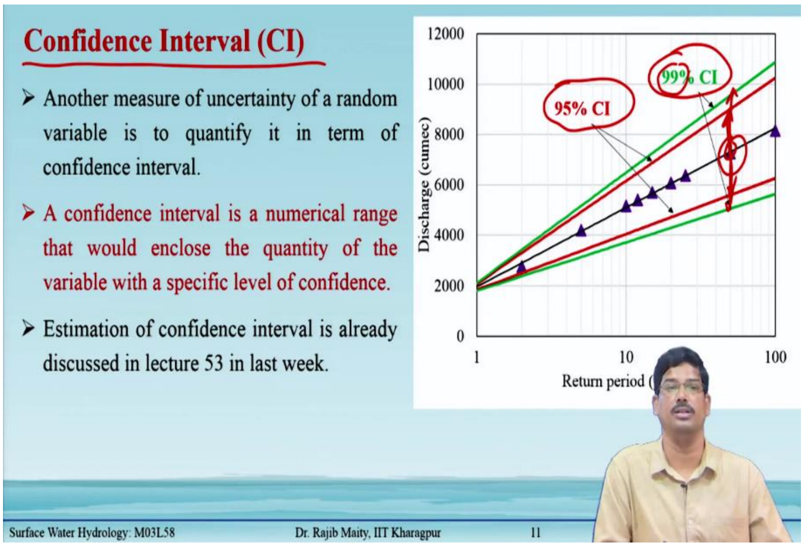


Standard Deviation or Variance

The one common way to measure the uncertainty is to use the different orders of statistical moments or the distribution, this variance is the second moment with respect to the mean. In particular, the 2nd order moment such as variance (hence the standard deviation also, which is the square root of variance) is the most commonly used measure of uncertainty.

Since the variance is a measure of the dispersion of a random variable, an increase in the variance of data implies an increase in the associated uncertainty.

(Refer Slide Time: 13:09)

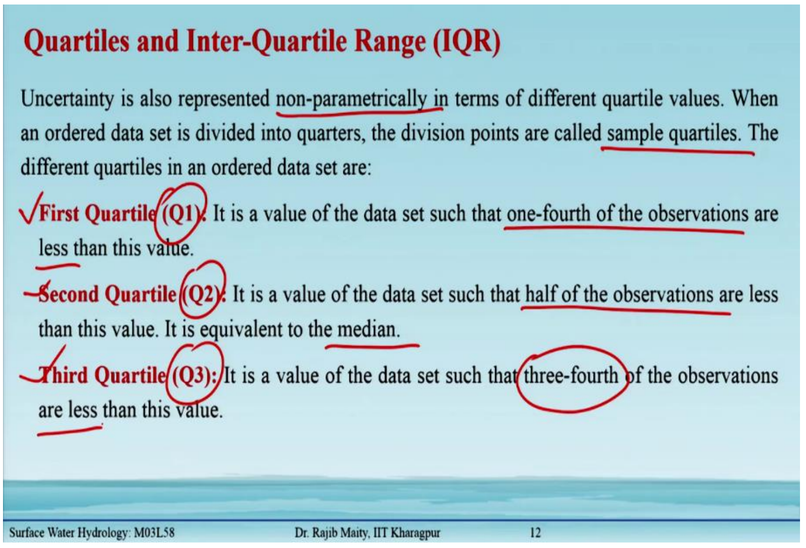


Confidence Interval (CI)

Another measure of uncertainty of a random variable is to quantify it in terms of the confidence interval.

A confidence interval is a numerical range that would enclose the quantity of the variable with a specific level of confidence.

(Refer Slide Time: 14:26)



Quartiles and Inter-Quartile Range (IQR)

