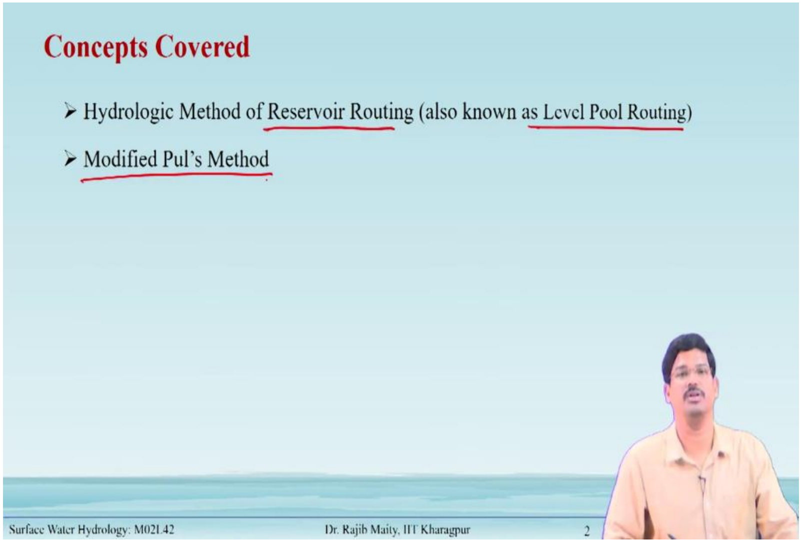
Surface Water Hydrology
Professor Rajib Maity
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture 42
Reservoir Routing: Modified Pul's Method

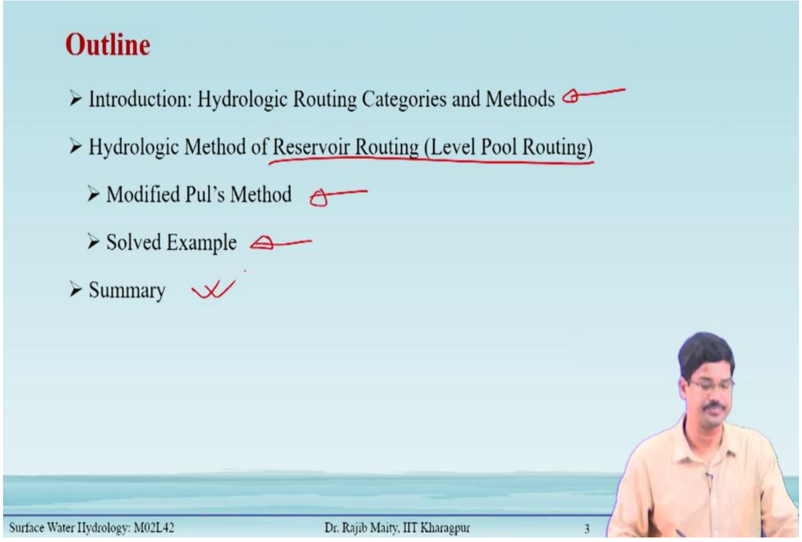
Today's lecture will discuss one of the popular desirable routing methods it is called the Modified Pul's method.

(Refer Slide Time: 00:32)



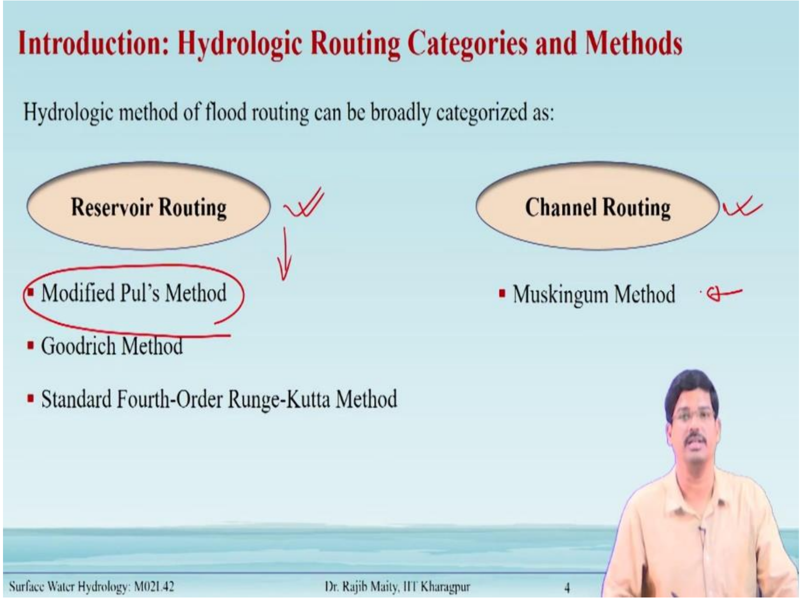
Under this concept cover, it is one of the Hydrological methods of reservoir routing. This reservoir routing is also known as the Level Pool Routing. And the method that we are discussing today is the Modified Pul's method.

