Surface Water Hydrology
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Lecture 33
Unit Hydrograph of Different Durations:
Method of S - Curve

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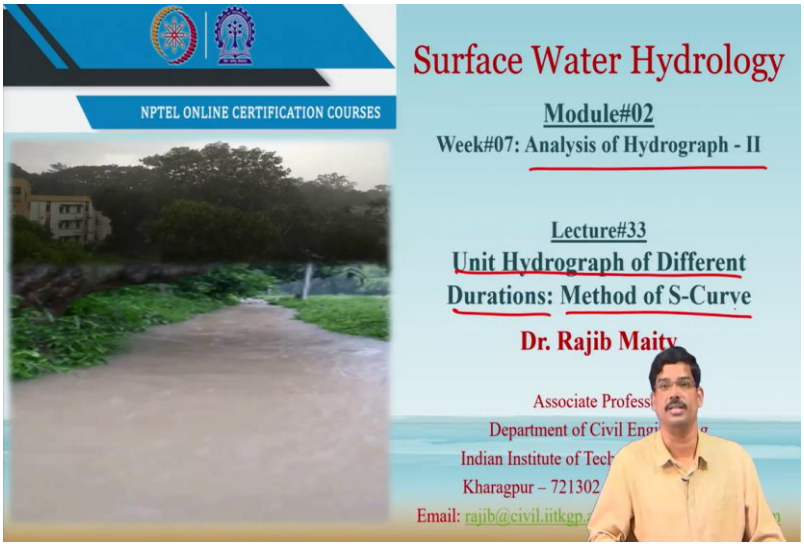

In this lecture, we are learning Unit hydrograph of different duration: method of S-curve.

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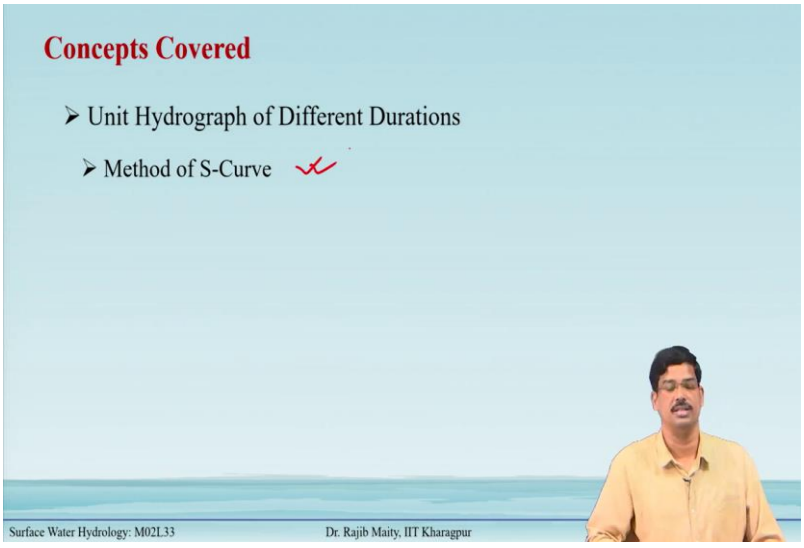

The concept covered the unit hydrograph of different durations and under this one method that we are discussing that is method of S-curve.

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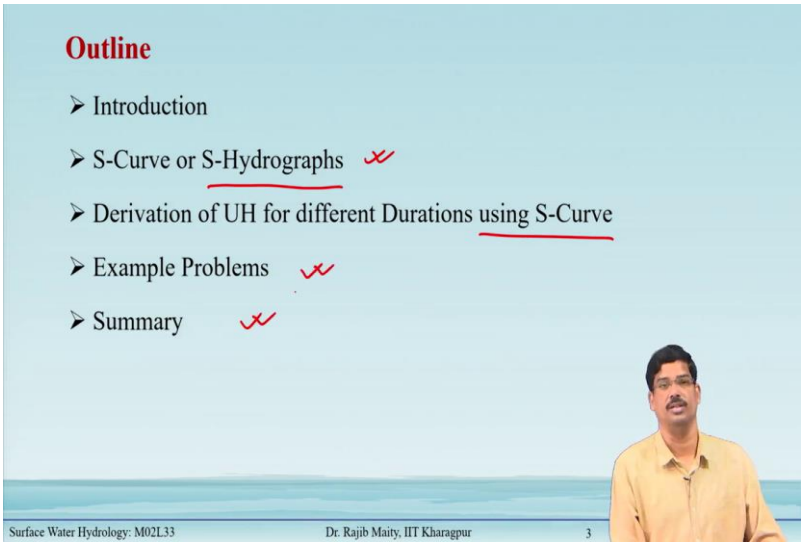

The outline of this lecture goes like this the introduction first and then we should know in detail what is of S-curve, which is also known as S-Hydrograph. So, that we should know first how is it being developed and after that using this S-curve using this S-curve, how we can develop the unit hydrograph for different duration. We will also discuss some example problems before I go to the summary.

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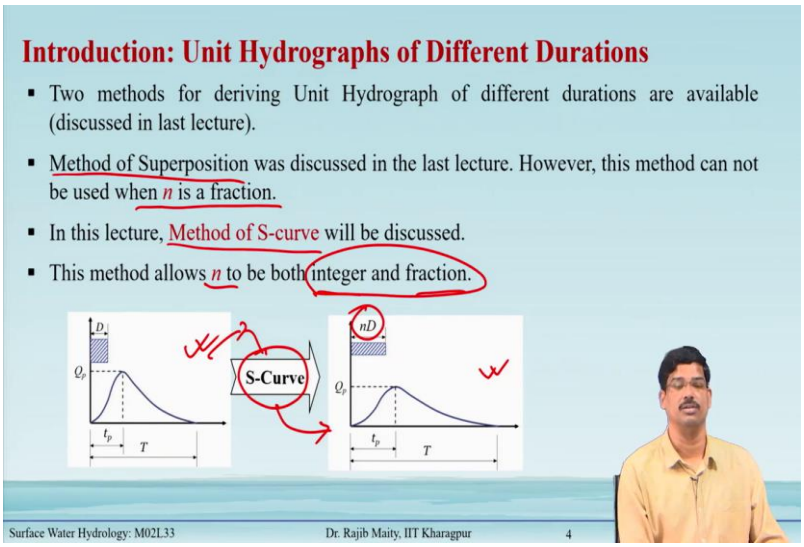

Introduction: Unit Hydrographs of Different Durations

Two methods for deriving this unit hydrograph for different durations are available. And method of superposition was there in the last class where it was this method cannot be used when n is a fraction. So, it is this method is good for when n is an integer. In this lecture, we are adopting the method of S-curve and in this method, n can be both integer as well as the fraction.

D hour unit hydrograph for a catchment, which is develop from the field data or some other method and once we have this is our input, they enter this D hour unit hydrograph is our input for this method, we use the method of S-curve to identify another unit hydrograph where we designate it as the nD hour unit in hydrograph. So, here n can be integer of or fraction both.