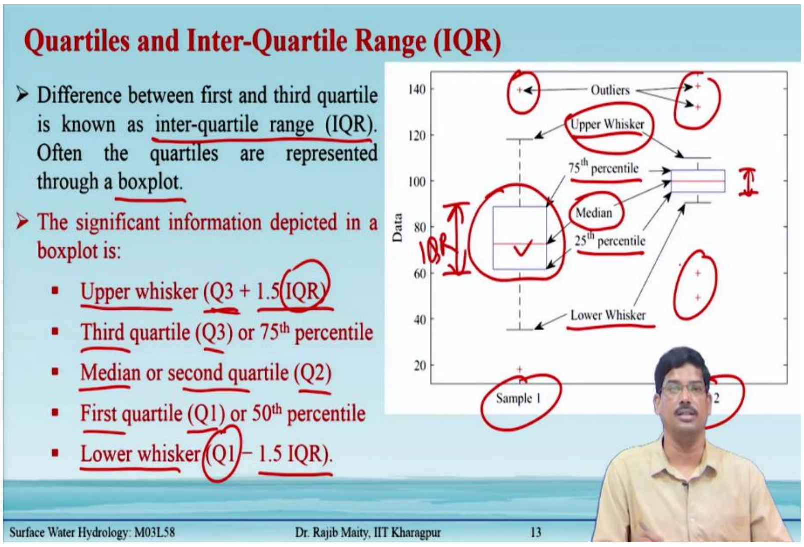
Uncertainty is also represented non-parametrically in terms of different quartile values. When an ordered data set is divided into quarters, the division points are called sample quartiles. The different quartiles in an ordered data set are:

First Quartile (Q1): It is a value of the data set such that one-fourth of the observations are less than this value.

Second Quartile (Q2): It is a value of the data set such that half of the observations are less than this value. It is equivalent to the median.

Third Quartile (Q3): It is a value of the data set such that three-fourths of the observations are less than this value.

(Refer Slide Time: 15:26)



➢ The difference between the first and third quartile is known as an inter-quartile range (IQR). Often the quartiles are represented through a boxplot.

➢ The significant information depicted in a boxplot is (shown in fig.1):

- Upper whisker (Q3 + 1.5 IQR)
- Third quartile (Q3) or 75^{th} percentile
- Median or second quartile (Q2)
- The first quartile (Q1) or 50^{th} percentile
- Lower whisker (Q1 − 1.5 IQR).

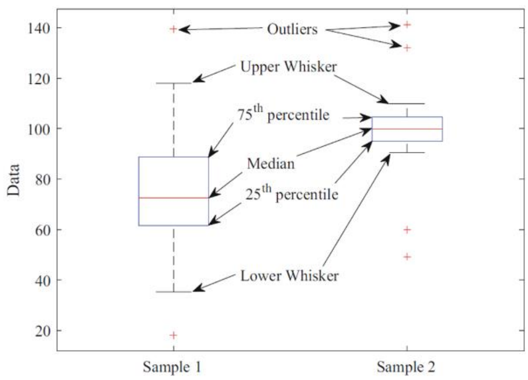
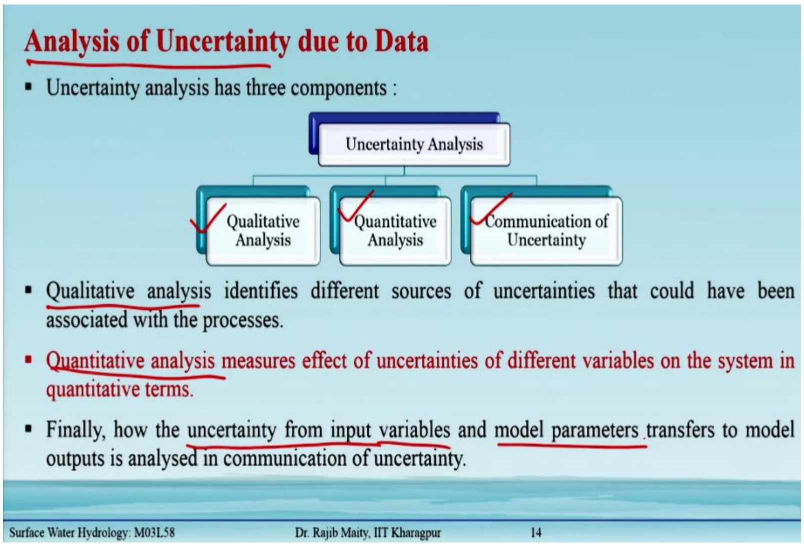


Figure 1 shows the representation of the box plot

(Refer Slide Time: 17:08)



