Surface Water Hydrology
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Lecture 45
Channel Routing: Muskingum Method and Hydraulic Flood Routing

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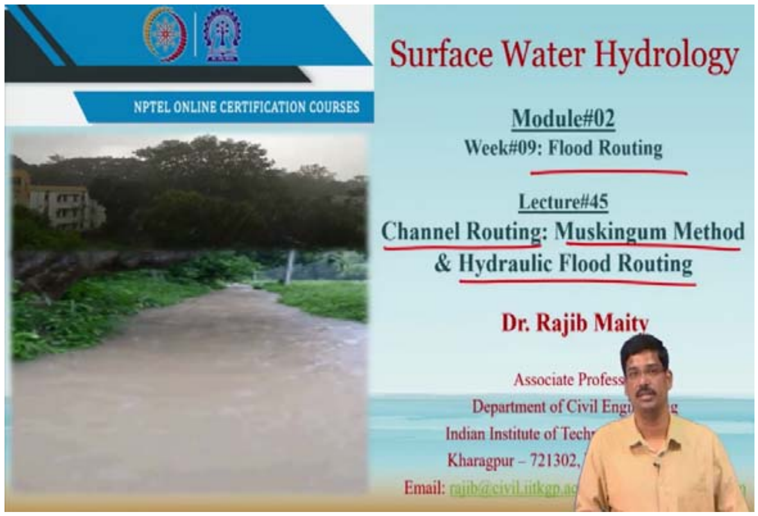

In this particular lecture, we will know the Muskingum method and Hydraulic Flood Routing method under the category of channel routing.

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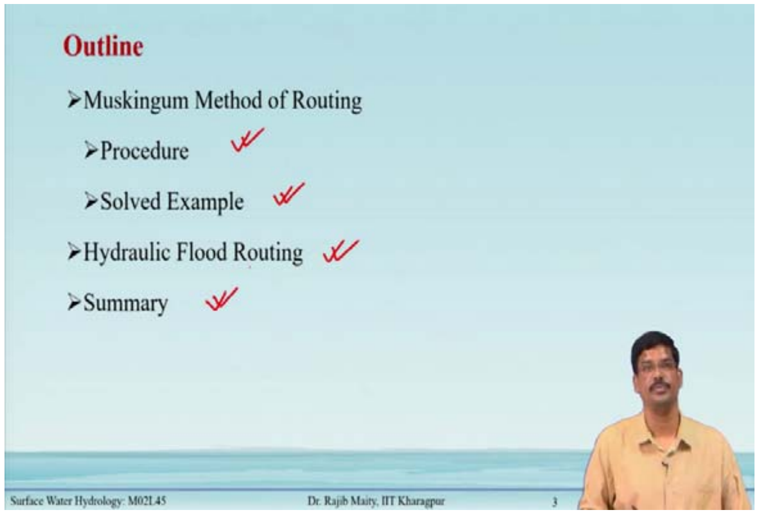

The two concepts are there. One first one as I mentioned, the Muskingum method of routing and second one is the hydraulic flood routing. Both concepts will be covered in this lecture. The outline of this lecture goes like this. So, Muskingum method of routing, is procedure, how to proceed with this method and with a solved example. And then some brief discussion on this hydraulic fluid routing before going to the summary.

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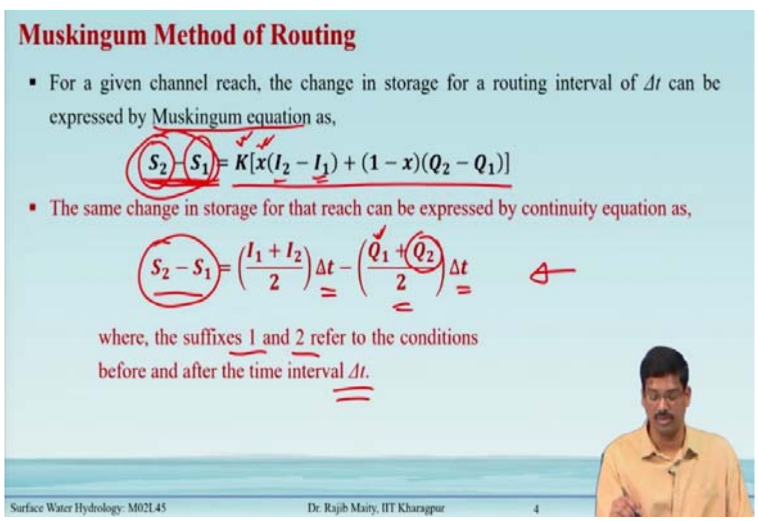

