**Surface Water Hydrology**
**Professor Rajib Maity**
**Department of Civil Engineering**
**Indian Institute of Technology, Kharagpur**
**Lecture – 23**
**Estimation of Runoff Volume: Empirical Models**

In lecture number 23 we are considering the Estimation of Runoff Volume: empirical model; some of the empirical models

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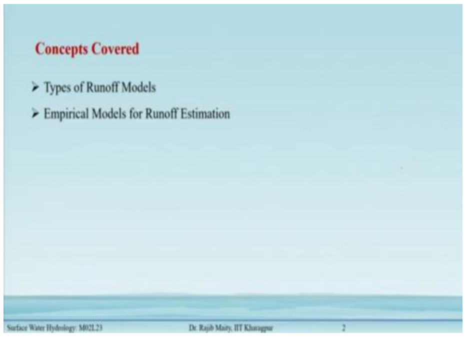


We are covering different types of runoff models and then mainly we are focusing on empirical models for runoff estimation in this particular lecture.

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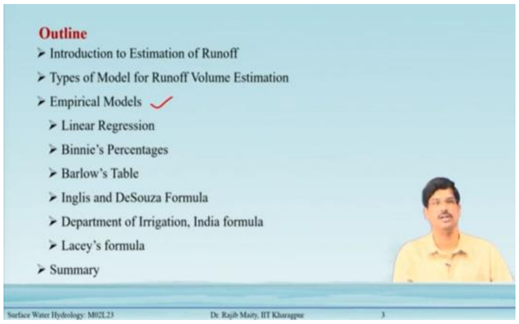


The outline of this lecture goes like this; introduction to the estimation of runoff, then types of models for runoff volume estimation. So, the outline of this lecture goes like this. First, we will give some introduction to the estimation of runoff, then different types of models for runoff volume estimation.

So, under these empirical models, linear regression, Binnie's percentage, Barlow's table, Inglis and DeSouza Formula. Then, one department of irrigation, India formula, and Lacey's formula we will cover; and then we summarize the lecture.

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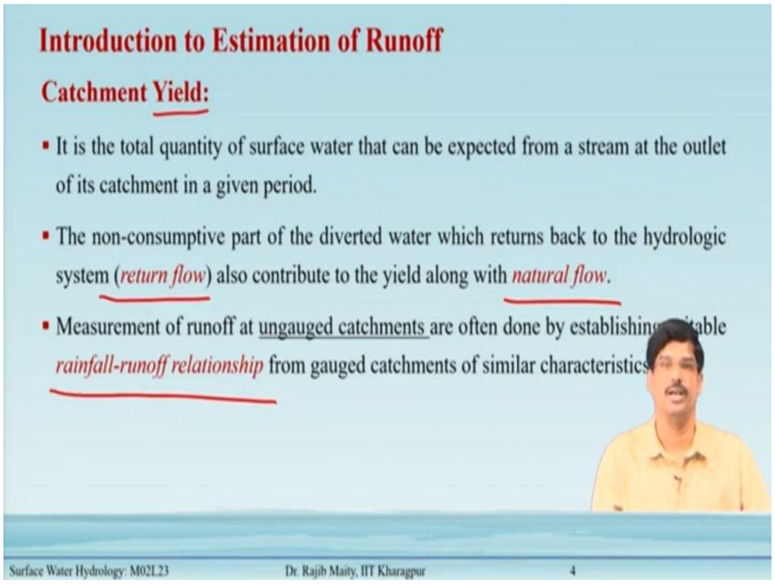


**Introduction to Estimation of Runoff**

**Catchment Yield:**

It is the total quantity of surface water that can be expected from a stream at the outlet of its catchment in a given period of time. Generally, if we consider that time to be one year, then it is the annual yield from the catchment. So, some non-consumptive parts of the diverted water return to the hydrologic system, which is called the return flow.

So, return flow also contributes to the yield along with the natural flow. So, that is supposed to come, and then the water that has been diverted for some different use for irrigation, or the industrial purpose, or domestic purpose that also come comes back and join. So, these two are together considered in the catchment yield. Now, these things are very useful, particularly for the ungauged catchment.

Sometimes we can develop some sort of rainfall and runoff relationship from some gauged catchment, and then other catchments which are of similar characteristics; then using that rainfall-runoff relationship, we can utilize that one to find out the yield for the new ungauged catchment. But the condition is that it should have some similar hydrometeorological characteristics.