Analysis of Uncertainty due to Data

It has three main components one is we can say that the

I. Qualitative analysis,

II. Quantitative analysis and

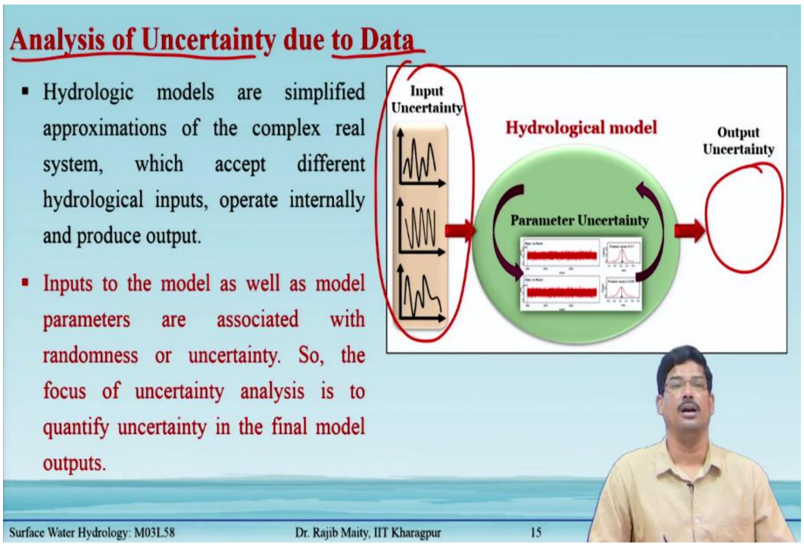
III. Communication of this uncertainty.

Qualitative analysis identifies different sources of uncertainties that could have been associated with the processes.

The quantitative analysis measures the effect of uncertainties of different variables on the system in quantitative terms.

Finally, how the uncertainty from input variables and model parameters transfers to model outputs is analyzed in the **communication of uncertainty**.

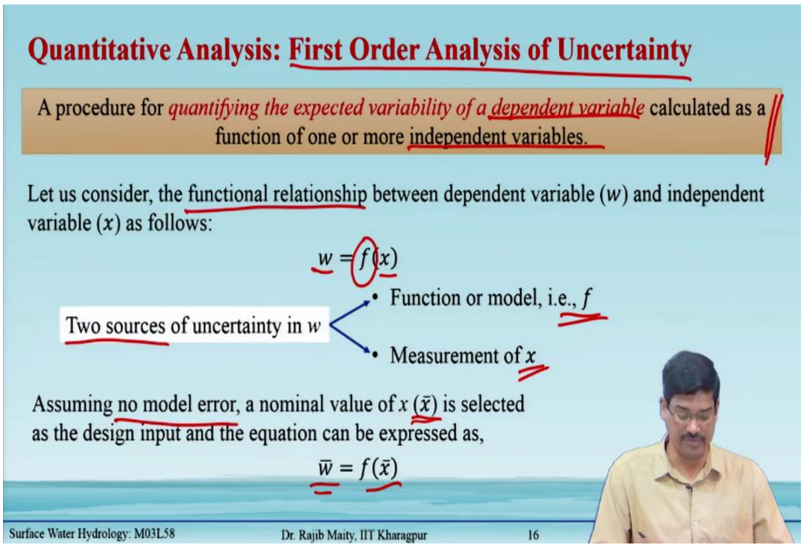
(Refer Slide Time: 17:58)



Hydrologic models are simplified approximations of the complex real system, which accept different hydrological inputs, operate internally and produce output.

Inputs to the model as well as model parameters are associated with randomness or uncertainty. So, the focus of uncertainty analysis is to quantify uncertainty in the final model outputs.

(Refer Slide Time: 18:50)



Quantitative Analysis: First Order Analysis of Uncertainty

There are different methods available here, we can just take up one that first-order uncertainty of first-order analysis of uncertainty, this is a procedure for quantifying the expected variability of a dependent variable calculated as a function of one or more independent variables the common.

