Surface Water Hydrology
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Lecture 36
Instantaneous Unit Hydrograph

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Today is the last topic that we will cover which is instantaneous unit hydrograph, so it is having some very special properties, and it is very useful in many applications.

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The concept covered in this today's lecture is the instantaneous unit hydrograph understanding and its derivation and applications through some example problems.

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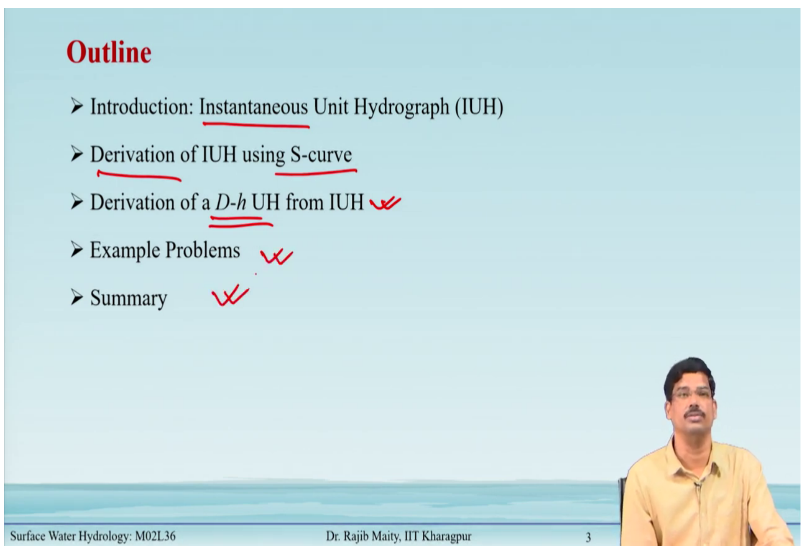

The outline for this lecture goes like this, first, we will give some introduction to this IUH, instantaneous unit hydrograph, and then derivation of this unit hydrograph using S-curve method lecture and then from there the derivation of D-hour unit hydrograph from the IUH. Some example problems will be taken and then finally, coming to the summary.

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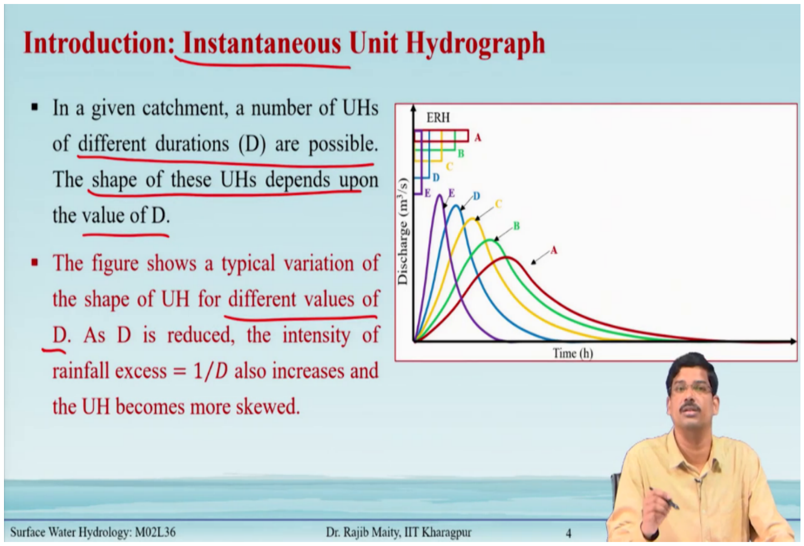

Introduction: Instantaneous Unit Hydrograph

In a given catchment a number of unit hydrographs of different durations are possible. From the field, data can get the most accurate estimate of the unit hydrograph. Now, if all the desired durations are not available, to get the unit hydrograph of different durations using two methods, method of superposition, S-curve method, all those things are discussed.

The different durations of the unit hydrograph for a specific catchment are there and the shape of this unit hydrograph depends on the value of the D. In figure1 a typical variation of the shape of the unit hydrograph for different values of D has been shown.

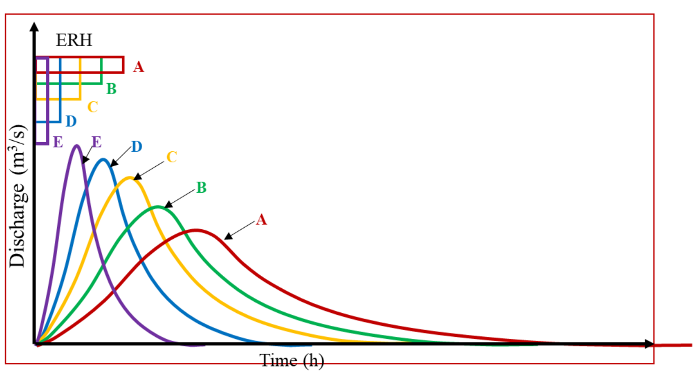

Fig.1shows a typical variation of the shape of UH for different values of D.

If this is the one-unit hydrograph that corresponds to this rainfall excess hydrograph is shown there at A. In fig.1 diagram shows all the properties of the unit hydrograph and here also the rainfall excess is 1 centimeter and only that duration is a specific duration of that 1-centimeter rainfall excess is shown here. So, far as the unit hydrograph A is concerned.

A is shown here in the excess rainfall and the corresponding unit hydrograph. Now if reduce the duration as shown in the green color box B there and the excess rainfall remains same 1 centimeter. So, if I reduce the duration intensity is increased that is why the corresponding typical change in the shape of the unit hydrograph is shown in B.

Typically, the time to peak is lesser, so the peak will occur faster and the peak will be more than the earlier one that is with respect to A. Now, if I further proceed take another one where the duration is further less. So, intensity is even more. So, this will go on, so that time to peak will be even faster and the peak discharge also will be more.