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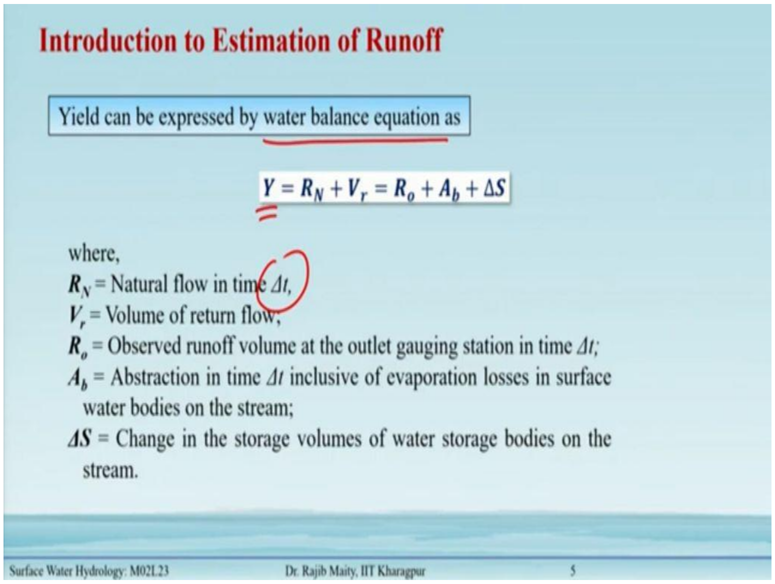


Now, to obtain the yield, this yield generally can be calculated from the water balance equation;

$$Y = R_N + V_r = R_o + A_b + \Delta S$$

where,

$R_N$ = Natural flow in time $\Delta t$,

$V_r$ = Volume of return flow;

$R_o$ = Observed runoff volume at the outlet gauging station in time $\Delta t$;

$A_b$ = Abstraction in time $\Delta t$ inclusive of evaporation losses in surface water bodies on the stream;

$\Delta S$ = Change in the storage volumes of water storage bodies on the stream.

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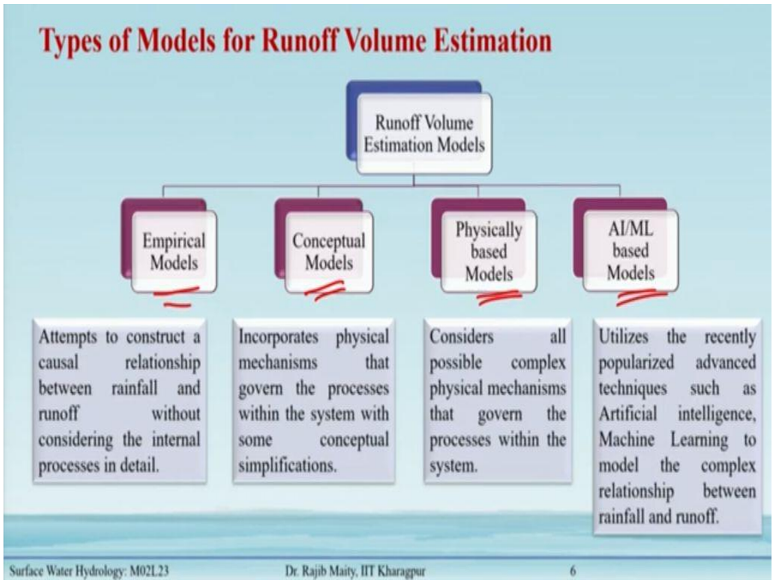


**Types of Models for Runoff Volume Estimation**

Now, there are different types of models available from which we can estimate the runoff, and broadly this can be categorized into four groups. The first one is the empirical model and then comes the conceptual model, then the physically-based model; and finally, more recently popularized artificial intelligence or machine learning-based abbreviated as AI/ML-based models.

*Empirical Models:* Attempts to construct a causal relationship between rainfall and runoff are considered. However, we do not consider the internal process in that much detail. We somehow try to establish some relationship; these are empirical models, so, it comes with the proper unit also. Sometimes, some of the empirical models are applicable only for that region itself. So, if we want to utilize it in some other region, some other parts of the world; then it needs to be properly calibrated again.