(Refer Slide Time: 00:55)



The outline goes like this, first, we will give some introduction to the Hydrologic Routing categories and their methods. The second Hydrologic method of Reservoir Routing in this category we will discuss is also known as the Level Pool Routing. And one of the methods is Modified Pul's method will be discussed with a solved example before we go to the summary.

(Refer Slide Time: 01:32)



Introduction: Hydrologic Routing Categories and Methods

The hydrologic method of flood routing can be broadly categorized as:

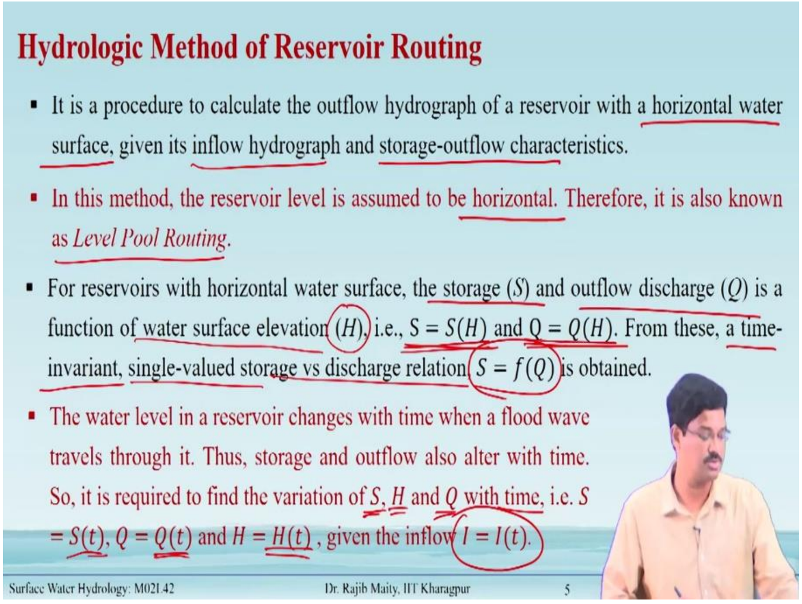
i. **Reservoir Routing**

Under this Reservoir Routing, the 3 methods that we will discuss are the Modified Pul's method Goodrich method and the Standard Fourth-Order Runge-Kutta method.

ii. **Channel Routing**

Under this Channel Routing also the popular method is the Muskingum method that all these methods will take upon after another in the subsequent lectures.

(Refer Slide Time: 02:24)



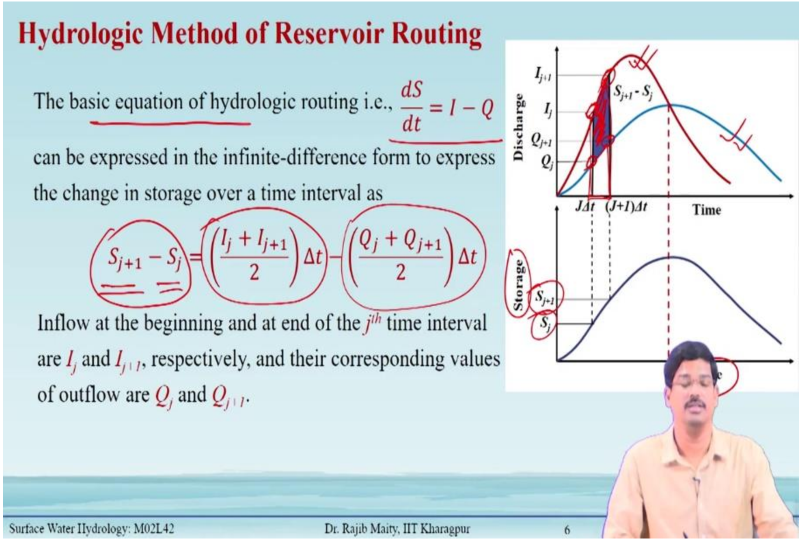
Hydrologic Method of Reservoir Routing

It is a procedure to calculate the outflow hydrograph of a reservoir with a horizontal water surface, given its inflow hydrograph and storage-outflow characteristics. In this method, the reservoir level is assumed to be horizontal. Therefore, it is also known as Level Pool Routing.

For reservoirs with a horizontal water surface, the storage (S) and outflow discharge (Q) is a function of water surface elevation (H), i.e., S=$S\ (H)$ and Q=$Q\ (H)$. From these, time-invariant, single-valued storage vs. discharge relation, S=$f\ (Q)$ is obtained.

The water level in a reservoir change with time when a flood wave travels through it. Thus, storage and outflow also alter with time. So, it is required to find the variation of S, H, and Q with time, i.e., $S = S\ (t)$, $Q = Q\ (t)$, and $H = H\ (t)$, given the inflow $I = I\ (t)$.