In this way if go ahead and just take that different unit hydrograph of different duration, means the intensity is increasing from hydrograph A to B to C to D. Similarly, the limiting condition is that the duration can tend to 0.

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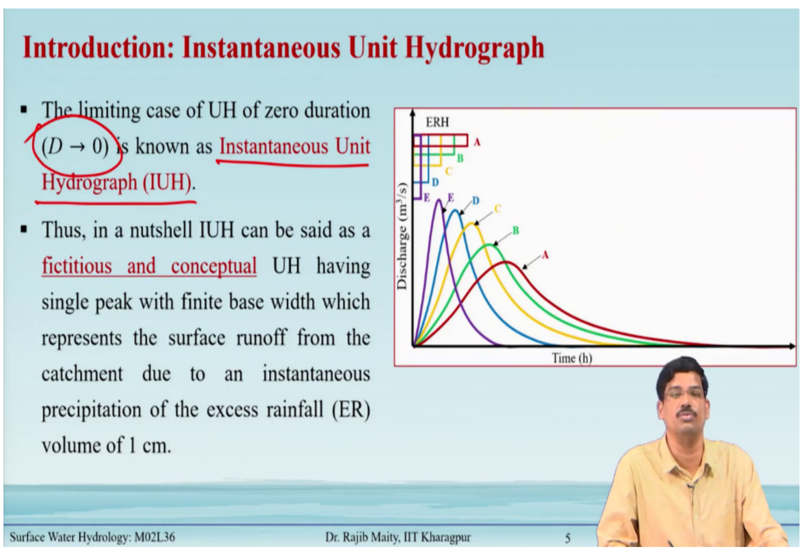

The limiting case of UH of zero duration (D → 0) is known as Instantaneous Unit Hydrograph (IUH).

Thus, in a nutshell, IUH can be said as a fictitious and conceptual UH having a single peak with finite base width which represents the surface runoff from the catchment due to instantaneous precipitation of the excess rainfall (ER) volume of 1 cm.

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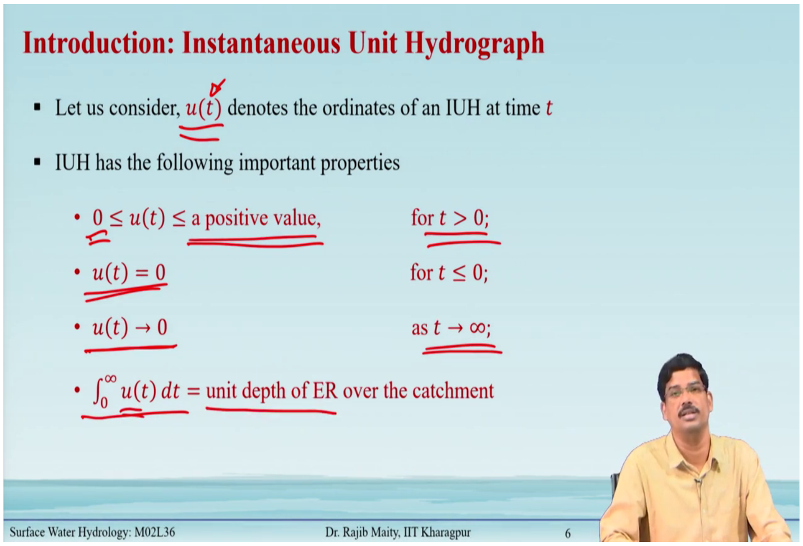

Let us consider, $u(t)$ denotes the ordinates of an IUH at time t

IUH has the following important properties

- $0 \leq u(t) \leq$ a positive value, for $t > 0$;
- $u(t) = 0$ for $t \leq 0$;
- $u(t) \to 0$ as $t \to \infty$;
- $\int_0^\infty u(t)\,dt =$ unit depth of ER over the catchment

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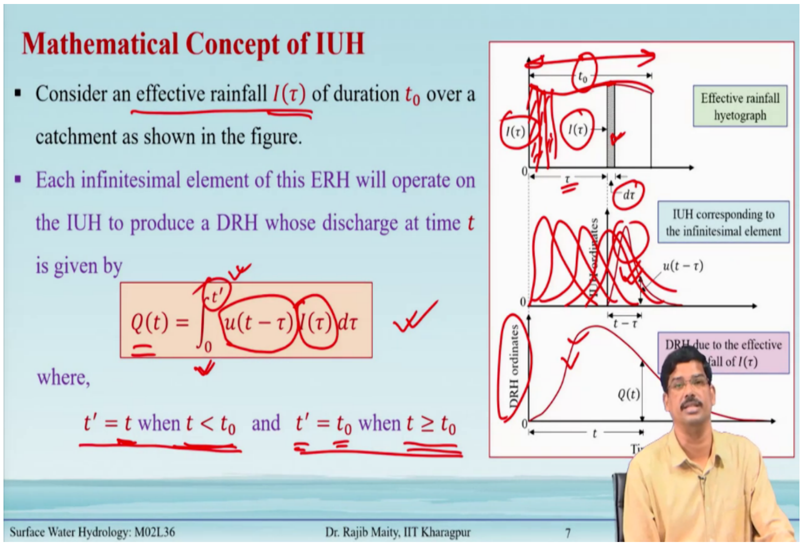

The concept of this unit hydrograph is concerned, now, we can see in this pictorial diagram in fig.2. Consider an effective rainfall $I(\tau)$ of duration t_0 over a catchment as shown in figure 2. τ is the intensity varies over the time and it is just changing continuously that is the most practical thing that it can change at any given time.

Assume one very small infinitesimal small-time unit which is dτ and for the small-time interval, the intensity is the same. Now, if I know that there is one instantaneous unit hydrograph then the contribution of this shaded area will come in the form of an instantaneous unit hydrograph like this.