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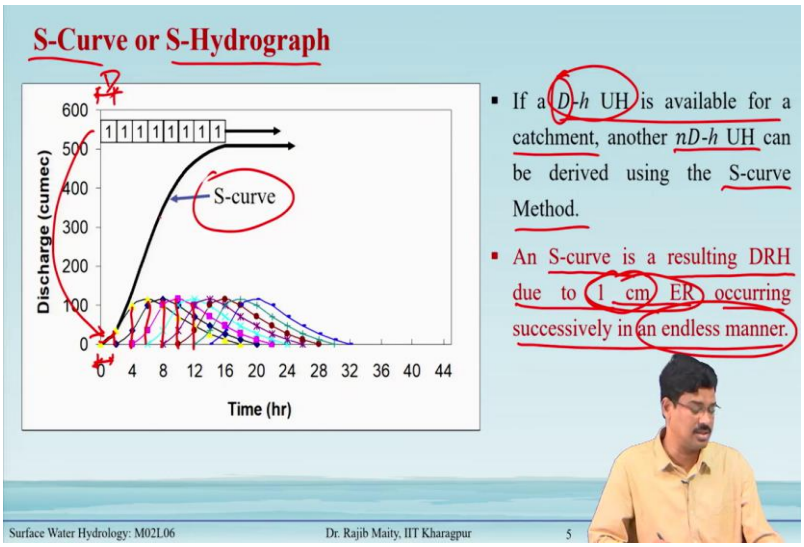

If a D-h UH is available for a catchment, another nD-h UH can be derived using the S-curve Method. An S-curve is a resulting direct runoff hydrograph due to 1 centimeter of rainfall excess occurring successively in an endless manner occurring successively in an endless manner.

The duration of 1 centimeter rainfall excess is D and for this one the resulting unit hydrograph which is shown in the fig.1. Now, if add one more such rainfall block of 1 centimeter rainfall excess and for that one get another unit hydrograph, the same unit hydrograph which basically now starting with the gap of D hour.

In this way keep on adding this 1 centimeter of rainfall excess for D hours as shown in fig.1 here one after another. Now, in this way it will go for an infinite time. After that all the ordinates just add and get a curve like this and which is known as the S-curve.

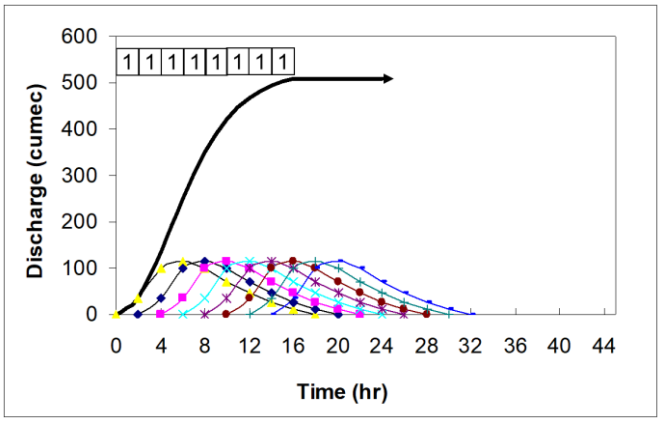

Fig.1 shows the construction of S-curve

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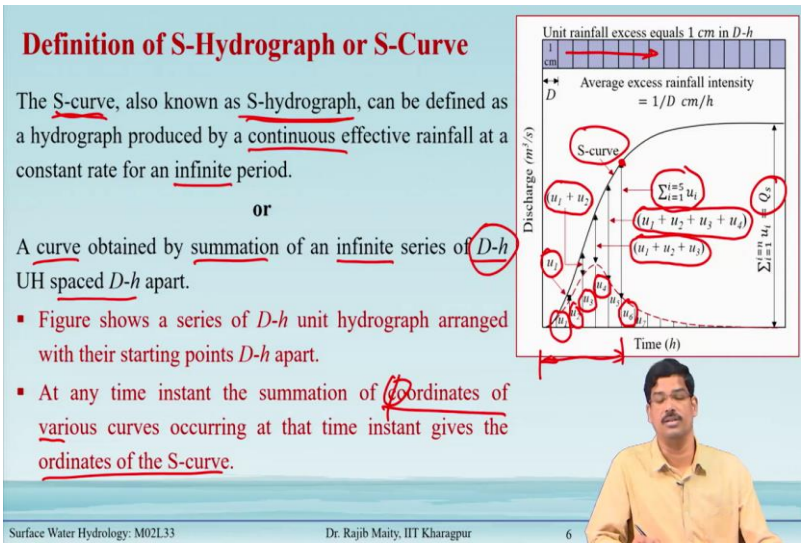

The S-curve, also known as S-hydrograph, can be defined as a hydrograph produced by a continuous effective rainfall at a constant rate for an infinite period or a curve obtained by summation of an infinite series of D-h UH spaced D-h apart.

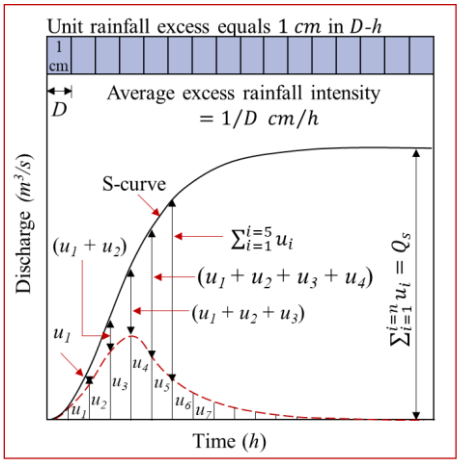

Fig.2 shows the S-curve

In fig.2 there is a series of D hour unit hydrograph arranged with their starting points at D hour apart at and at any time instant the summation of the ordinates of various curves occurring at the time instant gives the ordinance of these various curves occurring at a time instance gives the ordinate of this S-curve.

In fig.2 diagram that this curve resemblance, one S letter, so that is why the name is that S-curve. And other thing is that it is not going not increasing infinitely even if we are putting this series as infinite. So, at some point of time it reaches to a discharge which is called the equilibrium discharge. So, at that point, we can say that whatever is coming in as excess rainfall and everything is going out. So, that time it becomes stabilized. So, that we can say as a stable or the equilibrium discharge.