For a given channel reach, the change in storage for a routing interval of Δt can be expressed by Muskingum equation as,

$$S_2 - S_1 = K[x(I_2 - I_1) + (1 - x)(Q_2 - Q_1)]$$

The same change in storage for that reach can be expressed by continuity equation as,

$$S_2 - S_1 = \left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t$$

where, the suffixes 1 and 2 refer to the conditions before and after the time interval *Δt*.

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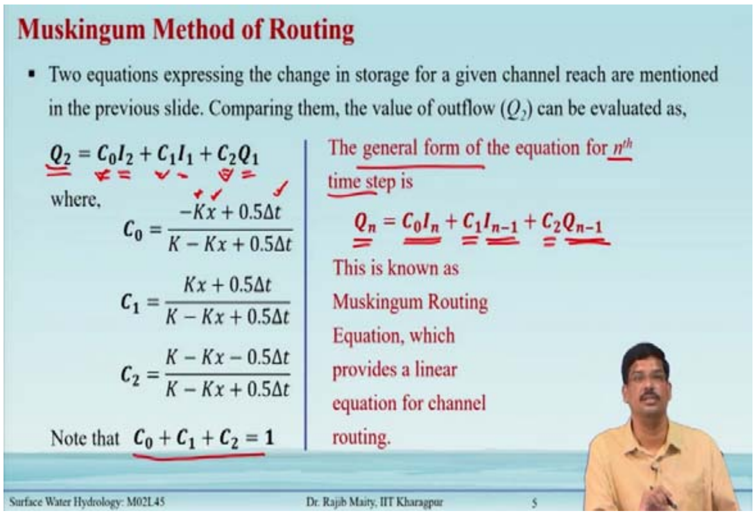

Muskingum Method of Routing

Two equations expressing the change in storage for a given channel reach are mentioned in the previous slide. Comparing them, the value of outflow (Q2) can be evaluated as,

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

where,

$$C_0 = \frac{-Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_1 = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

$$C_2 = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t}$$

It may be noted that $C_0 + C_1 + C_2 = 1$

The general form of the equation for n^{th} time step is

$$Q_n = C_0 I_n + C_1 I_{n-1} + C_2 Q_{n-1}$$

This is known as Muskingum Routing Equation, which provides a linear equation for channel routing.

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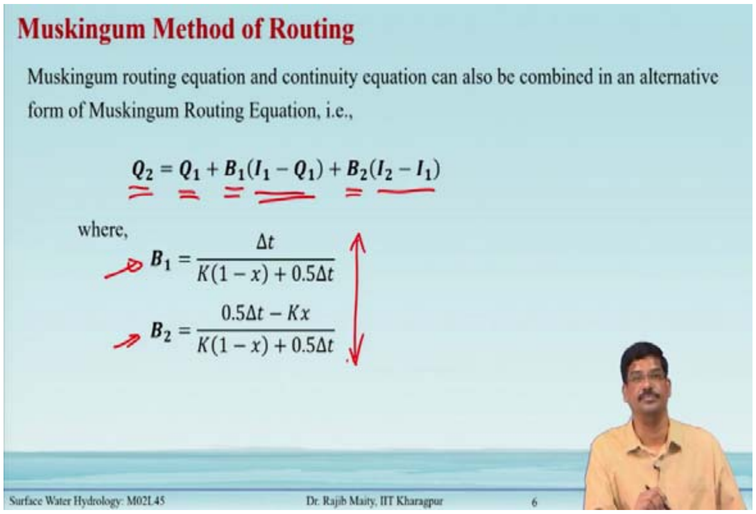