Let us consider, the functional relationship between the dependent variable (w) and independent variable (x) as follows:

$$w = f(x)$$

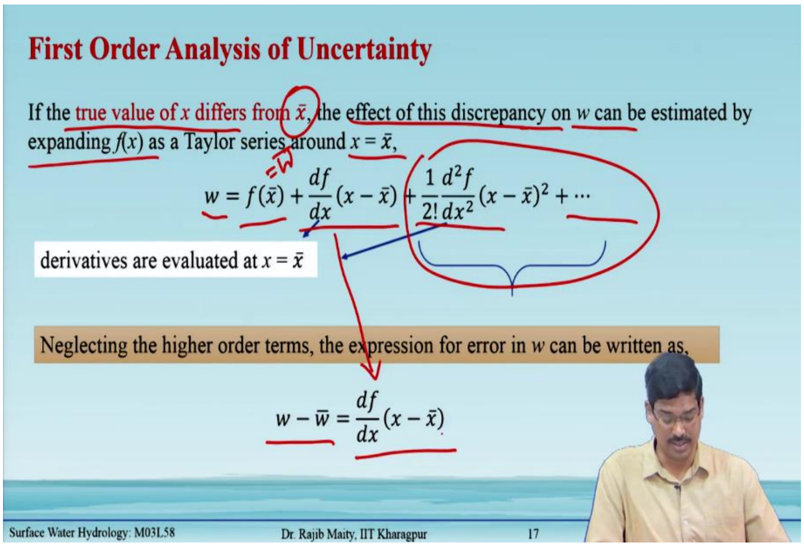
Two sources of uncertainty in *w*

- Function or model, i.e., f
- Measurement of x

Assuming no model error, a nominal value of x $(\bar{x})$ is selected as the design input and the equation can be expressed as,

$$\bar{w} = f(\bar{x})$$

(Refer Slide Time: 20:31)



First Order Analysis of Uncertainty

If the true value of x differs from $\bar{x}$, the effect of this discrepancy on w can be estimated by expanding $f(x)$ as a Taylor series around $x = \bar{x}$,

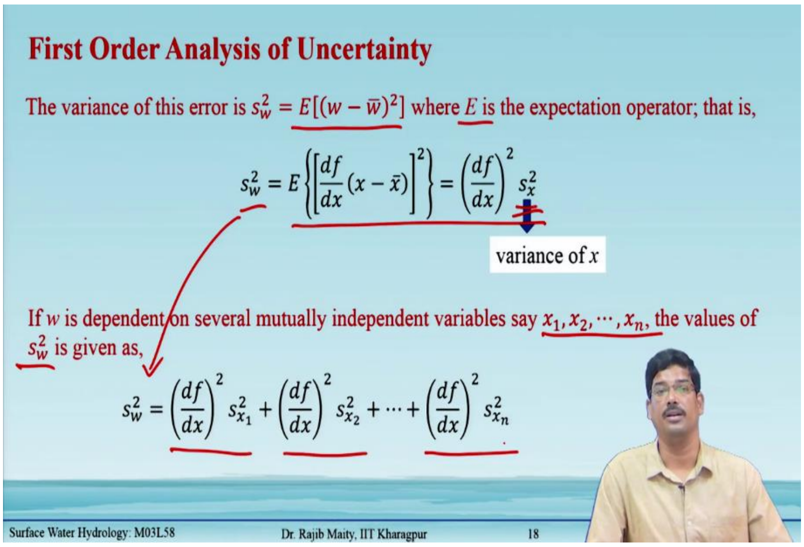
$$w = f(\bar{x}) + \frac{df}{dx}(x - \bar{x}) + \frac{1}{2!}\frac{d^2 f}{dx^2}(x - \bar{x})^2 + \cdots$$

Derivatives are evaluated at $x = \bar{x}$

Neglecting the higher-order terms, the expression for error in w can be written as,

$$w - \bar{w} = \frac{df}{dx}(x - \bar{x})$$

(Refer Slide Time: 21:57)



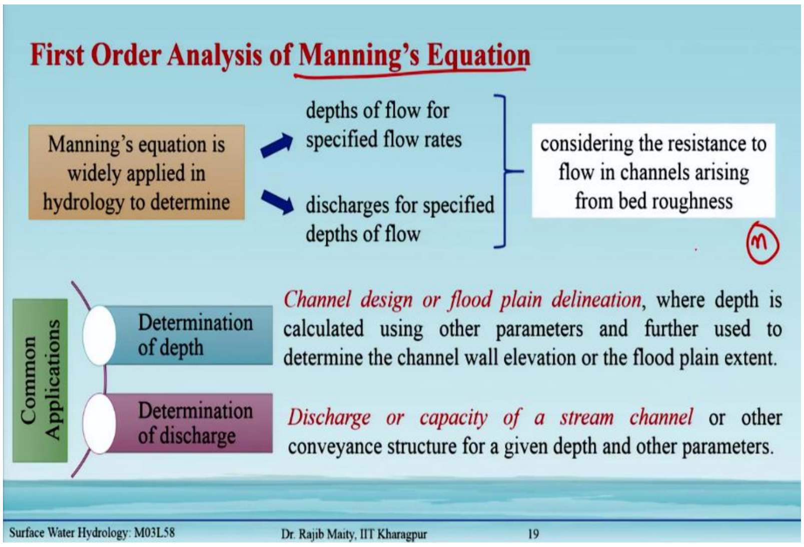
The variance of this error is $s_w^2 = E[(w - \bar{w})^2]$ where E is the expectation operator; that is,