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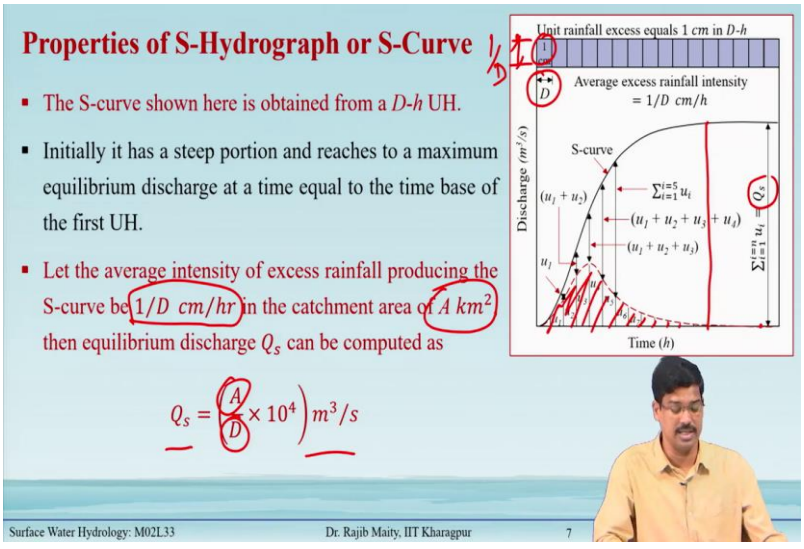

Properties of S-Hydrograph or S-Curve

- The S-curve shown here is obtained from a D-h UH.
- Initially it has a steep portion and reaches to a maximum equilibrium discharge at a time equal to the time base of the first UH.
- Let the average intensity of excess rainfall producing the S-curve be $1/D$ cm/h r in the catchment area of A km^2, then equilibrium discharge Q_s can be computed as

$$Q_s = \left(\frac{A}{D} \times 10^4\right) m^3/s$$

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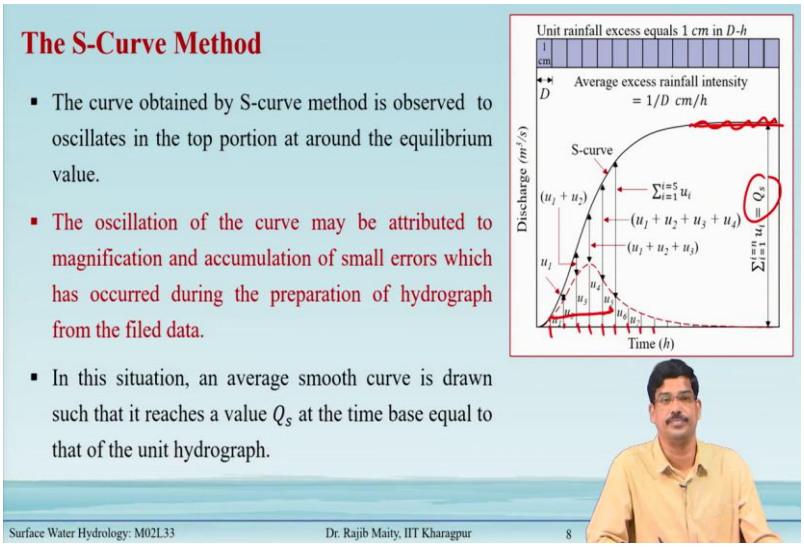

The S-Curve Method

- The curve obtained by S-curve method is observed to oscillate in the top portion at around the equilibrium value.
- The oscillation of the curve may be attributed to magnification and accumulation of small errors which has occurred during the preparation of hydrograph from the filed data.
- In this situation, an average smooth curve is drawn such that it reaches a value Q_s at the time base equal to that of the unit hydrograph.

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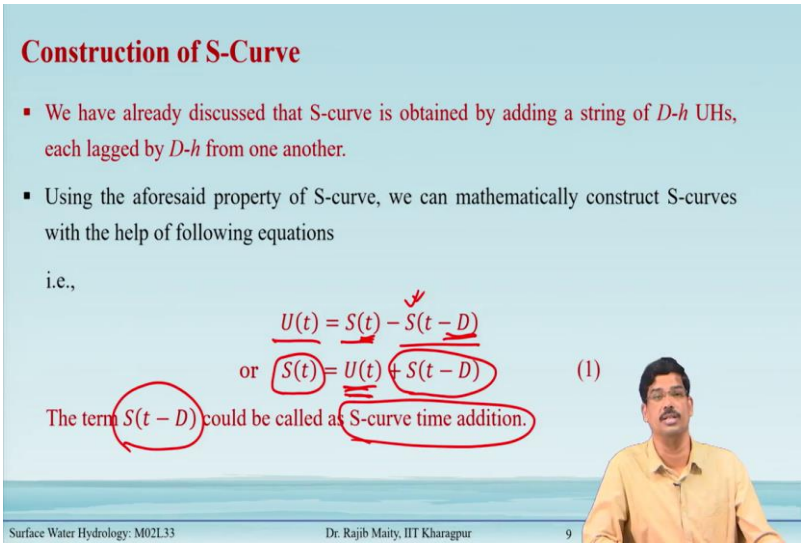

Construction of S-Curve

We have already discussed that S-curve is obtained by adding a string of D-h UHs, each lagged by D-h from one another.

Using the aforesaid property of S-curve, we can mathematically construct S-curves with the help of following equations, i. e.,

$$U(t) = S(t) - S(t-D)$$

$$\text{or} \quad S(t) = U(t) + S(t-D) \tag{1}$$

The term $S(t-D)$ could be called as S-curve time addition.

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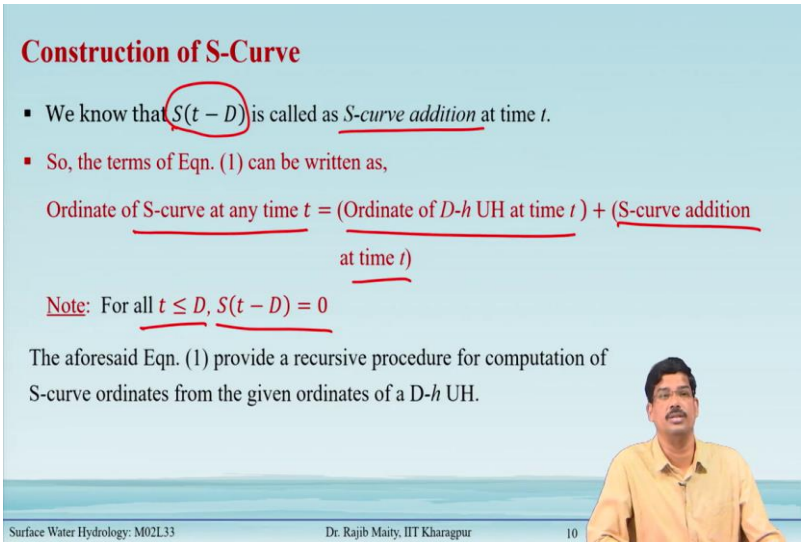

We know that $S(t-D)$ is called as S-curve addition at time t.

So, the terms of Eqn. (1) can be written as,

Ordinate of S-curve at any time t= (Ordinate of D-h UH at time t) + (S-curve addition at time t)

It may be noted that for all $t \leq D$, $S(t-D)$ =0

The aforesaid Eqn. (1) provide a recursive procedure for computation of S-curve ordinates from the given ordinates of a D-h UH. So, using this things procedure, this computation, we can get from the D hour unit hydrograph.