Muskingum routing equation and continuity equation can also be combined in an alternative form of Muskingum Routing Equation, i.e.,

$$\boldsymbol{Q_2} = \boldsymbol{Q_1} + \boldsymbol{B_1}(\boldsymbol{I_1} - \boldsymbol{Q_1}) + \boldsymbol{B_2}(\boldsymbol{I_2} - \boldsymbol{I_1})$$

where,

$$\boldsymbol{B_1} = \frac{\Delta t}{K(1-x) + 0.5\Delta t}$$

$$\boldsymbol{B_2} = \frac{0.5\Delta t - Kx}{K(1-x) + 0.5\Delta t}$$

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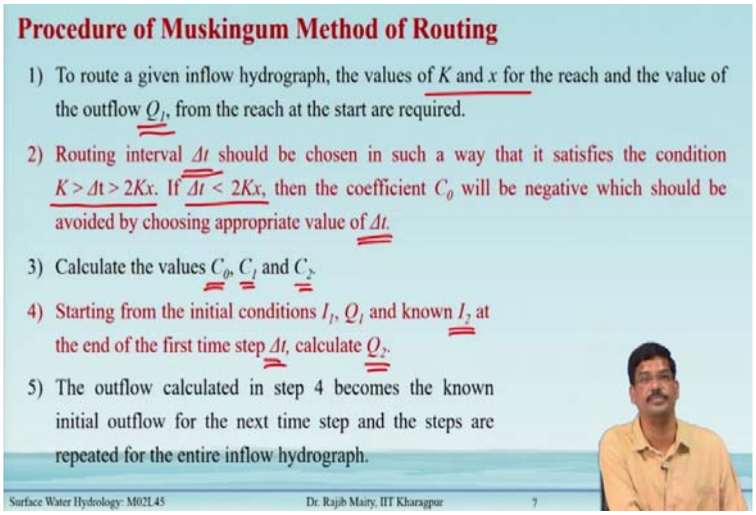

Procedure of Muskingum Method of Routing

1. To route a given inflow hydrograph, the values of K and x for the reach and the value of the outflow Q_1, from the reach at the start are required.

2. Routing interval Δt should be chosen in such a way that it satisfies the condition "$K > \Delta t > 2Kx$". If $\Delta t < 2Kx$, then the coefficient C_0 will be negative which should be avoided by choosing appropriate value of Δt.

3. Calculate the values C_0, C_1 and C_2.
4. Starting from the initial conditions I_1, Q_1 and known I_2 at the end of the first-time step Δt, calculate Q_2.
5. The outflow calculated in step 4 becomes the known initial outflow for the next time step and the steps are repeated for the entire inflow hydrograph.

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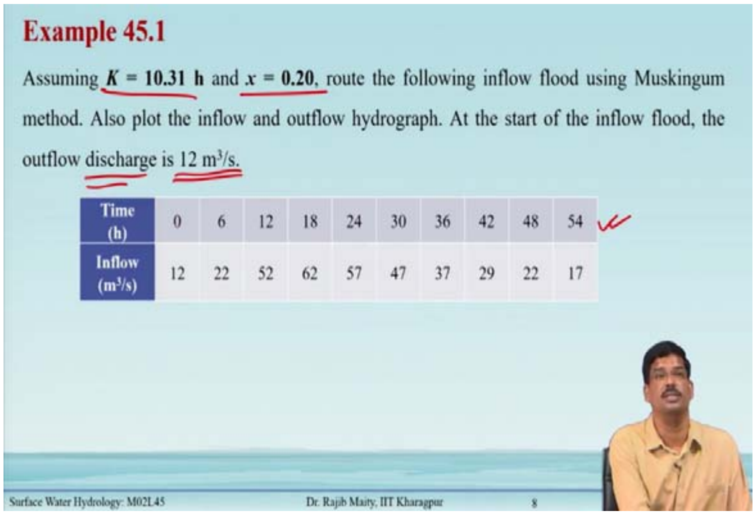

Example 45.1

Assuming K = 10.31 h and x = 0.20, route the following inflow flood using Muskingum method. Also plot the inflow and outflow hydrograph. At the start of the inflow flood, the outflow discharge is 12 m^3/s.