Each infinitesimal element of this ERH will operate on the IUH to produce a DRH whose discharge at time t is given by

$$Q(t) = \int_0^{t'} u(t-\tau)\, I(\tau)\, d\tau$$

Where,

$$t' = t \text{ when } t < t_0 \quad \text{and} \quad t' = t_0 \text{ when } t \geq t_0$$

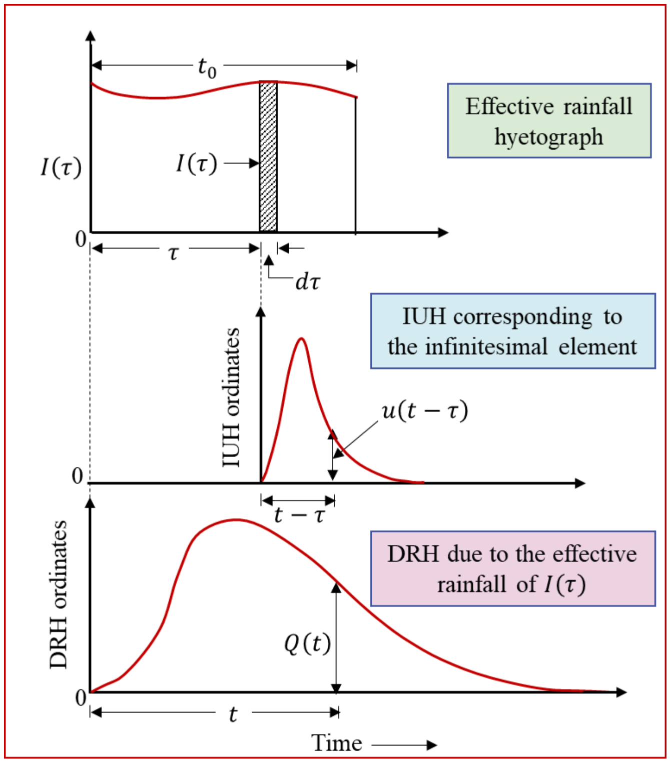

Fig.2 shows the convolution of I (τ) and IUH

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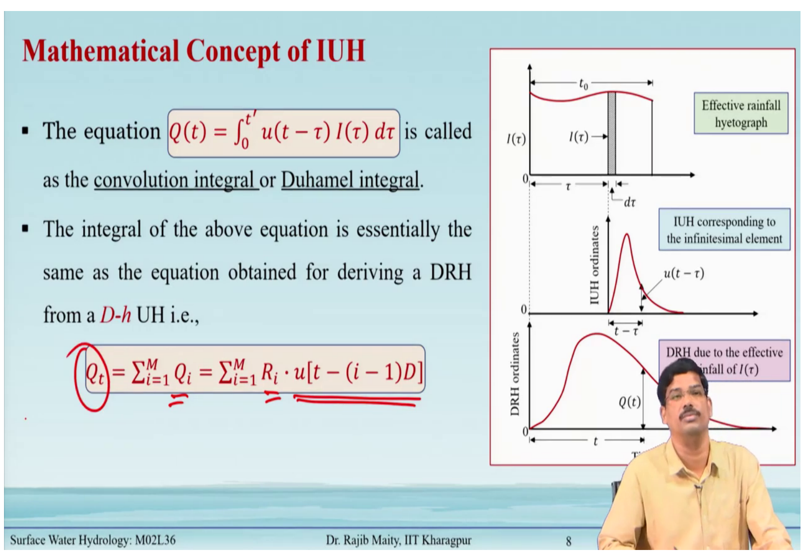

Mathematical Concept of IUH

$$Q(t) = \int_0^{t'} u(t-\tau)\, I(\tau)\, d\tau$$

The equation is called the convolution integral or Duhamel integral.

The integral of the above equation is essentially the same as the equation obtained for deriving a DRH from a D-h UH i.e.,

$$Q_t = \sum_{i=1}^{M} Q_i = \sum_{i=1}^{M} R_i \cdot u[t-(i-1)D]$$

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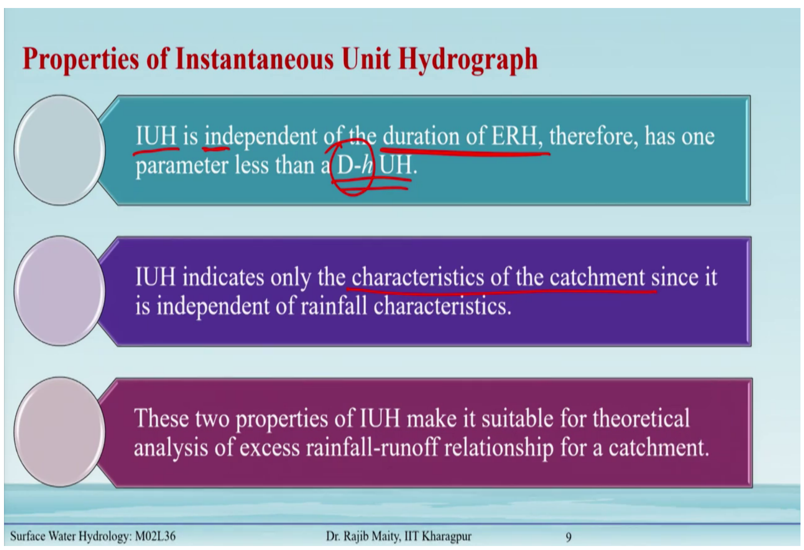

Properties of Instantaneous Unit Hydrograph

IUH is independent of the duration of ERH, therefore, has one parameter less than a D-*h* UH.

IUH indicates only the characteristics of the catchment since it is independent of rainfall characteristics.