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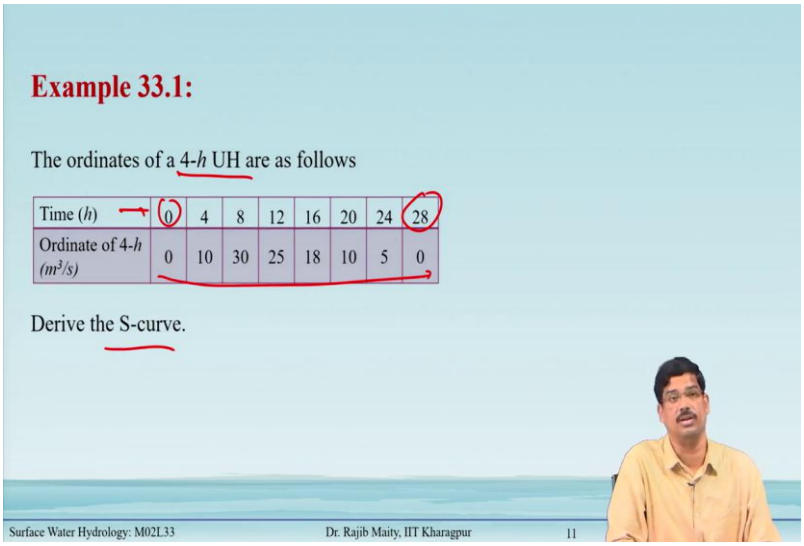

Example 33.1:

The ordinates of a 4-*h* UH are as follows

Time (*h*)	0	4	8	12	16	20	24	28
Ordinate of 4-*h* (m^3/s)	0	10	30	25	18	10	5	0

Derive the S-curve.

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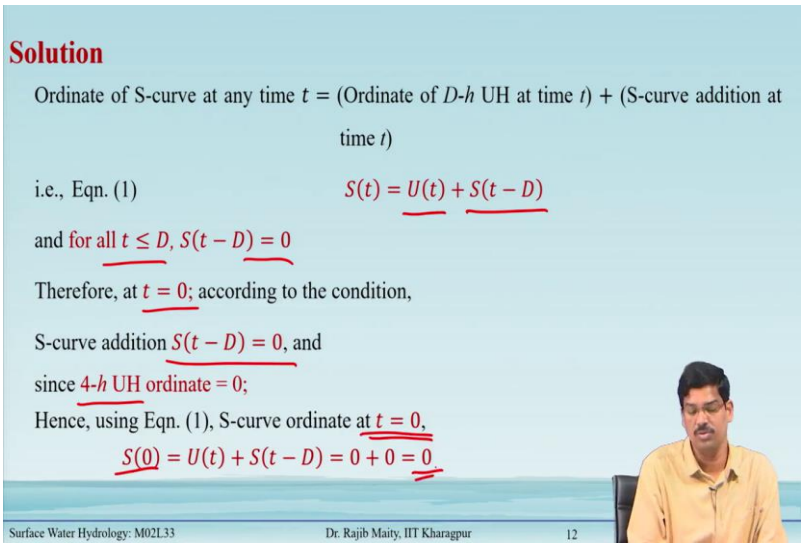

Solution

Ordinate of S-curve at any time t= (Ordinate of D-h UH at time t) + (S-curve addition at time t)

i.e., Eqn. (1) $S(t)=U(t)+S(t-D)$

And for all $t \leq D$, $S(t-D)$ =0

Therefore, at t=0; according to the condition,

S-curve addition $S(t-D)$ =0, and

Since 4-h UH ordinate = 0;

Hence, using Eqn. (1), S-curve ordinate at t=0,

$$S(0)=U(t)+S(t-D)=0+0=0$$

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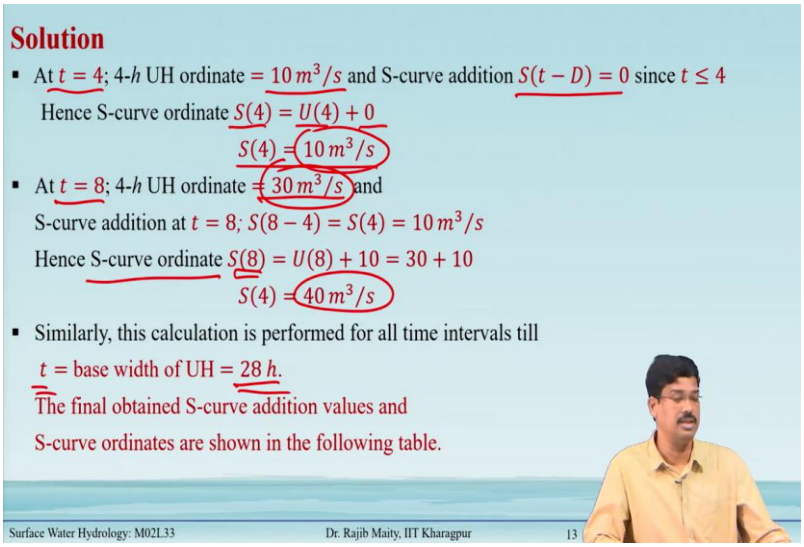

At t=4; 4-h UH ordinate =10 m^3⁄s and S-curve addition $S(t-D)$ =0 since $t\leq$4

Hence S-curve ordinate S (4) =U (4) +0

$$S\ (4) = 10\ m\text{^}3/s$$

At t=8; 4-h UH ordinate =30 m^3/s and

S-curve addition at t=8; S (8−4) =S (4) =10 m^3⁄s

Hence S-curve ordinate S (8) =U (8) +10=30+10

$$S\ (4) = 40\ m\text{^}3/s$$

Similarly, this calculation is performed for all time intervals till

t= base width of UH =28 h .

The final obtained S-curve addition values and

S-curve ordinates are shown in the following table.

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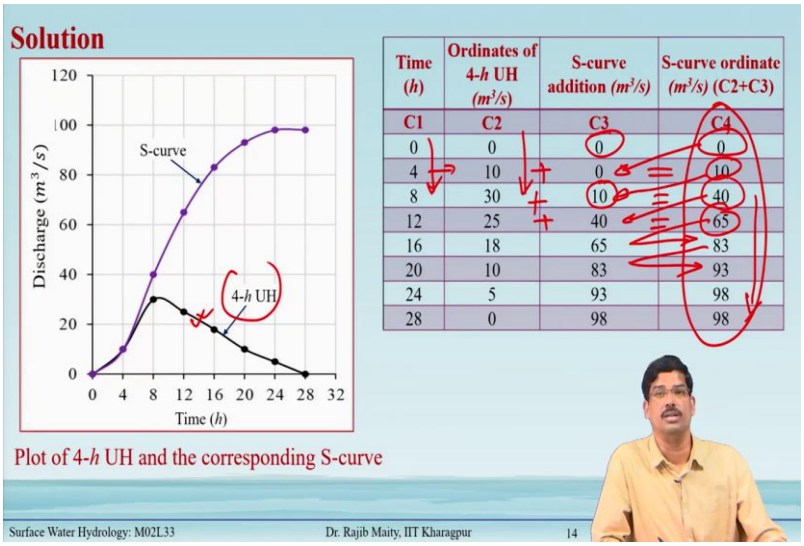

Time (*h*)	Ordinates of 4-*h* UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (m^3/s) (C2+C3)
C1	C2	C3	C4
0	0	0	0
4	10	0	10
8	30	10	40
12	25	40	65
16	18	65	83
20	10	83	93
24	5	93	98
28	0	98	98