(Refer Slide Time: 04:38)

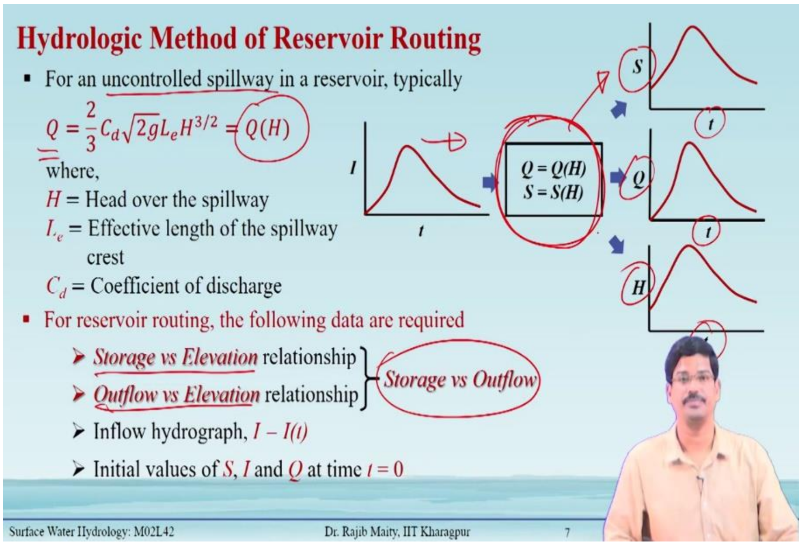


The basic equation of hydrologic routing i.e. $\frac{dS}{dt} = I - Q$ can be expressed in the infinite-difference form to express the change in storage over a time interval as

$$S_{j+1} - S_j = \left(\frac{I_j + I_{j+1}}{2}\right)\Delta t - \left(\frac{Q_j + Q_{j+1}}{2}\right)\Delta t$$

Inflow at the beginning and at end of the j^{th} time interval are I_j and I_{j+1}, respectively, and their corresponding values of outflow are Q_j and Q_{j+1}.

(Refer Slide Time: 06:46)



An uncontrolled spillway in a reservoir, typically

$$Q = \frac{2}{3} C_d \sqrt{2g} L_e H^{3/2} = Q(H)$$

where,

H = Head over the spillway

L_e = Effective length of the spillway crest

C_d = Coefficient of discharge

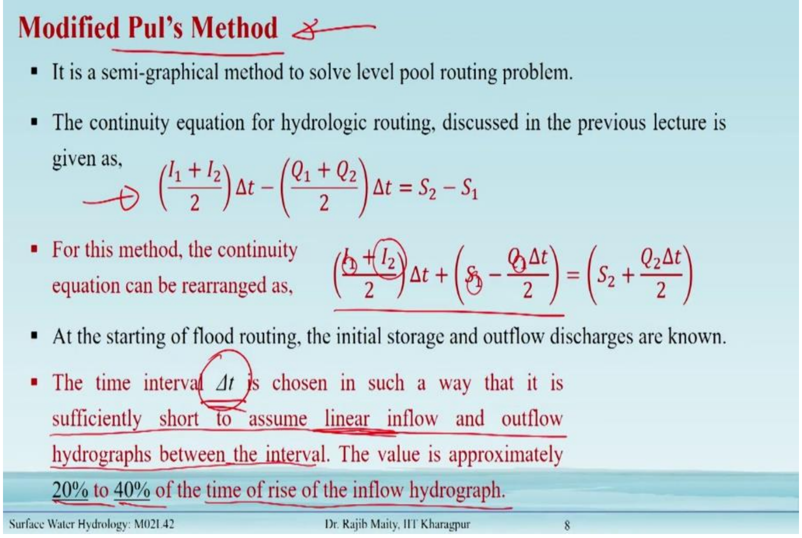
For reservoir routing, the following data are required

- Storage versus elevation relationship,
- Outflow versus elevation relationship.

The above two things are known and that gives the relationship the storage versus outflow.

- Inflow hydrograph, *I = I (t)*
- Initial values *S, I,* and *Q* at time *t=0*

(Refer Slide Time: 07:43)



Modified Pul's Method

It is a semi-graphical method to solve the level pool routing problem. The continuity equation for hydrologic routing, discussed in the previous lecture is given as,

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t = S_2 - S_1$$

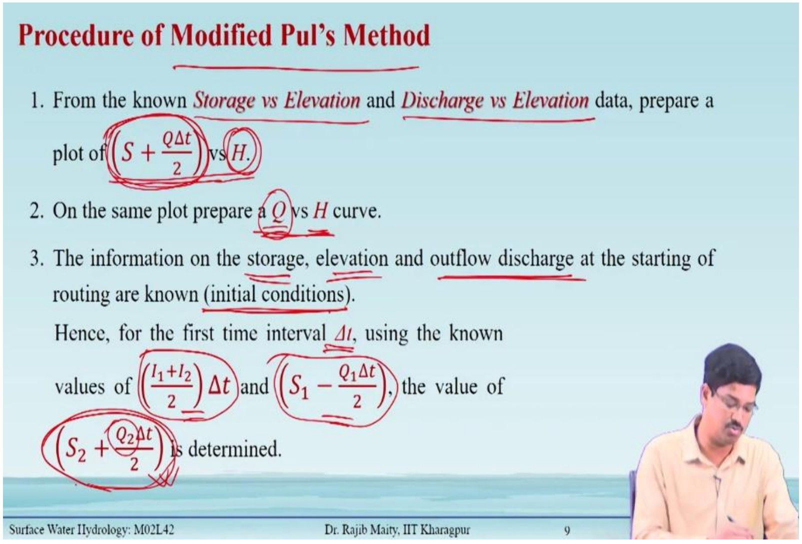
For this method, the continuity equation can be rearranged as,