These two properties of IUH make it eminently suitable for theoretical analysis of excess rainfall-runoff relationship for a catchment.

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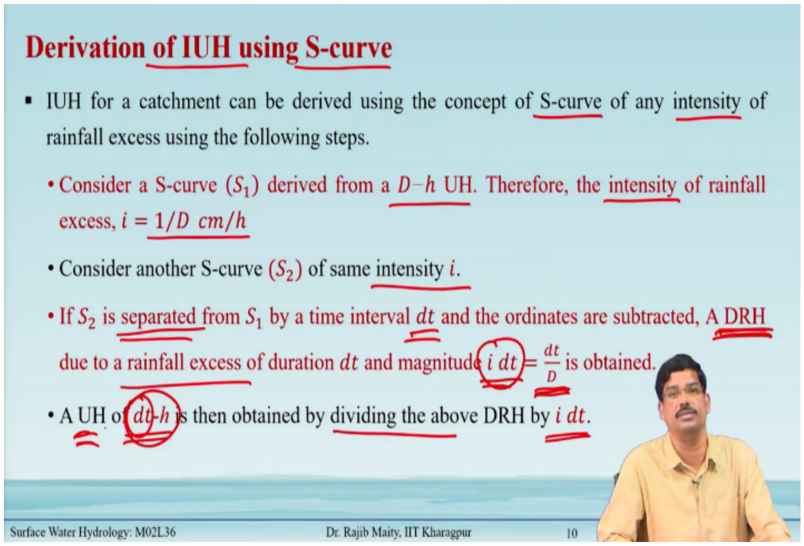

Derivation of IUH using S-curve

IUH for a catchment can be derived using the concept of an S-curve of any intensity of rainfall excess using the following steps.

Consider an S-curve (S_1)(S_1) derived from a D-h$D - h$ UH. Therefore, the intensity of rainfall excess, i =1/D cm/hr$i = \frac{1}{D}\frac{cm}{h}$

Consider another S-curve $S_{2(S_2)}$ of the same intensity *ii*.

If S_{2S_2} is separated from S_{1S_1} by a time interval dtdt and the ordinates are subtracted, A DRH due to a rainfall excess of duration dtdt and magnitude *i dt=dt/D*$i\ dt = \frac{dt}{D}$ is obtained.

A UH of *dtdt-h* is then obtained by dividing the above DRH by *i dti dt*

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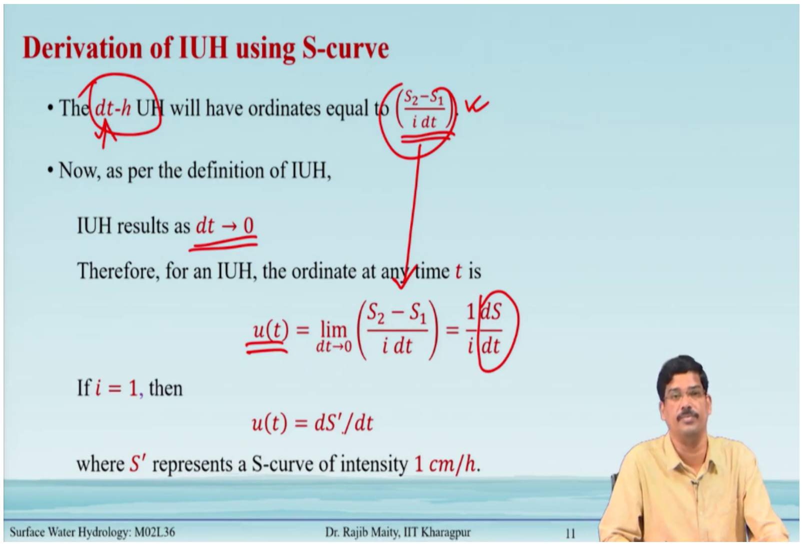

The dt-h UH will have ordinates equal to $\left(\frac{S_2 - S_1}{i\ dt}\right)$

Now, as per the definition of IUH,

IUH results as $dt \rightarrow 0$

Therefore, for an IUH, the ordinate at any time t is

$$u(t) = \lim_{dt\rightarrow 0}\left(\frac{S_2 - S_1}{i\ dt}\right) = \frac{1}{i}\frac{dS}{dt}$$

If i=1, then

$$u(t) = \frac{dS^{'}}{dt}$$

Where S' represents an S-curve of intensity 1 cm/h.

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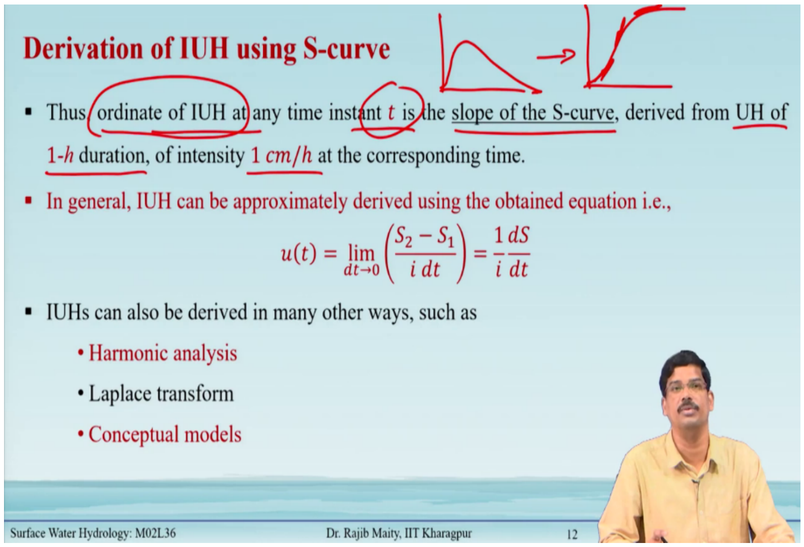

Thus, the ordinate of IUH at any time instant t is the slope of the S-curve, derived from UH of 1-h duration, of intensity 1 cm/h at the corresponding time.