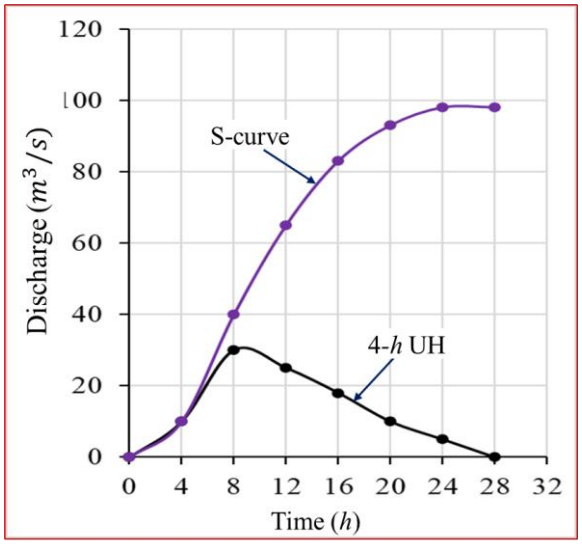

Fig.3 shows the Plot of 4-*h* UH and the corresponding S-curve of example 33.1

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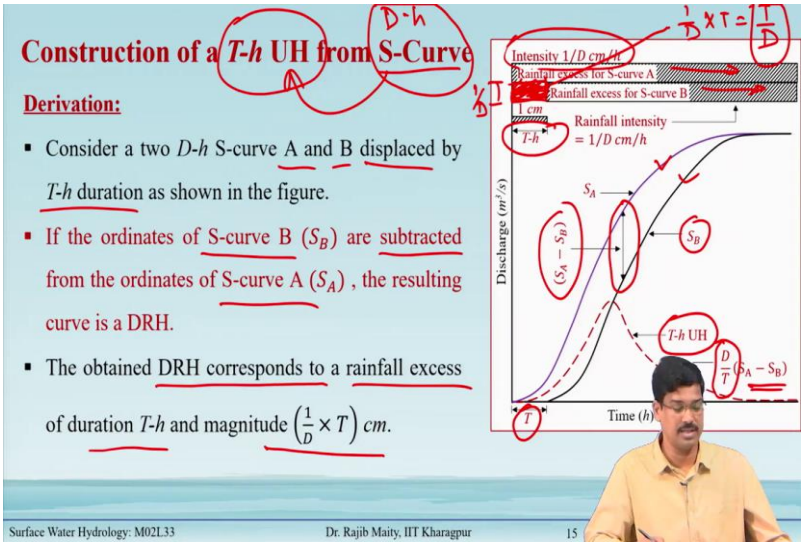

Construction of a *T-h* UH from S-Curve

Consider a two D-h S-curve A and B displaced by T-h duration as shown in the figure 4. If the ordinates of S-curve B (S_B), are subtracted from the ordinates of S-curve A (S_A), the resulting curve is a DRH. The obtained DRH corresponds to a rainfall excess of duration T-h and magnitude ($1/D \times T$) cm.

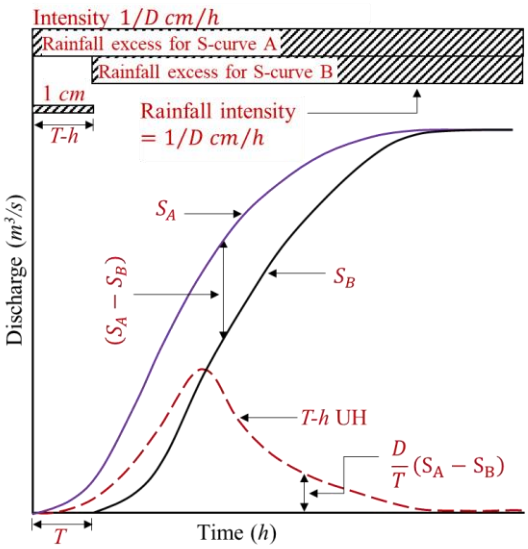

Fig.4 shows the Construction of a *T-h* UH from S-Curve

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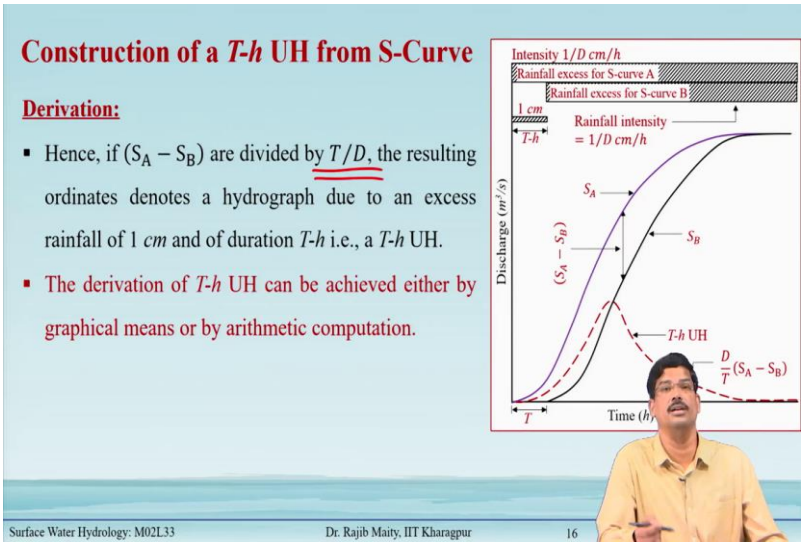

Hence, if (S_A−S_B) are divided by *T/D*, the resulting ordinates denotes a hydrograph due to an excess rainfall of 1 cm and of duration T-h i.e., a T-h UH.

The derivation of T-h UH can be achieved either by graphical means or by arithmetic computation.

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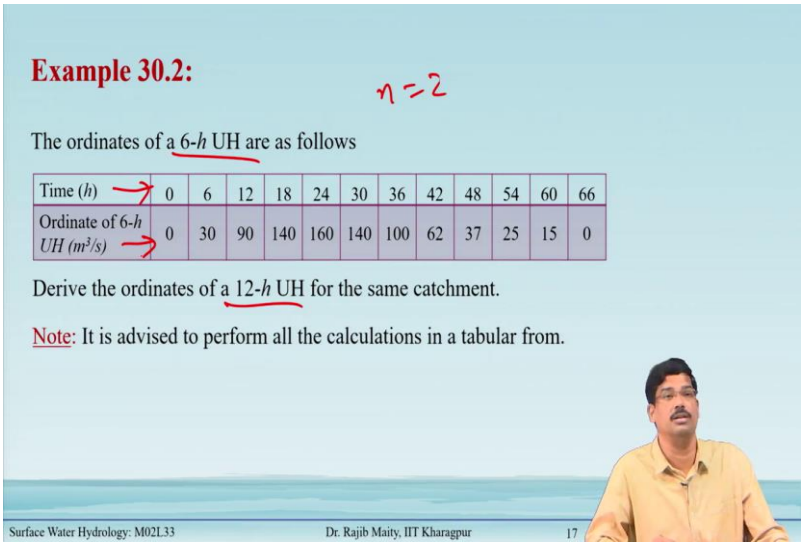

Example 30.2:

The ordinates of a 6-h UH are as follows

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Ordinate of 6-h UH (m^3/s)	0	30	90	140	160	140	100	62	37	25	15	0

Derive the ordinates of a 12-h UH for the same catchment.