Time (h)	0	6	12	18	24	30	36	42	48	54
Inflow (m^3/s)	12	22	52	62	57	47	37	29	22	17

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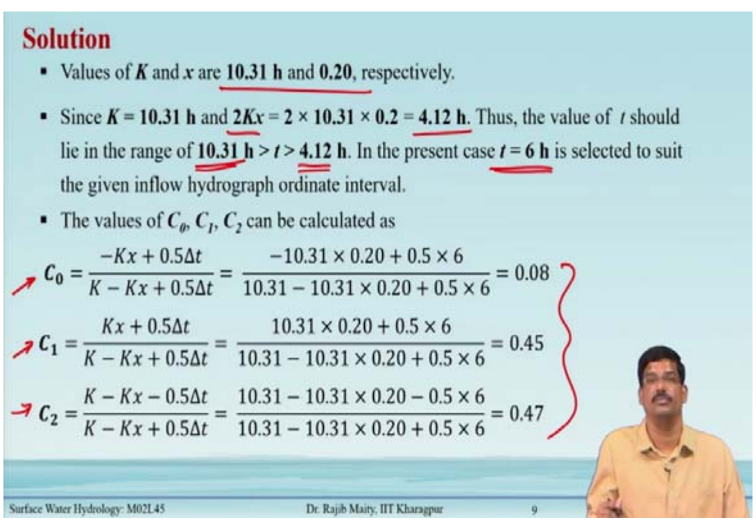

Solution

Values of K and x are 10.31 h and 0.20, respectively.

Since K = 10.31 h and 2Kx = 2 × 10.31 × 0.2 = 4.12 h. Thus, the value of t should lie in the range of 10.31 h > t > 4.12 h. In the present case t = 6 h is selected to suit the given inflow hydrograph ordinate interval.

The values of C_0, C_1, C_2 can be calculated as

$$\boldsymbol{C_0} = \frac{-Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t} = \frac{-10.31 \times 0.20 + 0.5 \times 6}{10.31 - 10.31 \times 0.20 + 0.5 \times 6} = 0.08$$

$$\boldsymbol{C_1} = \frac{Kx + 0.5\Delta t}{K - Kx + 0.5\Delta t} = \frac{10.31 \times 0.20 + 0.5 \times 6}{10.31 - 10.31 \times 0.20 + 0.5 \times 6} = 0.45$$

$$\boldsymbol{C_2} = \frac{K - Kx - 0.5\Delta t}{K - Kx + 0.5\Delta t} = \frac{10.31 - 10.31 \times 0.20 - 0.5 \times 6}{10.31 - 10.31 \times 0.20 + 0.5 \times 6} = 0.47$$

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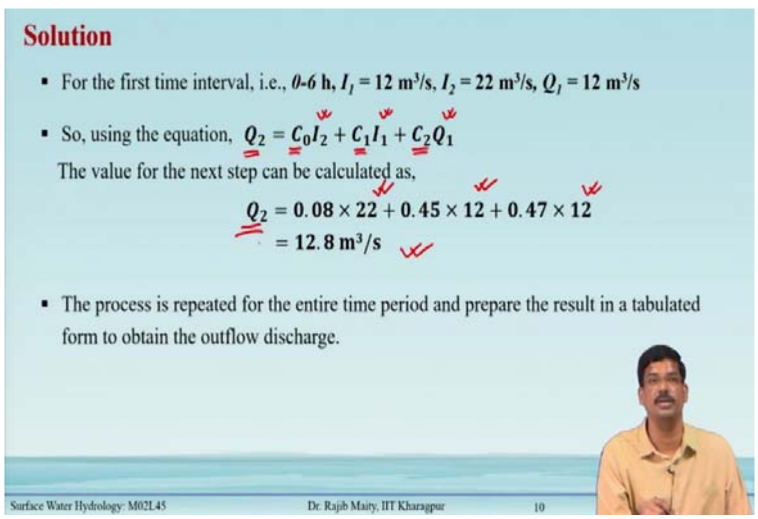

For the first time interval, i.e., ***0-6* h,** $\boldsymbol{I_1}$ **= 12 m**^{3}**/s**, $\boldsymbol{I_2}$ **= 22 m**^{3}**/s,** $\boldsymbol{Q_1}$ **= 12 m**^{3}**/s**

So, using the equation, $\boldsymbol{Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1}$

The value for the next step can be calculated as,

$$\boldsymbol{Q_2 = 0.08 \times 22 + 0.45 \times 12 + 0.47 \times 12}$$

$$\boldsymbol{= 12.8\ \mathrm{m}^3/\mathrm{s}}$$

The process is repeated for the entire time period and prepare the result in a tabulated form to obtain the outflow discharge.