In general, IUH can be approximately derived using the obtained equation i.e.,

$$u(t) = \left(\frac{S_2 - S_1}{i\,dt}\right) = \frac{1}{i}\frac{dS}{dt}$$

IUHs can also be derived in many other ways, such as

i. Harmonic analysis

ii. Laplace transform

iii. Conceptual models

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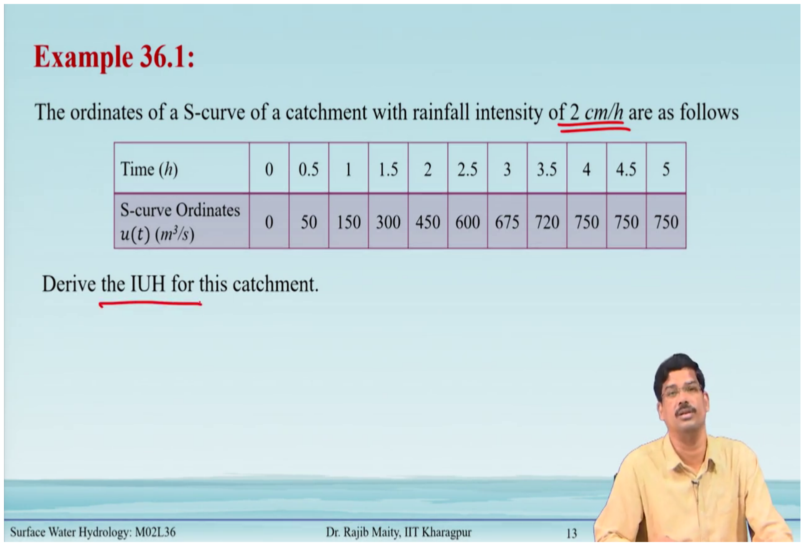

Example 36.1:

The ordinates of an S-curve of a catchment with rainfall intensity of 2 *cm/h* are as follows

Time (h)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
S-curve Ordinates $u(t)$ (m^3/s)	0	50	150	300	450	600	675	720	750	750	750

Derive the IUH for this catchment.

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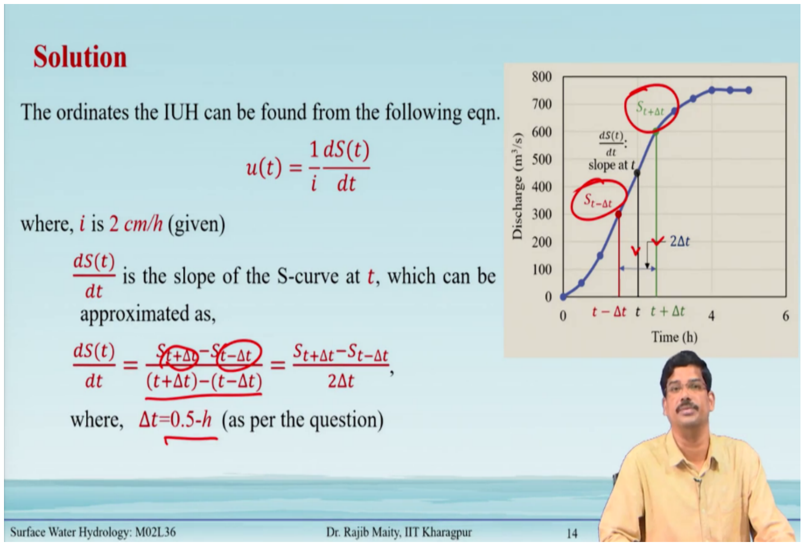

Solution

The ordinates of the IUH can be found from the following Eqn.

$$u(t) = \frac{1}{i}\frac{dS(t)}{dt}$$

Where, i is 2 cm/h (given)

$\frac{dS(t)}{dt}$ is the slope of the S-curve at t, which can be approximated as,

$$\frac{dS(t)}{dt} = \frac{S_{t+\Delta t} - S_{t-\Delta t}}{(t+\Delta t)-(t-\Delta t)} = \frac{S_{t+\Delta t} - S_{t-\Delta t}}{2\Delta t},$$

Where Δt=0.5-h (as per the question)