$$s_w^2 = E\left\{\left[\frac{df}{dx}(x - \bar{x})\right]^2\right\} = \left(\frac{df}{dx}\right)^2 s_x^2$$

If w is dependent on several mutually independent variables say $x_1, x_2, \cdots, x_n$, the values of s_w^2 is given as

$$s_w^2 = \left(\frac{df}{dx}\right)^2 s_{x_1}^2 + \left(\frac{df}{dx}\right)^2 s_{x_2}^2 + \cdots + \left(\frac{df}{dx}\right)^2 s_{x_n}^2$$

(Refer Slide Time: 22:53)



First Order Analysis of Manning's Equation

Manning's equation is widely applied in hydrology to determine

- Depths of flow for specified flow rates
- Discharges for specified depths of flow

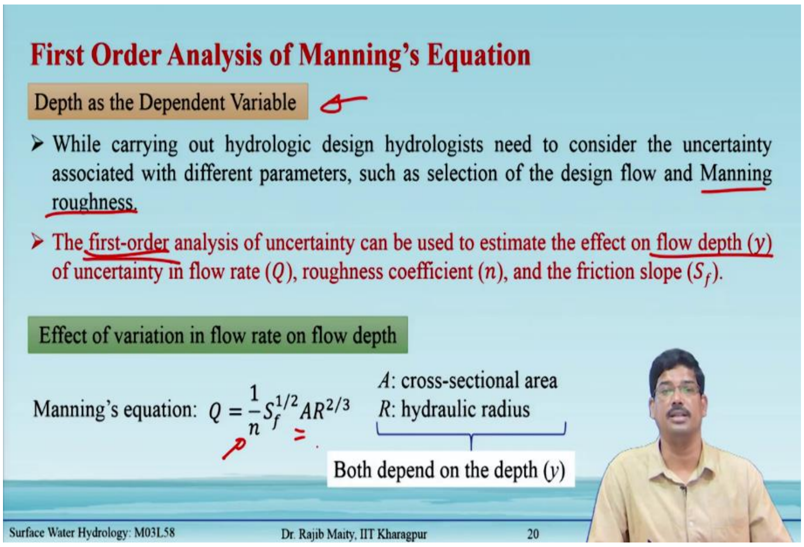
The flow depth, velocity, and discharge quantity that we can estimate considering the resistance of the flow to the channel through channels arising from the bed roughness that was that in the Manning's roughness.

The common applications generally determine what the depth of the flow is, what is the discharge, and the velocity of the flow.

Channel design or flood plain delineation, where depth is calculated using other parameters and further used to determine the channel wall elevation or the flood plain extent.

Discharge or capacity of a stream channel or other conveyance structure for a given depth and other parameters.

(Refer Slide Time: 23:55)



First Order Analysis of Manning's Equation

Depth as the Dependent Variable

While carrying out hydrologic design hydrologists need to consider the uncertainty associated with different parameters, such as selection of the design flow and Manning roughness.

The first-order analysis of uncertainty can be used to estimate the effect on flow depth (y) of uncertainty in flow rate (Q), roughness coefficient (n), and the friction slope (S_f).