Note: It is advised to perform all the calculations in a tabular from.

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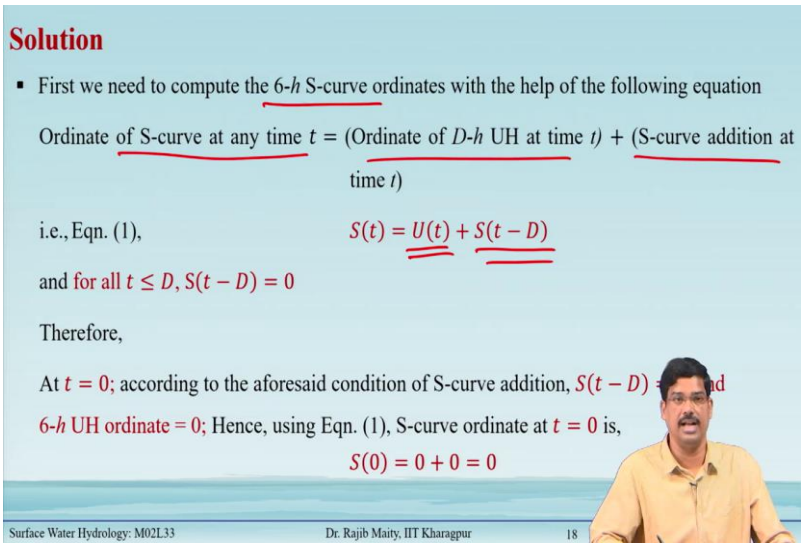

Solution

First, we need to compute the 6-h S-curve ordinates with the help of the following equation

Ordinate of S-curve at any time t= (Ordinate of D-h UH at time t) + (S-curve addition at time t)

i.e., Eqn. (1), $S(t)=U(t)+S(t-D)$

and for all $t \leq D$, S $(t-D)$ =0

Therefore, At t=0; according to the aforesaid condition of S-curve addition, $S(t-D)$ =0, and

6-h UH ordinate = 0; Hence, using Eqn. (1), S-curve ordinate at t=0 is,

$$S\ (0) =0+0=0$$

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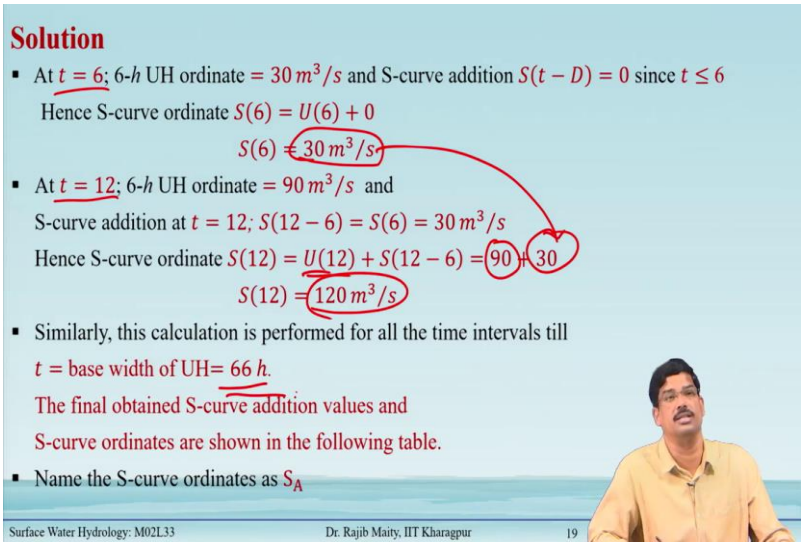

At t=6; 6-h UH ordinate =30 m^3⁄s and S-curve addition $S(t-D)$ =0 since $t\leq$ 6

Hence S-curve ordinate S (6) =U (6) +0

$$S\ (6) = 30\ m^3/s$$

At t=12; 6-h UH ordinate =90 m^3/s and

S-curve addition at t=12; S (12−6) =S (6) =30 m^3⁄s

Hence S-curve ordinate S (12) =U (12) +S (12−6) =90+30

$$S\ (12) = 120\ m^3/s$$

Similarly, this calculation is performed for all the time intervals till

t= base width of UH=66 h .

The final obtained S-curve addition values and

S-curve ordinates are shown in the following table.

Name the S-curve ordinates as S_A

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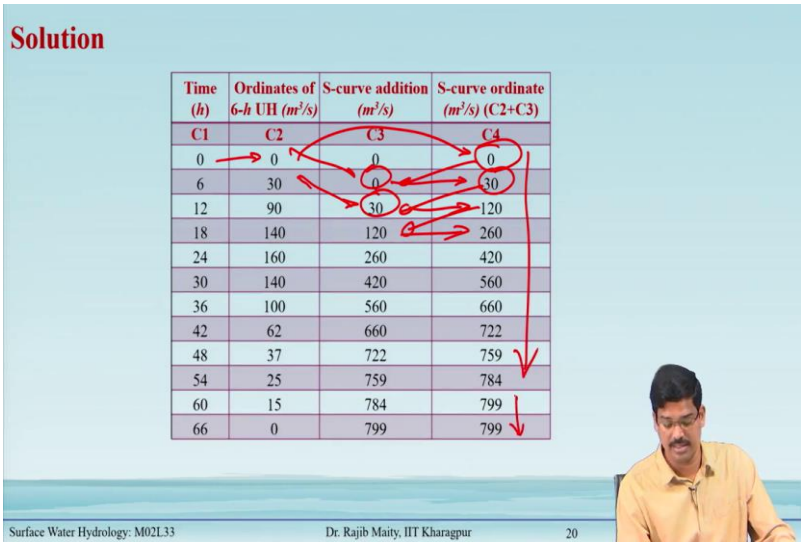

Time (h)	Ordinates of 6-h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (m^3/s) (C2+C3)
C1	C2	C3	C4
0	0	0	0
6	30	0	30
12	90	30	120
18	140	120	260
24	160	260	420
30	140	420	560
36	100	560	660
42	62	660	722
48	37	722	759
54	25	759	784
60	15	784	799
66	0	799	799

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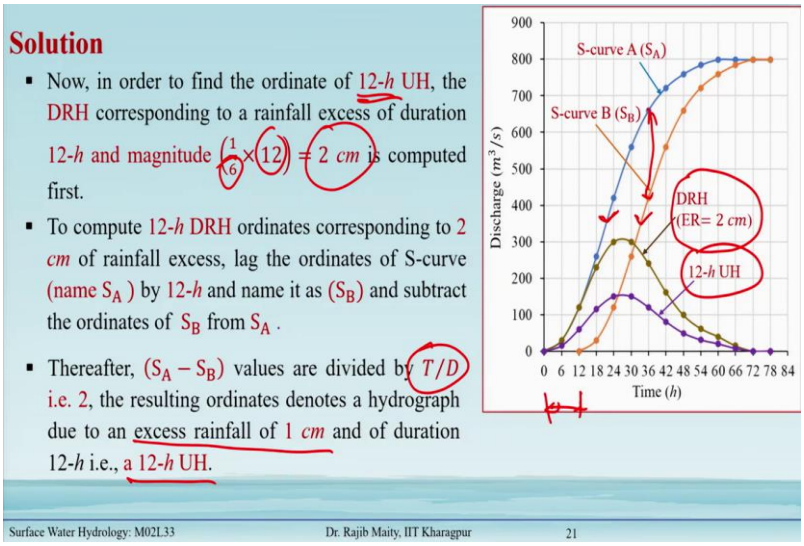

Now, in order to find the ordinate of 12-h UH, the DRH corresponding to a rainfall excess of duration 12-h and magnitude (1/6×12) =2 cm is computed first.