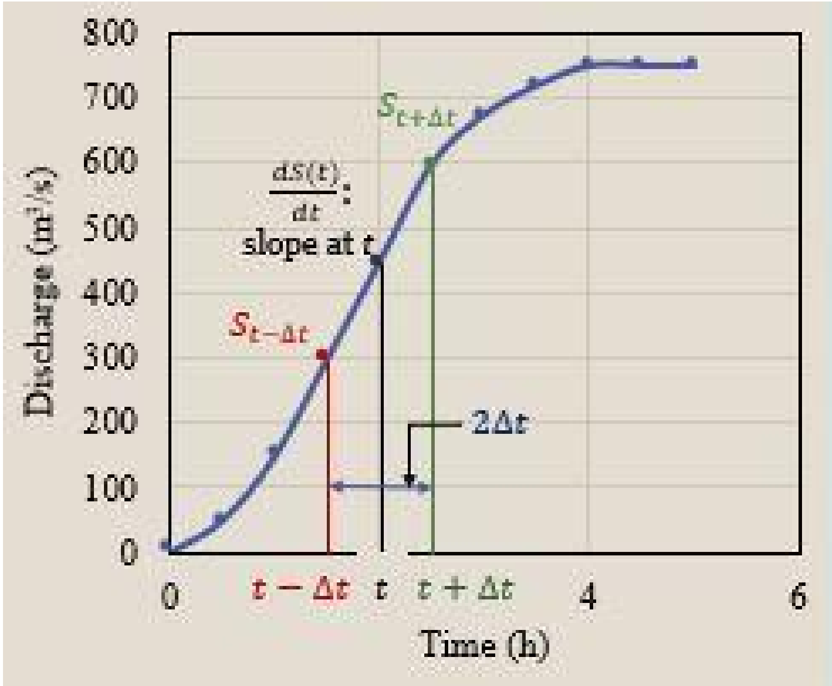

Fig.3 shows the ordinates of the S-curve of example 36.1

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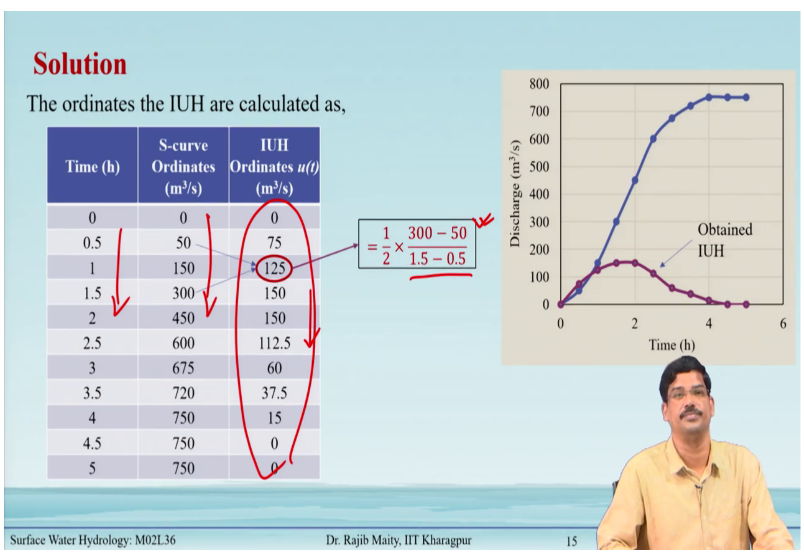

The ordinates the IUH are calculated as,

Time (h)	S-curve Ordinates (m³/s)	IUH Ordinates *u(t)* (m³/s)
0	0	0
0.5	50	75
1	150	125
1.5	300	150
2	450	150
2.5	600	112.5
3	675	60
3.5	720	37.5
4	750	15
4.5	750	0
5	750	0

$$= \frac{1}{2} \times \frac{300 - 50}{1.5 - 0.5}$$

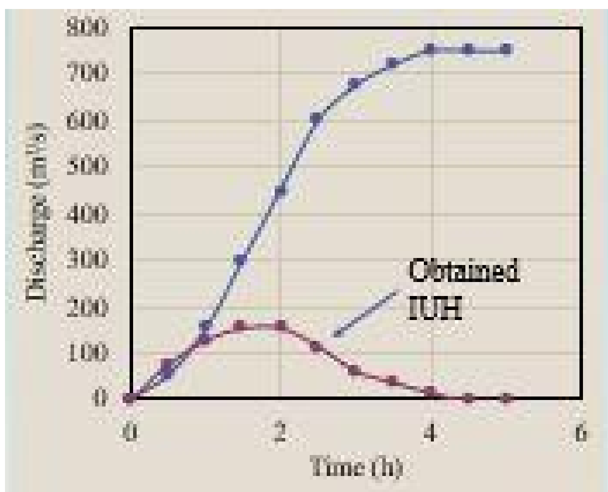

Fig.4 shows the ordinates of IUH of example 36.1

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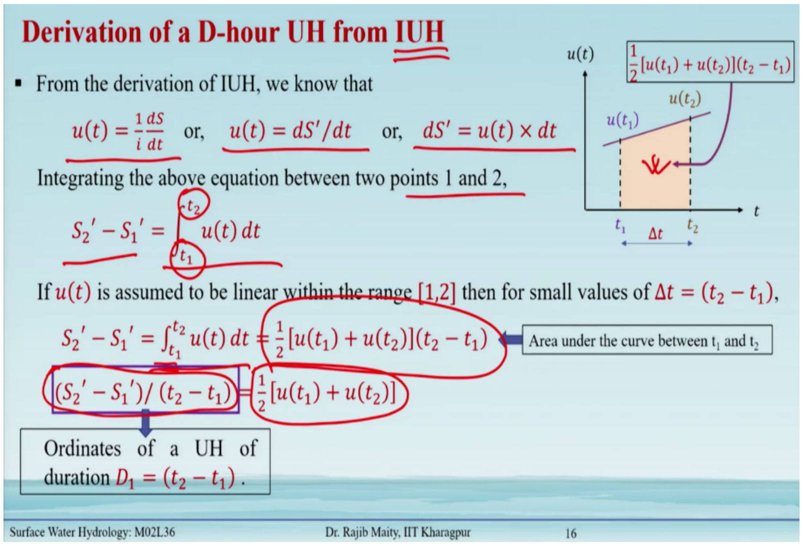

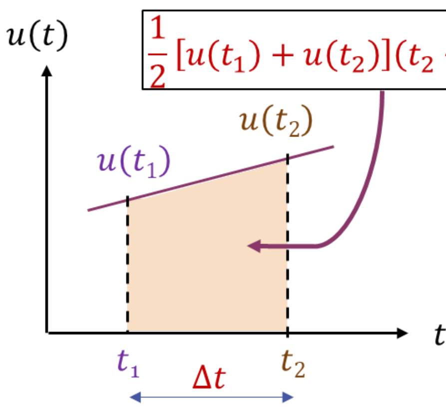

- From the derivation of IUH, we know that

$$u(t) = \frac{1}{i}\frac{dS}{dt} \quad \text{or,} \quad u(t) = dS'/dt \quad \text{or,} \quad dS' = u(t) \times dt$$