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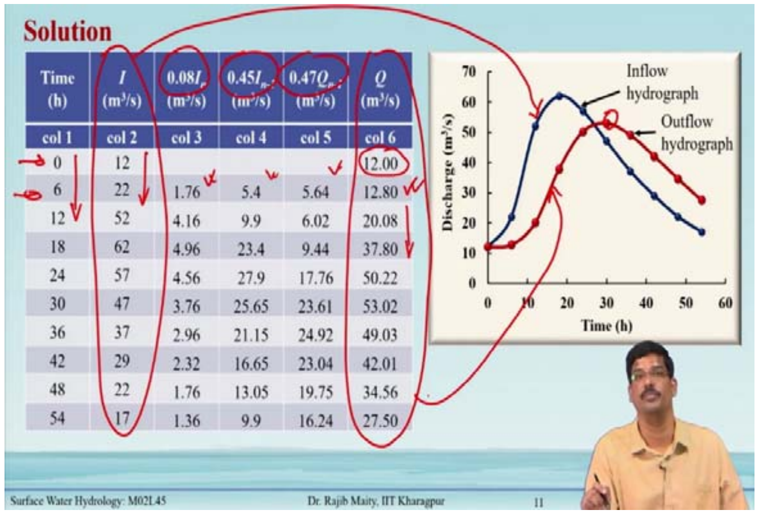

Time (h)	I (m³/s)	$0.08I_n$ (m³/s)	$0.45I_{n-1}$ (m³/s)	$0.47Q_{n-1}$ (m³/s)	Q (m³/s)
col 1	col 2	col 3	col 4	col 5	col 6
0	12				12.00
6	22	1.76	5.4	5.64	12.80
12	52	4.16	9.9	6.02	20.08
18	62	4.96	23.4	9.44	37.80
24	57	4.56	27.9	17.76	50.22
30	47	3.76	25.65	23.61	53.02
36	37	2.96	21.15	24.92	49.03
42	29	2.32	16.65	23.04	42.01
48	22	1.76	13.05	19.75	34.56
54	17	1.36	9.9	16.24	27.50

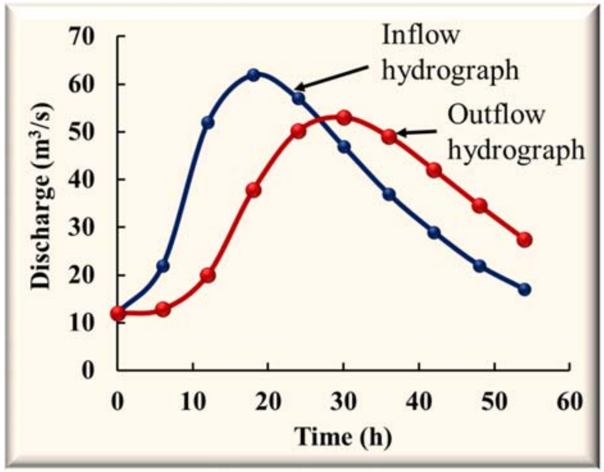

Figure 1: Hydrograph in channel routing of example 45.1

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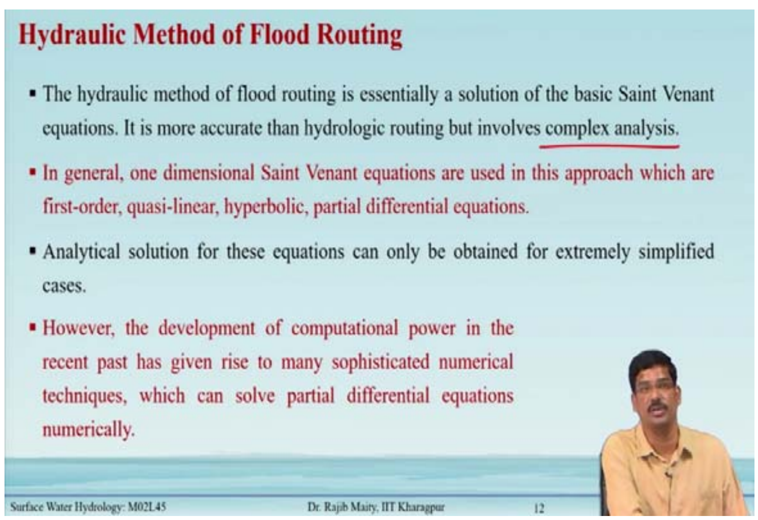

Hydraulic Method of Flood Routing

The hydraulic method of flood routing is essentially a solution of the basic Saint Venant equations. It is more accurate than hydrologic routing but involves complex analysis.