To compute 12-h DRH ordinates corresponding to 2 cm of rainfall excess, lag the ordinates of S-curve (name S_A) by 12-h and name it as S_B and subtract the ordinates of S_B from S_A.

Thereafter, (S_A–S_B) values are divided by *T/D* i.e., 2, the resulting ordinates denotes a hydrograph due to an excess rainfall of 1 cm and of duration 12-h i.e., a 12-h UH.

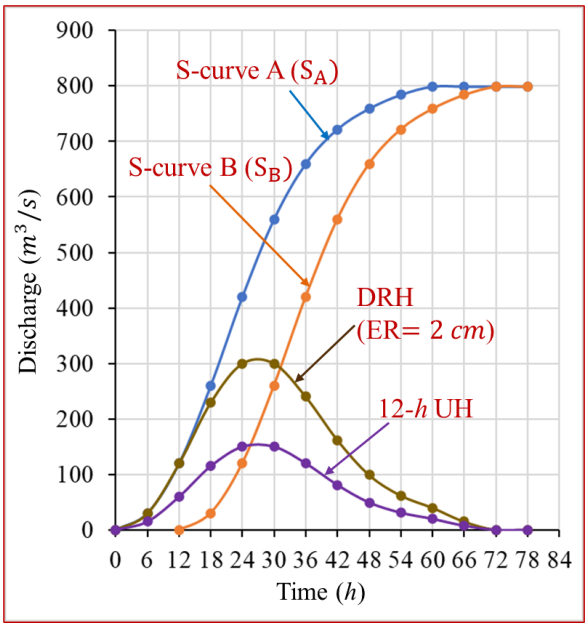

Fig.5 shows the ordinate of 12-h UH from a 6-h UH

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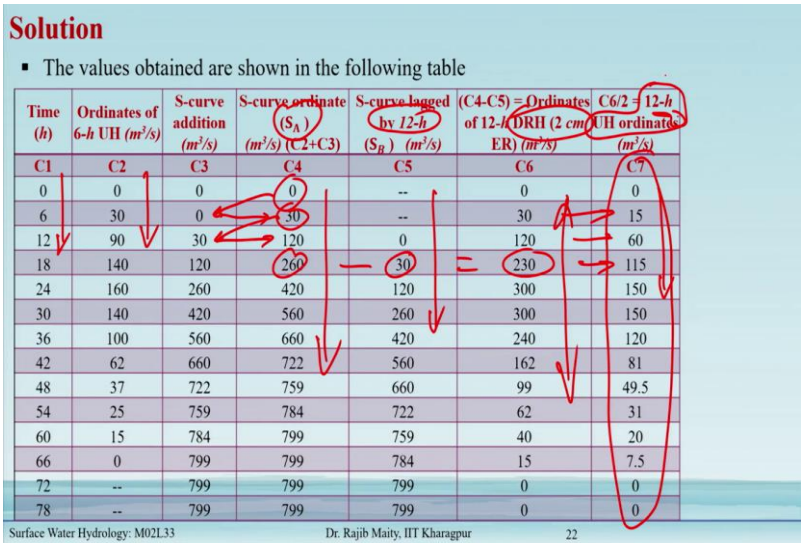

The values obtained are shown in the following table

Time (h)	Ordinates of 6-h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (S_A) (m^3/s) (C2+C3)	S-curve lagged by 12-h (S_B) (m^3/s)	(C4-C5) = Ordinates of 12-h DRH (2 cm ER) (m^3/s)	C6/2 = 12-h UH ordinates (m^3/s)
C1	C2	C3	C4	C5	C6	C7
0	0	0	0	--	0	0
6	30	0	30	--	30	15
12	90	30	120	0	120	60
18	140	120	260	30	230	115
24	160	260	420	120	300	150
30	140	420	560	260	300	150
36	100	560	660	420	240	120
42	62	660	722	560	162	81
48	37	722	759	660	99	49.5
54	25	759	784	722	62	31
60	15	784	799	759	40	20
66	0	799	799	784	15	7.5
72	--	799	799	799	0	0
78	--	799	799	799	0	0

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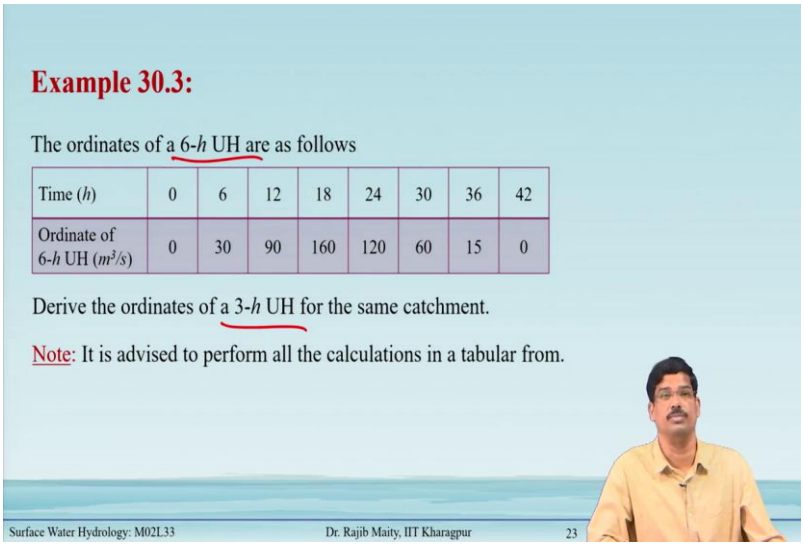

Example 30.3:

The ordinates of a 6-*h* UH are as follows

Time (*h*)	0	6	12	18	24	30	36	42
Ordinate of 6-*h* UH (m^3/s)	0	30	90	160	120	60	15	0

Derive the ordinates of a 3-*h* UH for the same catchment.

Note: It is advised to perform all the calculations in a tabular from.