Integrating the above equation between two points 1 and 2,

$$S_2{}' - S_1{}' = \int_{t_1}^{t_2} u(t)\, dt$$

If $u(t)$ is assumed to be linear within the range $[1,2]$ then for small values of $\Delta t = (t_2 - t_1)$,

$$S_2{}' - S_1{}' = \int_{t_1}^{t_2} u(t)\, dt = \frac{1}{2}[u(t_1) + u(t_2)](t_2 - t_1)$$

$$(S_2{}' - S_1{}')/\ (t_2 - t_1) = \frac{1}{2}[u(t_1) + u(t_2)]$$

$(t_2 - t_1)$ = Area under the curve between t_1 and t_2

$(S_2' - S_1')/\ (t_2 - t_1)$ =Ordinates of a UH of duration $D1 = (t_2 - t_1)$.

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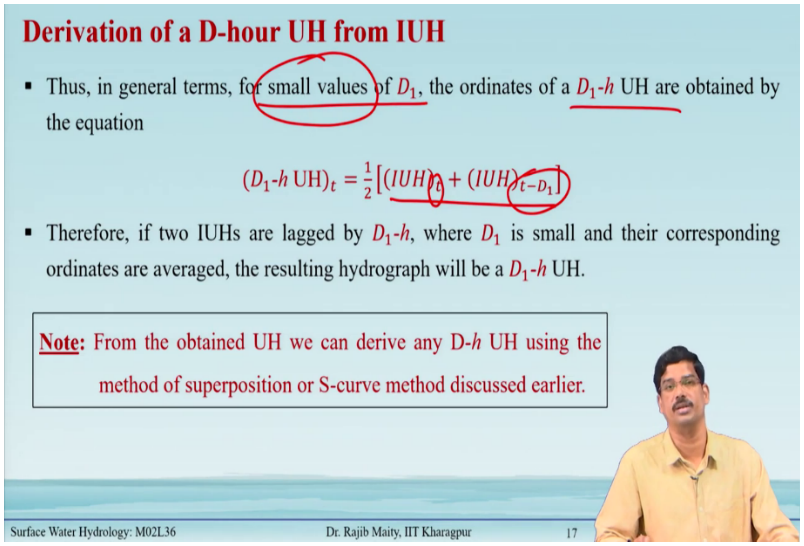

Thus, in general terms, for small values of D_1, the ordinates of a D_1-h UH are obtained by the equation

$$(D_1\text{-}h\ \text{UH})_t = \frac{1}{2}\left[(IUH)_t + (IUH)_{t-D_1}\right]$$

Therefore, if two IUHs are lagged by D_1-h, where D_1 is small and their corresponding ordinates are averaged, the resulting hydrograph will be a D_1-h UH.

It may be noted that from the obtained UH we can derive any D-*h,* UH using the method of superposition or S-curve method discussed earlier.

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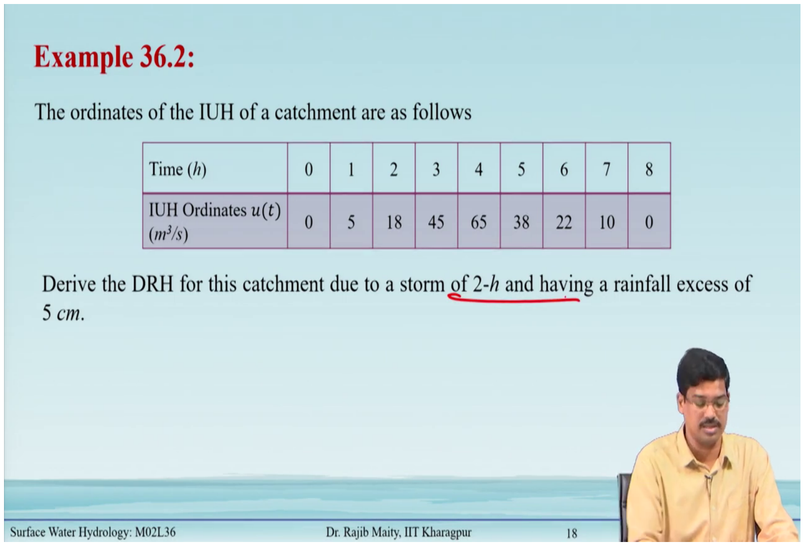

Example 36.2:

The ordinates of the IUH of a catchment are as follows

Time (h)	0	1	2	3	4	5	6	7	8
IUH Ordinates $u(t)$ (m^3/s)	0	5	18	45	65	38	22	10	0

Derive the DRH for this catchment due to a storm of 2-h and having a rainfall excess of 5 cm.

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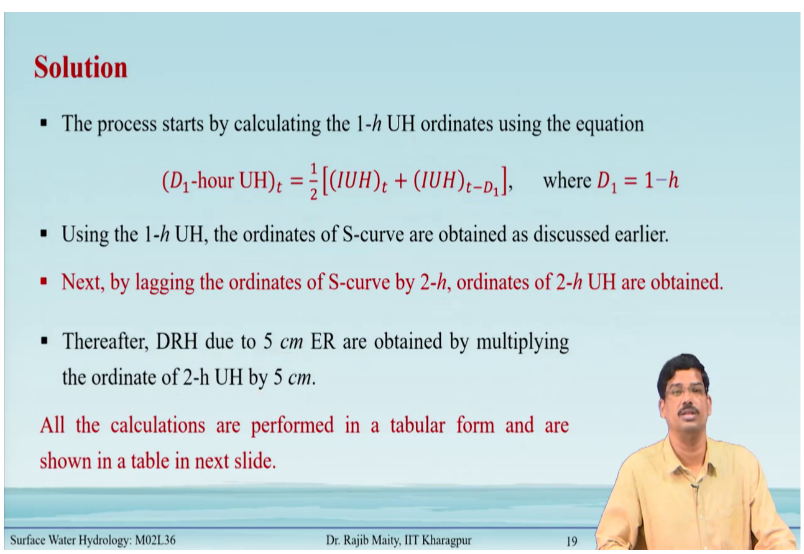

The process starts by calculating the 1-h UH ordinates using the equation

$(D_1\text{-hour UH})_t = \frac{1}{2}\left[(IUH)_t + (IUH)_{t-D_1}\right],$ where $D_1 = 1{-}h$

Using the 1-h UH, the ordinates of the S-curve are obtained as discussed earlier.