In general, one-dimensional Saint Venant equations are used in this approach which are first-order, quasi-linear, hyperbolic, partial differential equations.

Analytical solution for these equations can only be obtained for extremely simplified cases.

However, the development of computational power in the recent past has given rise to many sophisticated numerical techniques, which can solve partial differential equations numerically.

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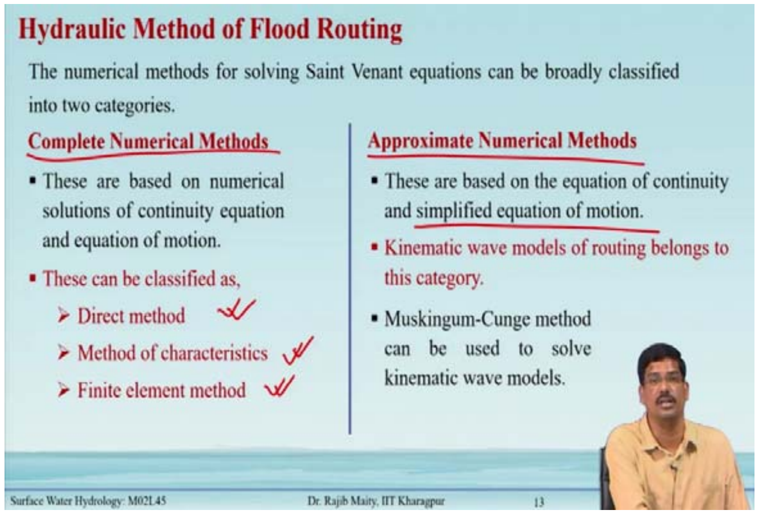

The numerical methods for solving Saint Venant equations can be broadly classified into two categories.

1. **Complete Numerical Methods**

- These are based on numerical solutions of continuity equation and equation of motion.
- These can be classified as,
  - Direct method
  - Method of characteristics
  - Finite element method

2. **Approximate Numerical Methods**

- These are based on the equation of continuity and simplified equation of motion.
- Kinematic wave models of routing belong to this category.
- Muskingum-Cunge method can be used to solve kinematic wave models.

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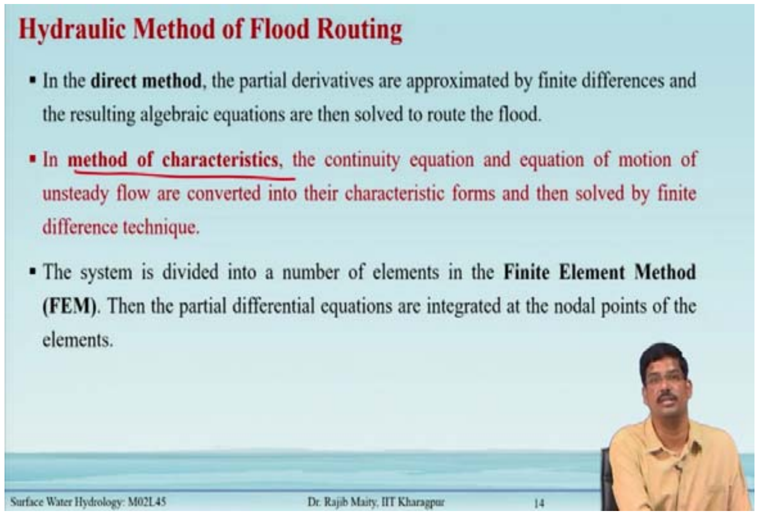

In the direct method, the partial derivatives are approximated by finite difference and resulting algebraic equations are then solved to route the flow. In case of the method of characteristics, the continuity equation and the equation of motion of the unsteady flow are converted to their characteristics form first and then solved by this finite difference technique.

The system is divided into number of elements in the Finite Element Method, FEM sometimes abbreviated as. Then the partial differential equations are integrated at the nodal point of the elements.