Effect of variation in flow rate on flow depth

Manning's equation:

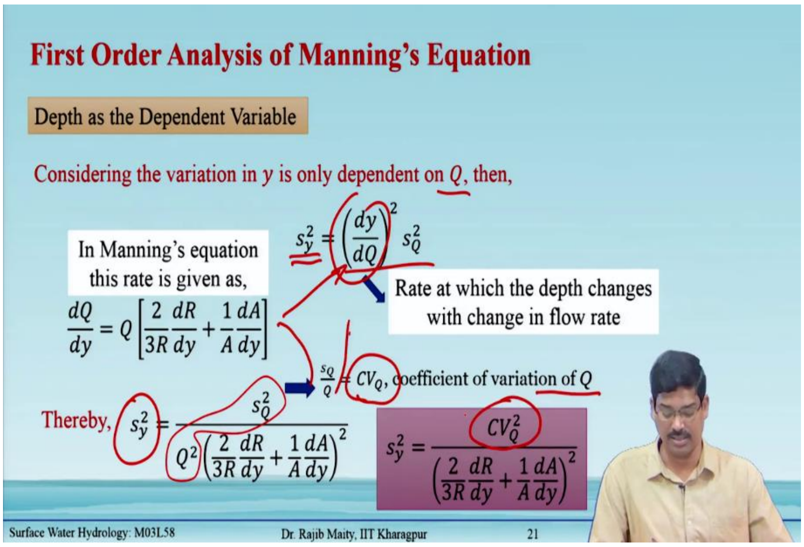
$$Q = \frac{1}{n} S_f^{1/2} A R^{2/3}$$

Where, A: cross-sectional area

R: hydraulic radius

It may be noted that both depend on the depth (y)

(Refer Slide Time: 25:22)



Depth as the Dependent Variable

Considering the variation in y is only dependent on$Q$, then,

$$s_y^2 = \left(\frac{dy}{dQ}\right)^2 s_Q^2$$

Where, $\frac{dy}{dQ}$ is the rate at which the depth changes with a change in flow rate.

In Manning's equation, this rate is given as,

$$\frac{dQ}{dy} = Q\left[\frac{2}{3R}\frac{dR}{dy} + \frac{1}{A}\frac{dA}{dy}\right]$$

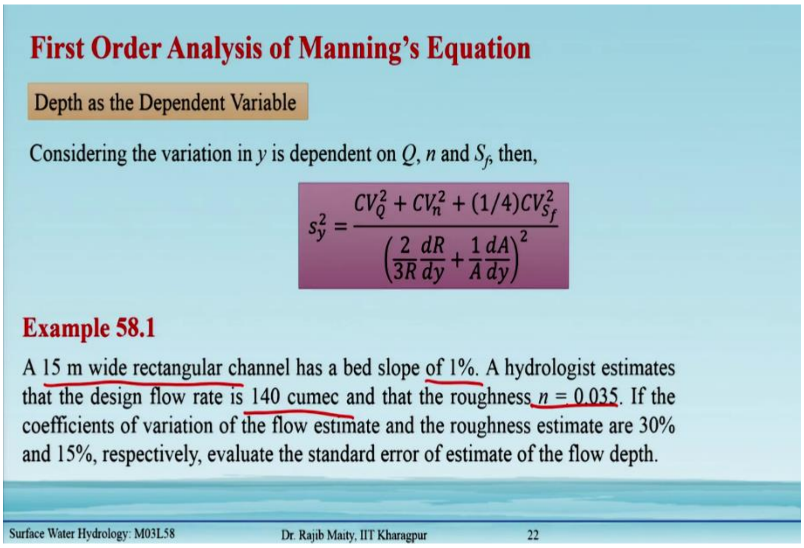
Thereby,

$$s_y^2 = \frac{s_Q^2}{Q^2\left(\frac{2}{3R}\frac{dR}{dy} + \frac{1}{A}\frac{dA}{dy}\right)^2}$$

$\frac{s_Q}{Q} = CV_Q$, coefficient of variation of Q

$$s_y^2 = \frac{CV_Q^2}{\left(\frac{2}{3R}\frac{dR}{dy} + \frac{1}{A}\frac{dA}{dy}\right)^2}$$

(Refer Slide Time: 26:41)