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t + \left(S_1 - \frac{Q_1 \Delta t}{2}\right) = \left(S_2 + \frac{Q_2 \Delta t}{2}\right)$$

At the start of flood routing, the initial storage and outflow discharges are known.

The time interval Δt is chosen in such a way that it is sufficiently short to assume linear inflow and outflow hydrographs between the interval. The value is approximately 20% to 40% of the time of the rise of the inflow hydrograph.

(Refer Slide Time: 09:06)

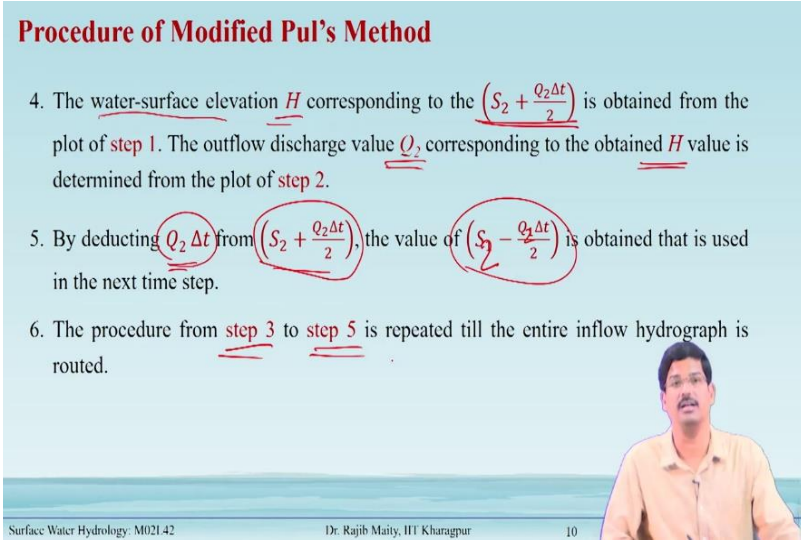


The procedure of Modified Pul's Method

1. From the known Storage vs. Elevation and Discharge vs Elevation data, prepare a plot of $(S+Q\Delta t/2)$ vs. H.
2. On the same plot prepare a Q vs. H curve.
3. The information on the storage, elevation and outflow discharge at the start of routing are known (initial conditions).

Hence, for the first-time interval Δt, using the known values of $\left(\frac{I_1+I_2}{2}\right)\Delta t$ and, the $\left(S_1 - \frac{Q_1\Delta t}{2}\right)$ value of is $\left(S_2 + \frac{Q_2\Delta t}{2}\right)$ determined.

(Refer Slide Time: 11:35)

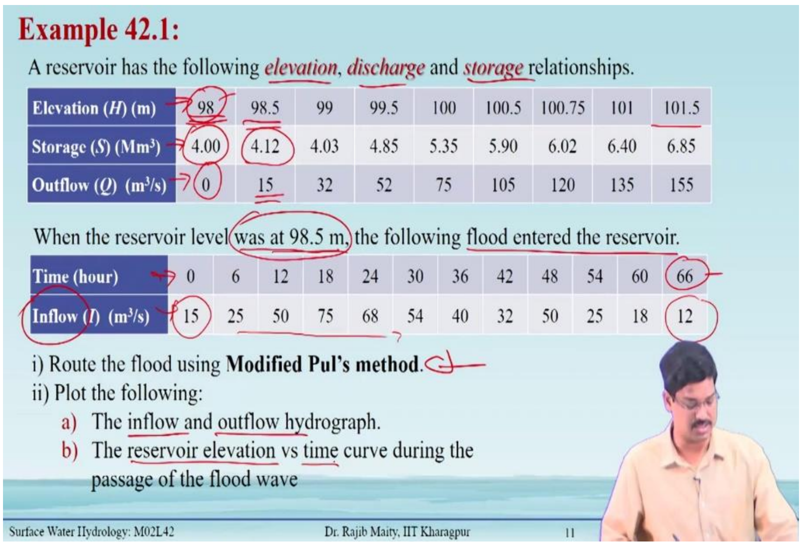


4. The water-surface elevation H corresponding to the $\left(S_2 + \frac{Q_2 \Delta t}{2}\right)$ is obtained from the plot of step 1. The outflow discharge value Q_2 corresponding to the obtained H value is determined from the plot of step 2.