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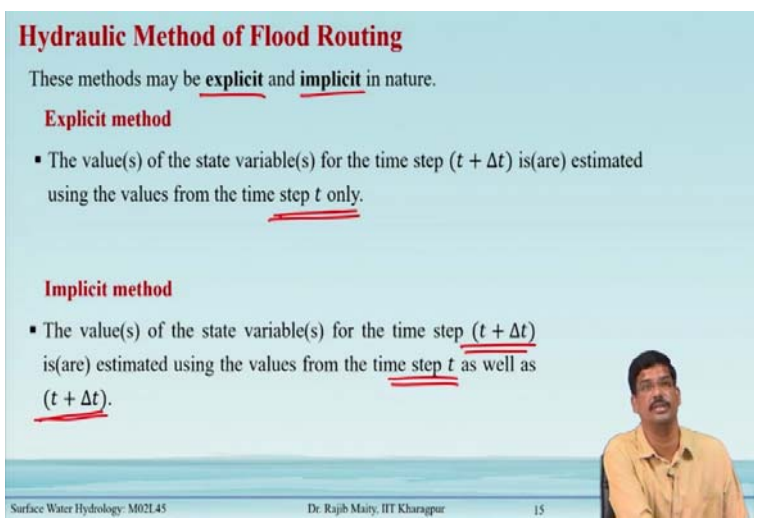

Now, while we exercise this finite element method, there could be two schemes: one is the explicit scheme, other one is the implicit scheme again.

Explicit method

The value(s) of the state variable(s) for the time step (t+Δt) is(are) estimated using the values from the time step t only.

Implicit method

The value(s) of the state variable(s) for the time step (t+Δt) is(are) estimated using the values from the time step t as well as (t+Δt).

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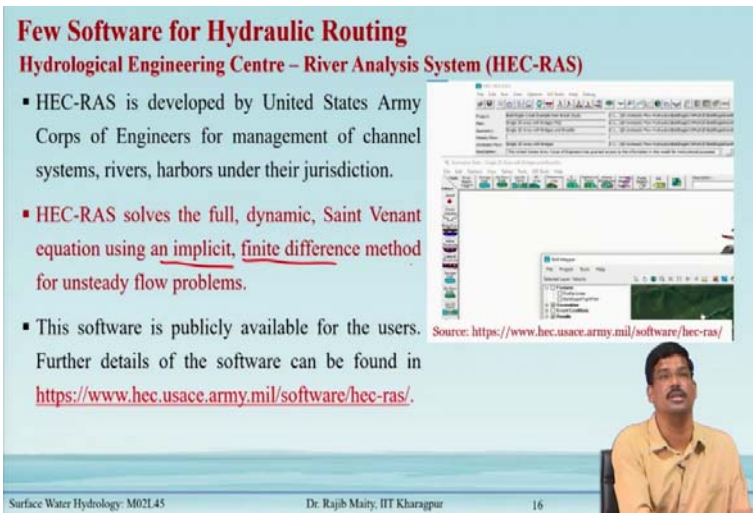

Few Software for Hydraulic Routing

Hydrological Engineering Centre – River Analysis System (HEC-RAS)

- HEC-RAS is developed by United States Army Corps of Engineers for management of channel systems, rivers, harbors under their jurisdiction.
- HEC-RAS solves the full, dynamic, Saint Venant equation using an implicit, finite difference method for unsteady flow problems.
- This software is publicly available for the users. Further details of the software can be found in https://www.hec.usace.army.mil/software/hec-ras/.

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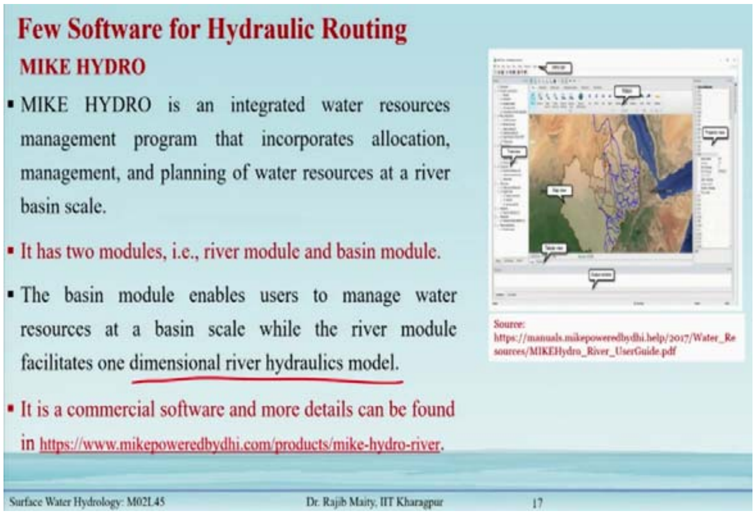