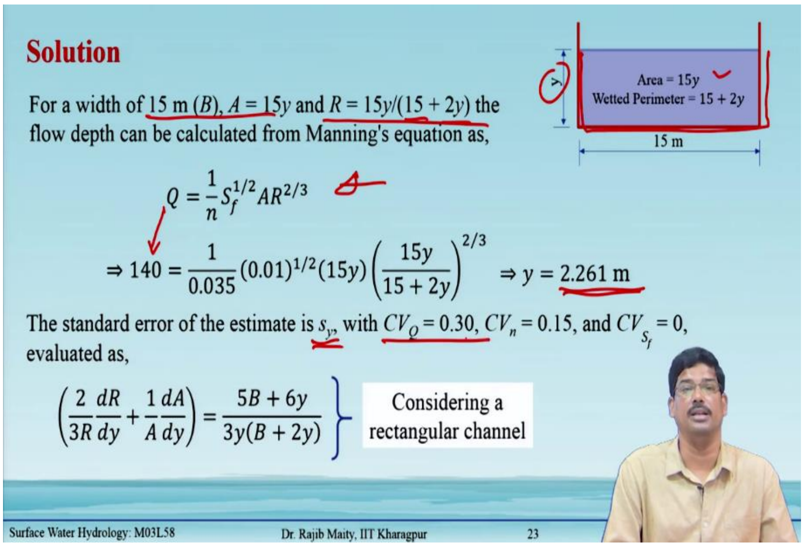
Considering the variation in y is dependent on Q, n, and S_f, then,

$$s_y^2 = \frac{CV_Q^2 + CV_n^2 + (1/4)CV_{S_f}^2}{\left(\frac{2}{3R}\frac{dR}{dy} + \frac{1}{A}\frac{dA}{dy}\right)^2}$$

Example 58.1

A 15 m wide rectangular channel has a bed slope of 1%. A hydrologist estimates that the design flow rate is 140 cumec and that the roughness n = 0.035. If the coefficients of variation of the flow estimate and the roughness estimate are 30% and 15%, respectively, evaluate the standard error of estimate of the flow depth.

(Refer Slide Time: 27:23)



Solution

For a width of 15 m (B), A = 15y and R = 15y/(15 + 2y) the flow depth can be calculated from Manning's equation as,

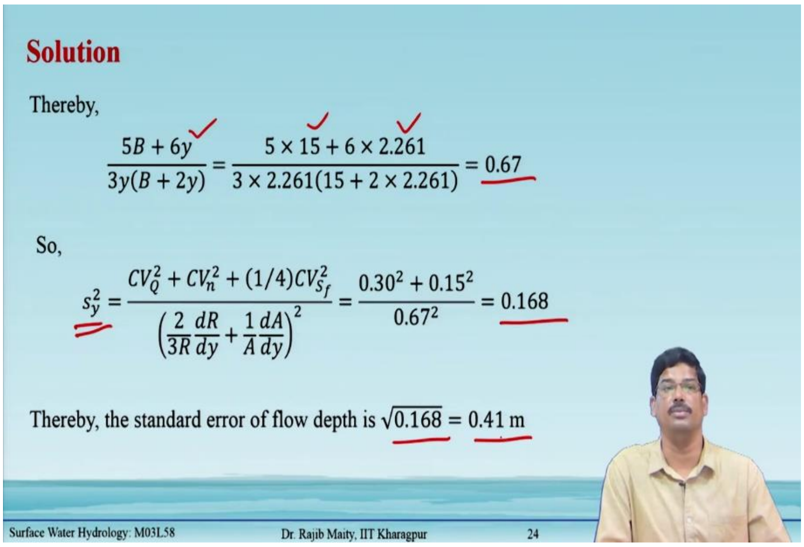
$$Q = \frac{1}{n} S_f^{1/2} A R^{2/3}$$

$$\Rightarrow 140 = \frac{1}{0.035}(0.01)^{1/2}(15y)\left(\frac{15y}{15+2y}\right)^{2/3} \Rightarrow y = 2.261 \text{ m}$$

The standard error of the estimate is s_y, with CV_Q = 0.30, CV_n = 0.15, and CVS_f = 0, evaluated as,

$$\left(\frac{2}{3R}\frac{dR}{dy} + \frac{1}{A}\frac{dA}{dy}\right) = \frac{5B + 6y}{3y(B + 2y)}$$

(Refer Slide Time: 28:38)