5. By deducting $Q_2\, \Delta t$ from $\left(S_2 + \frac{Q_2 \Delta t}{2}\right)$, the value of $\left(S_2 - \frac{Q_2 \Delta t}{2}\right)$ is obtained that is used in the next time step.

The procedure from step 3 to step 5 is repeated till the entire inflow hydrograph is routed.

(Refer Slide Time: 12:18)



Example 42.1:

A reservoir has the following elevation, discharge, and storage relationships.

Elevation (H) (m)	98	98.5	99	99.5	100	100.5	100.75	101	101.5
Storage (S) (Mm³)	4.00	4.12	4.03	4.85	5.35	5.90	6.02	6.40	6.85
Outflow (Q) (m³/s)	0	15	32	52	75	105	120	135	155

When the reservoir level was at 98.5 m, the following flood entered the reservoir.

Time (hour)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (I) (m³/s)	15	25	50	75	68	54	40	32	50	25	18	12

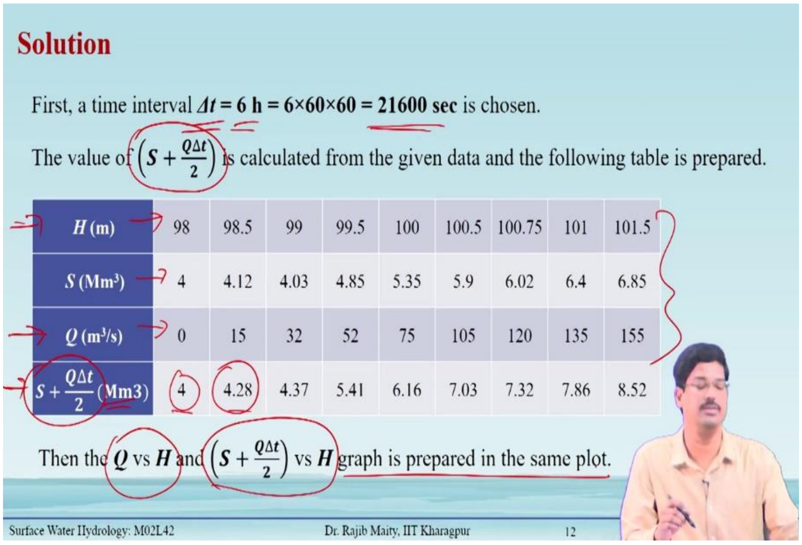
i) Route the flood using Modified Pul's method.

ii) Plot the following:

a) The inflow and outflow hydrograph.

b) The reservoir elevation vs time curve during the passage of the flood wave

(Refer Slide Time: 14:15)



Solution

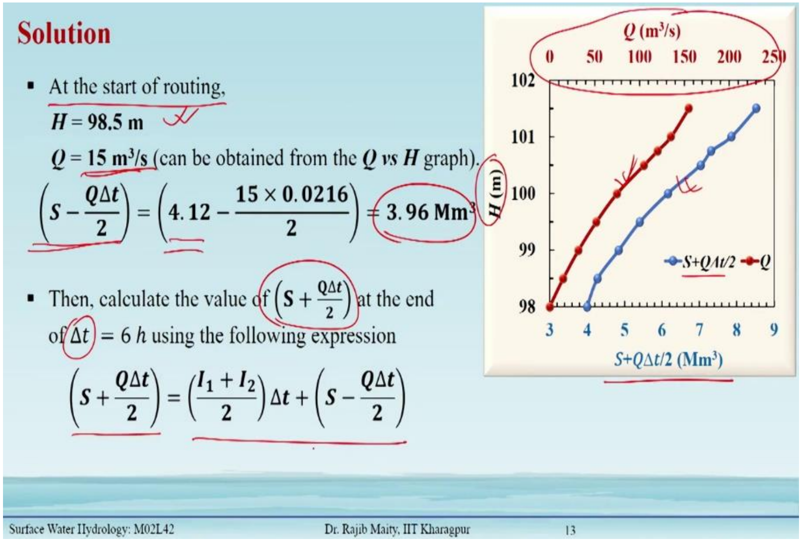
First, a time interval $\Delta t = 6$ h $= 6\times60\times60 = 21600$ sec is chosen.

The value $\left(S + \frac{Q\Delta t}{2}\right)$ is calculated from the given data and the following table is prepared.

H (m)	98	98.5	99	99.5	100	100.5	100.75	101	101.5
S (Mm^3)	4	4.12	4.03	4.85	5.35	5.9	6.02	6.4	6.85
Q (m^3/s)	0	15	32	52	75	105	120	135	155
$S+\frac{Q\Delta t}{2}$ (Mm3)	4	4.28	4.37	5.41	6.16	7.03	7.32	7.86	8.52

Then the $\boldsymbol{Q}$ vs $\boldsymbol{H}$ and $\left(\boldsymbol{S}+\frac{\boldsymbol{Q\Delta t}}{\boldsymbol{2}}\right)$ vs $\boldsymbol{H}$ graph is prepared in the same plot.