Next, by lagging the ordinates of the S-curve by 2-h, ordinates of 2-h UH are obtained.

Thereafter, DRH due to 5 cm ER is obtained by multiplying the ordinate of 2-h UH by 5 cm.

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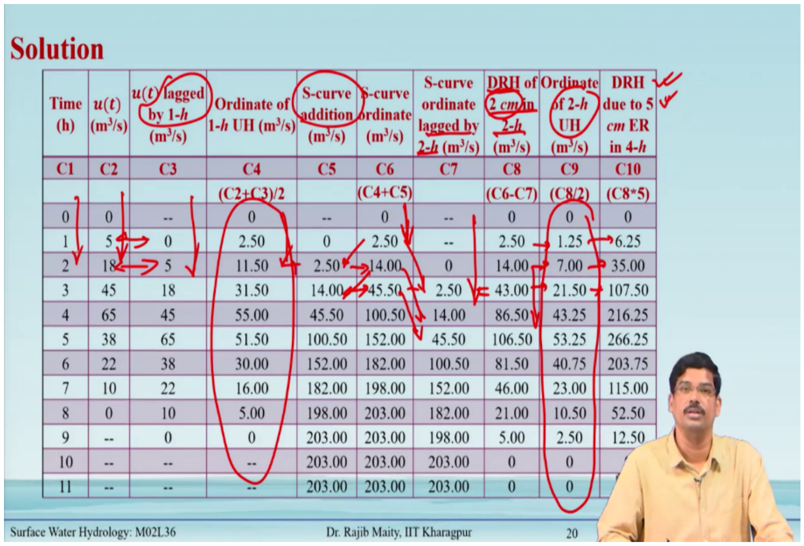

Time (h)	$u(t)$ (m³/s)	$u(t)$ lagged by 1-*h* (m³/s)	Ordinate of 1-*h* UH (m³/s)	S-curve addition (m³/s)	S-curve ordinate (m³/s)	S-curve ordinate lagged by 2-*h* (m³/s)	DRH of 2 *cm* in 2-*h* (m³/s)	Ordinate of 2-*h* UH (m³/s)	DRH due to 5 *cm* ER in 4-*h*
C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
			(C2+C3)/2		(C4+C5)		(C6-C7)	(C8/2)	(C8*5)
0	0	--	0	--	0	--	0	0	0
1	5	0	2.50	0	2.50	--	2.50	1.25	6.25
2	18	5	11.50	2.50	14.00	0	14.00	7.00	35.00
3	45	18	31.50	14.00	45.50	2.50	43.00	21.50	107.50
4	65	45	55.00	45.50	100.50	14.00	86.50	43.25	216.25
5	38	65	51.50	100.50	152.00	45.50	106.50	53.25	266.25
6	22	38	30.00	152.00	182.00	100.50	81.50	40.75	203.75
7	10	22	16.00	182.00	198.00	152.00	46.00	23.00	115.00
8	0	10	5.00	198.00	203.00	182.00	21.00	10.50	52.50
9	--	0	0	203.00	203.00	198.00	5.00	2.50	12.50
10	--	--	--	203.00	203.00	203.00	0	0	0
11	--	--	--	203.00	203.00	203.00	0	0	0

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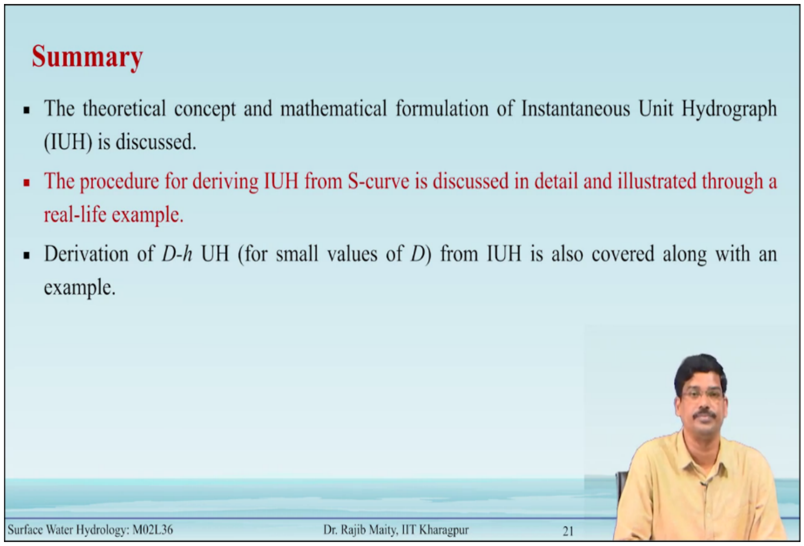

Summary

In summary, we learned the following points from this lecture:

The theoretical concept and mathematical formulation of Instantaneous Unit Hydrograph (IUH) are discussed.

The procedure for deriving IUH from S-curve is discussed in detail and illustrated through a real-life example.

Derivation of *D-h* UH (for small values of *D*) from IUH is also covered along with an example.

In the next lecture, a few conceptual models of developing IUH for a catchment will be discussed.