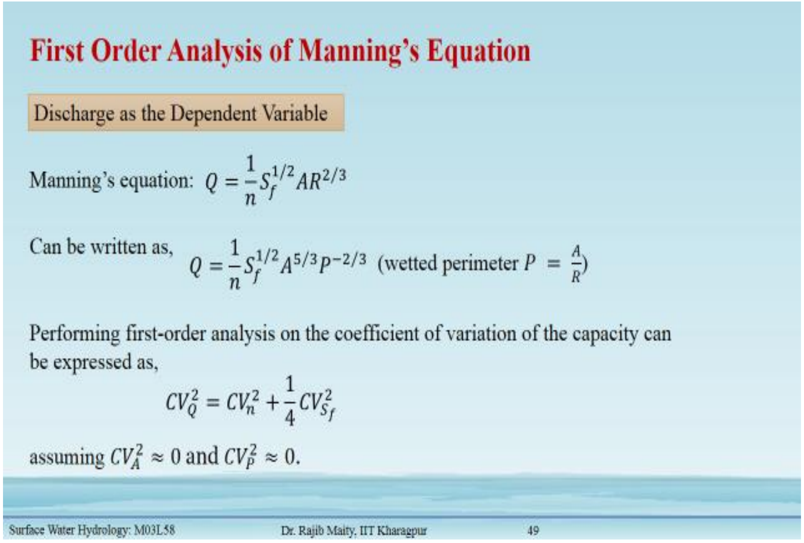
Thereby,

$$\frac{5B+6y}{3y(B+2y)} = \frac{5\times 15 + 6\times 2.261}{3\times 2.261(15+2\times 2.261)} = 0.67$$

So,

$$s_y^2 = \frac{CV_Q^2 + CV_n^2 + (1/4)CV_{S_f}^2}{\left(\frac{2}{3R}\frac{dR}{dy}+\frac{1}{A}\frac{dA}{dy}\right)^2} = \frac{0.30^2+0.15^2}{0.67^2} = 0.168$$

Thereby, the standard error of flow depth is $\sqrt{0.168} = 0.41$ m



Discharge as the Dependent Variable

Manning's equation: $Q = \frac{1}{n} S_f^{1/2} A R^{2/3}$

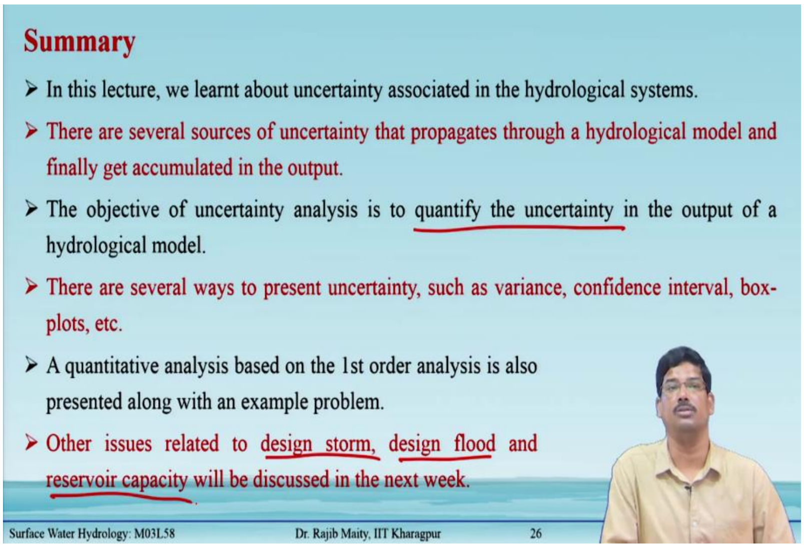
Can be written as, $Q = \frac{1}{n} S_f^{1/2} A^{5/3} P^{-2/3}$ (wetted perimeter $P = \frac{A}{R}$)

Performing first-order analysis on the coefficient of variation of the capacity can be expressed as,

$$CV_Q^2 = CV_n^2 + \frac{1}{4} CV_{S_f}^2$$

Assuming $CV_A^2 \approx 0$ and $CV_P^2 \approx 0$.

(Refer Slide Time: 29:14)



Summary

In summary, we learned the following points from this lecture:

- In this lecture, we learned about the uncertainty associated with hydrological systems.
- There are several sources of uncertainty that propagates through a hydrological model and finally get accumulated in the output.
- The objective of uncertainty analysis is to quantify the uncertainty in the output of a hydrological model.
- There are several ways to present uncertainty, such as variance, confidence interval, box plots, etc.

- A quantitative analysis based on the 1st order analysis is also presented along with an example problem.
- Other issues related to design storm, design flood, and reservoir capacity will be discussed in the next week.