(Refer Slide Time: 15:39)



At the start of routing,

H = 98.5 m

Q = 15 m^3/s (can be obtained from the Q vs. H graph).

$$\left(S-\frac{Q\Delta t}{2}\right)=\left(4.12-\frac{15\times 0.0216}{2}\right)=3.96\ \mathrm{Mm^3}$$

Then, calculate the value $\left(S+\frac{Q\Delta t}{2}\right)$ at the end of $\Delta t\ =6\ h$ using the following expression

$$\left(S+\frac{Q\Delta t}{2}\right)=\left(\frac{I_1+I_2}{2}\right)\Delta t+\left(S-\frac{Q\Delta t}{2}\right)$$

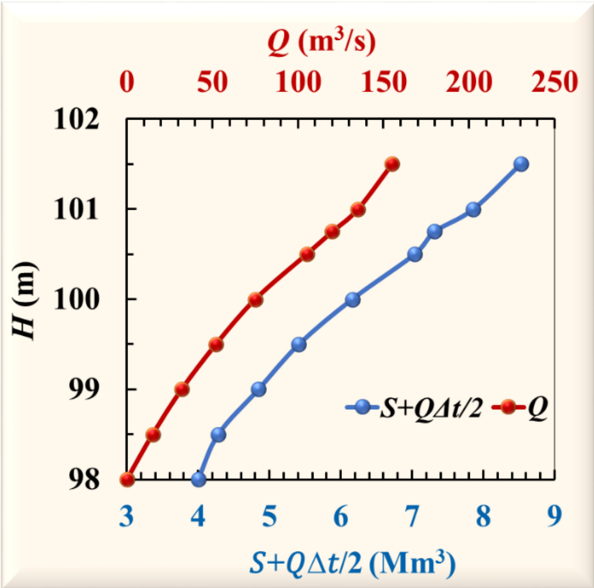
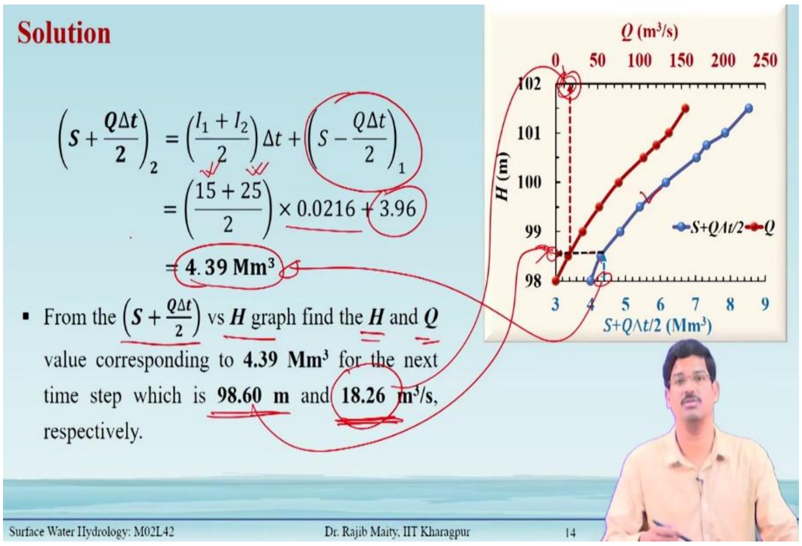


Figure 3: Modified Pul's Method of Storage routing

(Refer Slide Time: 17:00)



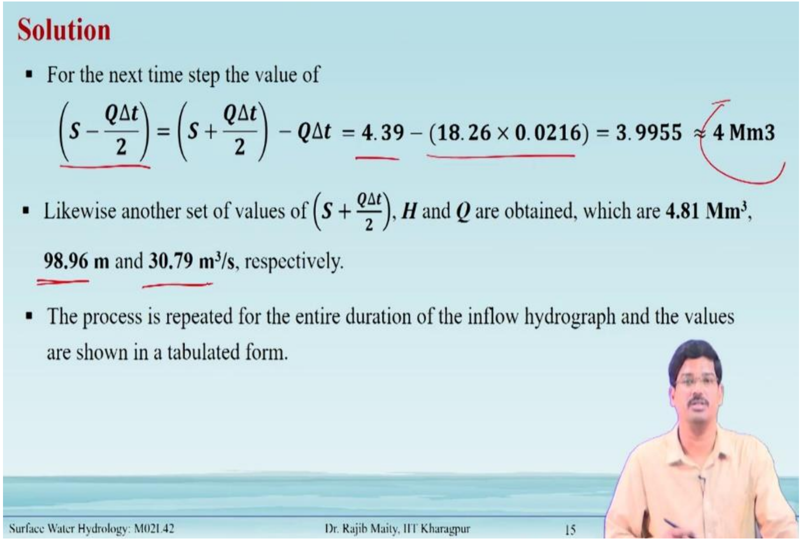
$$\left(\boldsymbol{S}+\frac{\boldsymbol{Q\Delta t}}{\boldsymbol{2}}\right)_{\boldsymbol{2}}=\left(\frac{I_1+I_2}{2}\right)\Delta t+\left(S-\frac{Q\Delta t}{2}\right)_1$$