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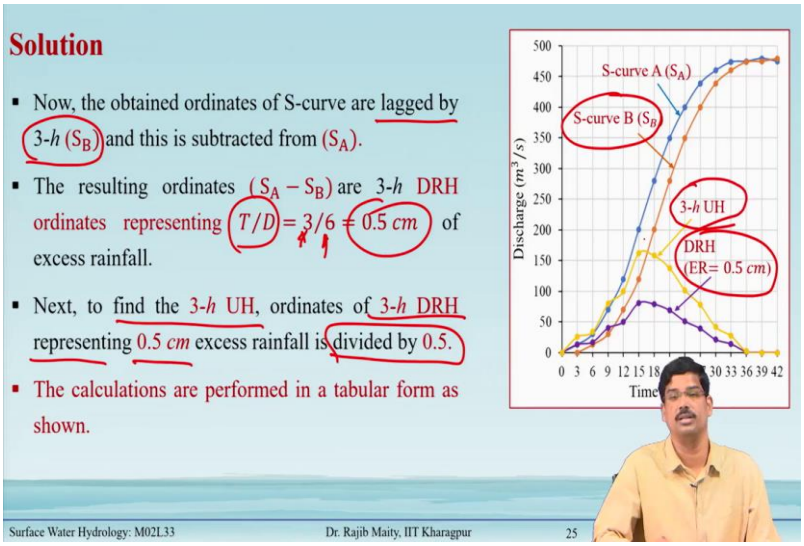

In this example a time interval of 6-h UH ordinates are given.

To derive a UH of D-h duration it is necessary that ordinates of UH are known at least at time interval of D-h.

Therefore, the first step is to plot the given 6-h UH and find the ordinates at 3-h interval.

Thereafter, the value of S-curve addition and S-curve ordinates (S_A) are obtained using the same procedure as discussed in the previous problem.

Now, the obtained ordinates of S-curve are lagged by 3-h (S_B) and this is subtracted from (S_A).

The resulting ordinates (S_A – S_B) are 3-h DRH ordinates representing $T/D=3/6=0.5$ *cm* of excess rainfall.

Next, to find the 3-h UH, ordinates of 3-h DRH representing 0.5 cm excess rainfall is divided by 0.5.

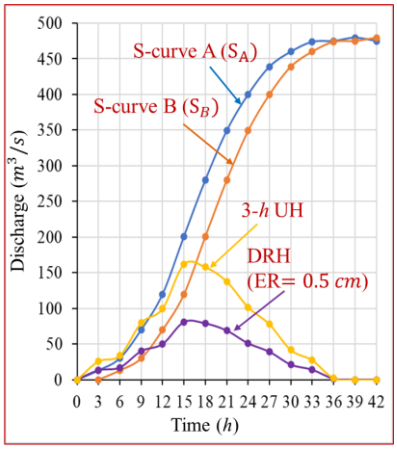

Fig.6 shows the ordinate of 3-h UH of example 30.3

The calculations are performed in a tabular form as shown.

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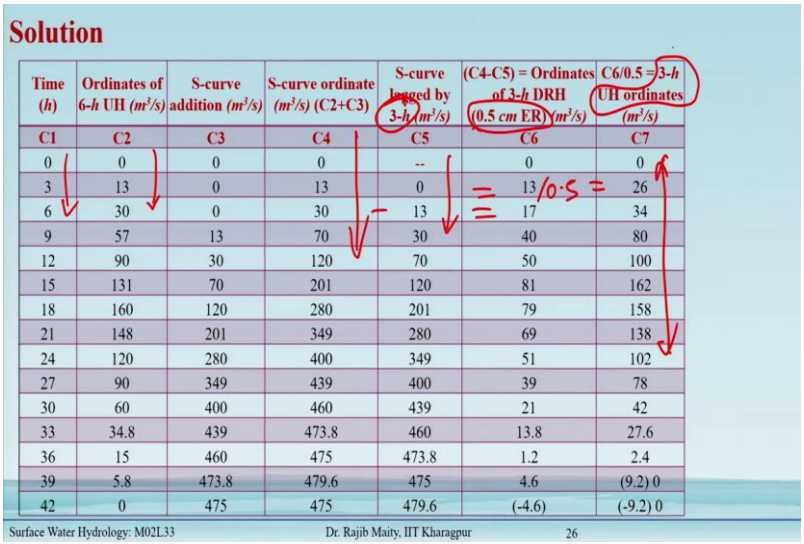

Time (h)	Ordinates of 6-h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate (m^3/s) (C2+C3)	S-curve lagged by 3-h (m^3/s)	(C4-C5) = Ordinates of 3-h DRH (0.5 cm ER) (m^3/s)	C6/0.5 = 3-h UH ordinates (m^3/s)
C1	C2	C3	C4	C5	C6	C7
0	0	0	0	--	0	0
3	13	0	13	0	13	26
6	30	0	30	13	17	34
9	57	13	70	30	40	80
12	90	30	120	70	50	100
15	131	70	201	120	81	162
18	160	120	280	201	79	158
21	148	201	349	280	69	138
24	120	280	400	349	51	102
27	90	349	439	400	39	78
30	60	400	460	439	21	42
33	34.8	439	473.8	460	13.8	27.6
36	15	460	475	473.8	1.2	2.4
39	5.8	473.8	479.6	475	4.6	(9.2) 0
42	0	475	475	479.6	(-4.6)	(-9.2) 0

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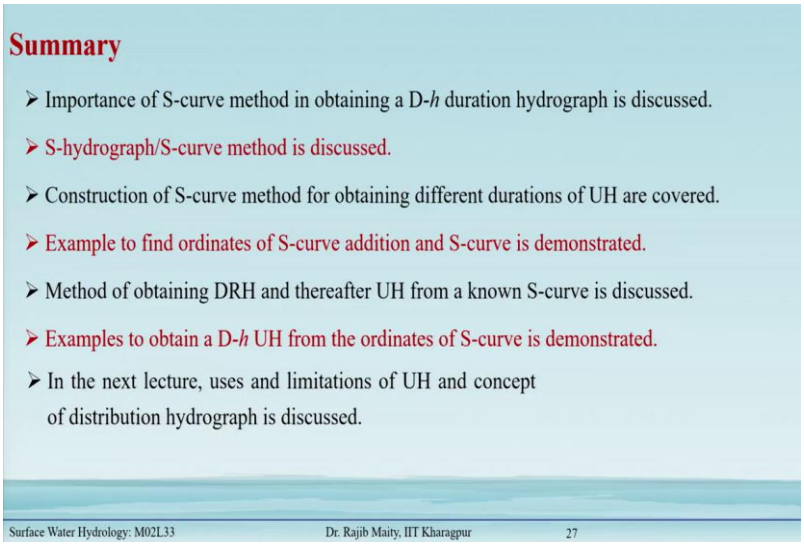

Summary

In summary, we learned the following points from this lecture:

- Importance of S-curve method in obtaining a D-*h* duration hydrograph is discussed.
- S-hydrograph/S-curve method is discussed.
- Construction of S-curve method for obtaining different durations of UH are covered.
- Example to find ordinates of S-curve addition and S-curve is demonstrated.
- Method of obtaining DRH and thereafter UH from a known S-curve is discussed.
- Examples to obtain a D-*h* UH from the ordinates of S-curve is demonstrated.
- In the next lecture, uses and limitations of UH and concept of distribution hydrograph is discussed