MIKE HYDRO

- MIKE HYDRO is an integrated water resources management program that incorporates allocation, management, and planning of water resources at a river basin scale.
- It has two modules, i.e., river module and basin module.
- The basin module enables users to manage water resources at a basin scale while the river module facilitates one dimensional river hydraulics model.
- It is a commercial software and more details can be found in https://www.mikepoweredbydhi.com/products/mike-hydro-river.

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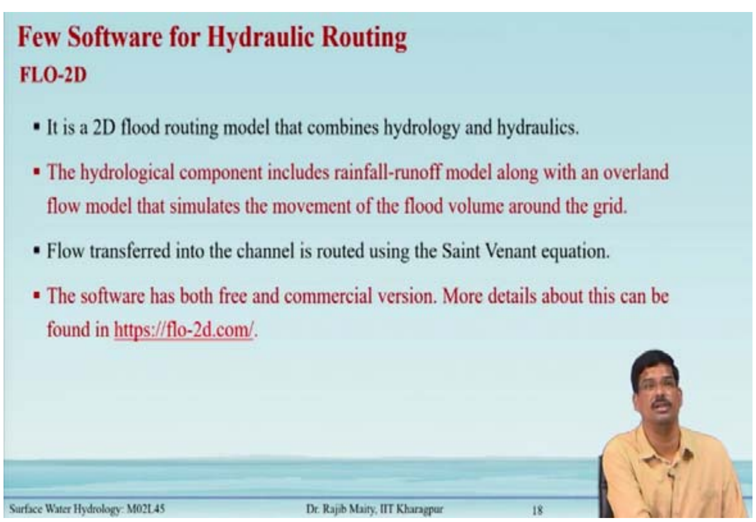

FLO-2D

- It is a 2D flood routing model that combines hydrology and hydraulics.
- The hydrological component includes rainfall-runoff model along with an overland flow model that simulates the movement of the flood volume around the grid.
- Flow transferred into the channel is routed using the Saint Venant equation.
- The software has both free and commercial version. More details about this can be found in https://flo-2d.com/.

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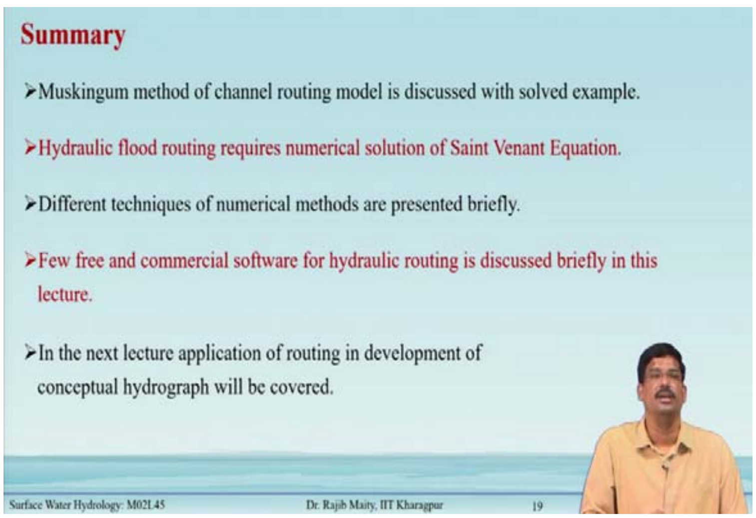

Summary

In summary, we learned the following points from this lecture:

- Muskingum method of channel routing model is discussed with solved example.
- Hydraulic flood routing requires numerical solution of Saint Venant Equation.
- Different techniques of numerical methods are presented briefly.
- Few free and commercial software for hydraulic routing is discussed briefly in this lecture.
- In the next lecture application of routing in development of conceptual hydrograph will be covered.