$$=\left(\frac{15+25}{2}\right)\times 0.0216+3.96$$

$$=\mathbf{4.39\ Mm^3}$$

From the $\left(\boldsymbol{S}+\frac{\boldsymbol{Q\Delta t}}{\boldsymbol{2}}\right)$ vs. $\boldsymbol{H}$ graph find the $\boldsymbol{H}$ and $\boldsymbol{Q}$ value corresponding to **4.39 Mm³** for the next time step which is **98.60 m** and **18.26 m³/s**, respectively.

(Refer Slide Time: 18.35)

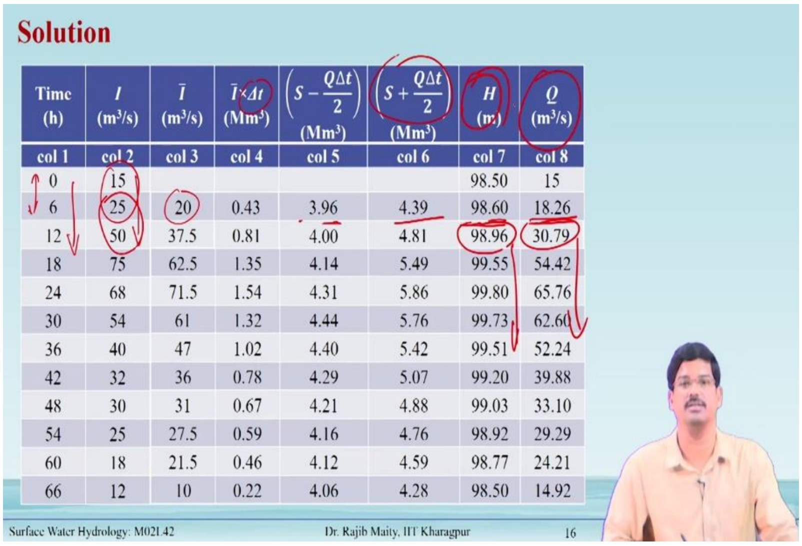


For the next time step the value of

$$\left(S-\frac{Q\Delta t}{2}\right)=\left(S+\frac{Q\Delta t}{2}\right)-Q\Delta t=4.39-(18.26\times 0.0216)=3.9955\approx 4\ \mathrm{Mm3}$$

Likewise, another set of values of $\left(S+\frac{Q\Delta t}{2}\right)$, H and Q are obtained, which are **4.81 Mm³**, **98.96 m,** and **30.79 m³/s**, respectively.

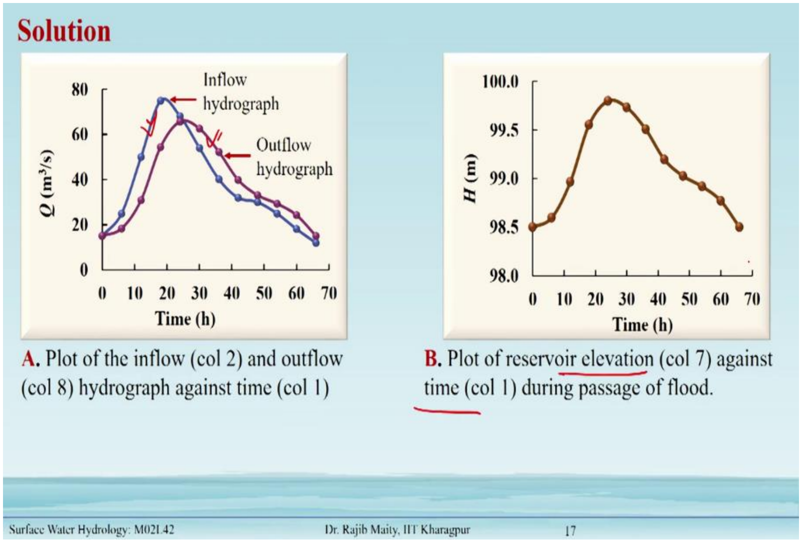
(Refer Slide Time: 19:02)

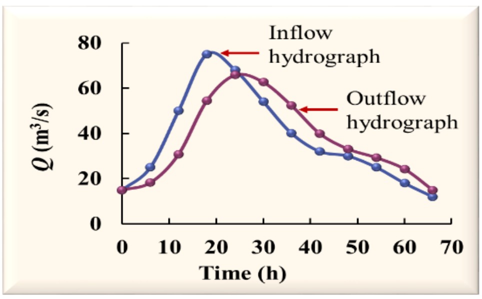


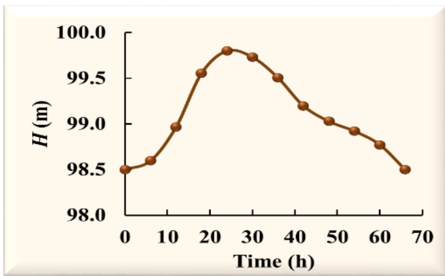
Time (h)	I (m³/s)	$\bar{I}$ (m³/s)	$\bar{I}\times\Delta t$ (Mm³)	$\left(S-\frac{Q\Delta t}{2}\right)$ (Mm³)	$\left(S+\frac{Q\Delta t}{2}\right)$ (Mm³)	H (m)	Q (m³/s)
col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8
0	15					98.50	15
6	25	20	0.43	3.96	4.39	98.60	18.26
12	50	37.5	0.81	4.00	4.81	98.96	30.79
18	75	62.5	1.35	4.14	5.49	99.55	54.42
24	68	71.5	1.54	4.31	5.86	99.80	65.76
30	54	61	1.32	4.44	5.76	99.73	62.60
36	40	47	1.02	4.40	5.42	99.51	52.24
42	32	36	0.78	4.29	5.07	99.20	39.88
48	30	31	0.67	4.21	4.88	99.03	33.10
54	25	27.5	0.59	4.16	4.76	98.92	29.29
60	18	21.5	0.46	4.12	4.59	98.77	24.21
66	12	10	0.22	4.06	4.28	98.50	14.92

Time (h)	I (m^3/s)	$\bar{I}$ (m^3/s)	$\bar{I}\times\Delta t$ (Mm^3)	$\left(S-\frac{Q\Delta t}{2}\right)$ (Mm^3)	$\left(S+\frac{Q\Delta t}{2}\right)$ (Mm^3)	H (m)	Q (m^3/s)
col 1	col 2	col 3	col 4	col 5	col 6	col 7	col 8
0	15					98.50	15
6	25	20	0.43	3.96	4.39	98.60	18.26
12	50	37.5	0.81	4.00	4.81	98.96	30.79
18	75	62.5	1.35	4.14	5.49	99.55	54.42
24	68	71.5	1.54	4.31	5.86	99.80	65.76
30	54	61	1.32	4.44	5.76	99.73	62.60
36	40	47	1.02	4.40	5.42	99.51	52.24
42	32	36	0.78	4.29	5.07	99.20	39.88
48	30	31	0.67	4.21	4.88	99.03	33.10
54	25	27.5	0.59	4.16	4.76	98.92	29.29
60	18	21.5	0.46	4.12	4.59	98.77	24.21
66	12	10	0.22	4.06	4.28	98.50	14.92

(Refer Slide Time: 20:53)



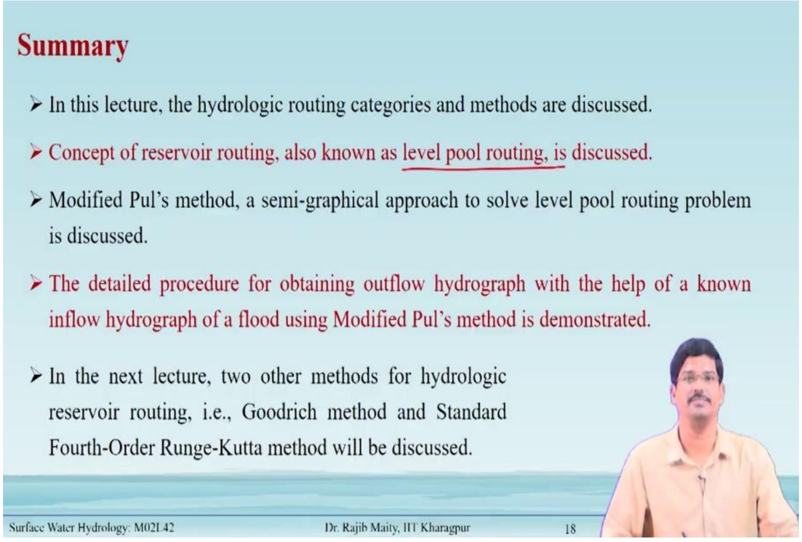




A. Plot of the inflow (col 2) and outflow (col 8) hydrograph against time (col 1)

B. Plot of reservoir elevation (col 7) against time (col 1) during the passage of flood.

(Refer Slide Time: 21:27)



Summary

In summary, we learned the following points from this lecture:

- In this lecture, the hydrologic routing categories and methods are discussed.
- The concept of reservoir routing, also known as level pool routing, is discussed.
- Modified Pul's method, a semi-graphical approach to solving level pool routing problems is discussed.
- The detailed procedure for obtaining an outflow hydrograph with the help of a known inflow hydrograph of a flood using Modified Pul's method is demonstrated.
- In the next lecture, two other methods for hydrologic reservoir routing, i.e., the Goodrich method and the Standard Fourth-Order Runge-Kutta method will be discussed.