*Conceptual Models:* It is not as simplified as the empirical model; at the same time it is not as detailed as the physically-based model. So, under this conceptual model, it incorporates the physical mechanism that governs the process within the system. However, these processes are highly simplified. Sometimes it has been seen that even if these processes are simplified, it gives a reasonably good; or sometimes even much acceptable result as compared to the other models where the competition is highly intensive.

*Physically-based Models:* It is one of the computationally demanding models groups. Here are all the possible complex physical mechanisms that govern the process within the system that is considered. And it has different types like that, whether the time variation or the special variation, they sometimes considered.

*AI/ML-based Models:* In the last category that AI/ML-based model, it generally utilized the recent, it is recently popularized; and these are some advanced techniques such as artificial intelligence and machine learning. Under this also there, there are different categories are there; and it helps to model the complex relationship between rainfall and runoff. So, sometimes it is found to be very fruitful. However, one thing is that it needs to be calibrated again and again for different catchments. And sometimes it needs to be calibrated over time also, or after sometime when the catchment characteristics have been changed; which is true for the other models such as empirical and the conceptual models also.

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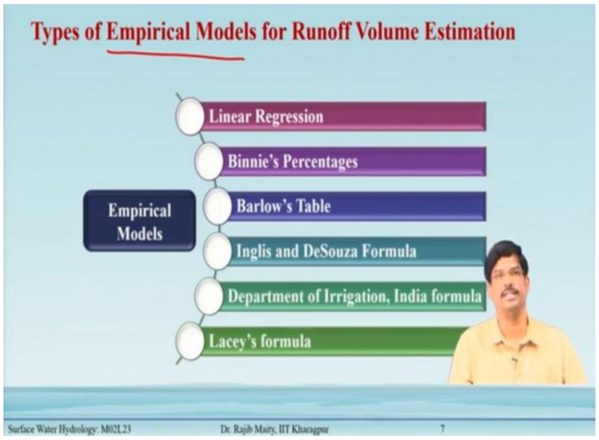


**Types of Empirical Models for Runoff Volume Estimation**

- Linear Regression
- Binnie's Percentages
- Barlow's Table

- Inglis and DeSouza Formula
- Department of Irrigation, India formula
- Lacey's formula

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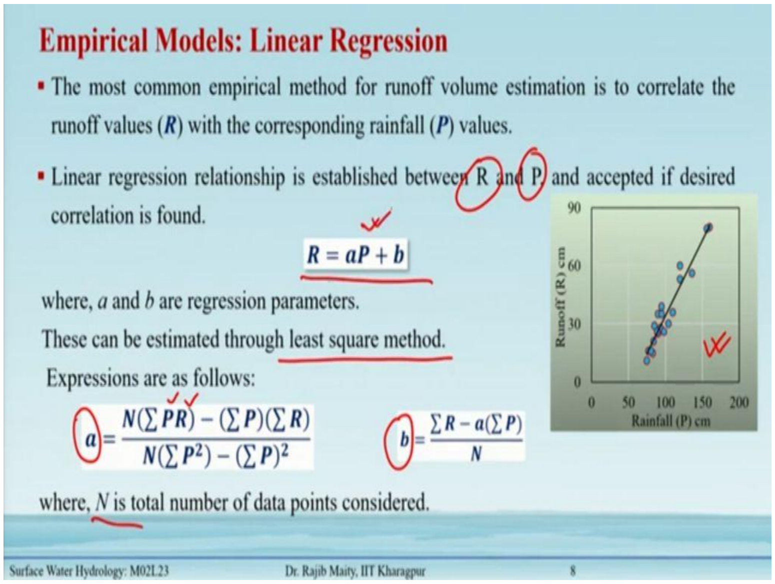


**Empirical Models: Linear Regression**

The most common empirical method for runoff volume estimation is to correlate the runoff values (R) with the corresponding rainfall (P) values. Linear regression relationship is established between R and P and accepted if the desired correlation is found.

$$R = aP + b$$

where, a and b, these are the regression parameters;

These can be estimated through the least square method. Expressions are as follows:

$$a = \frac{N(\sum PR) - (\sum P)(\sum R)}{N(\sum P^2) - (\sum P)^2} \qquad b = \frac{\sum R - a(\sum P)}{N}$$

where $N$ is the total number of data points considered.

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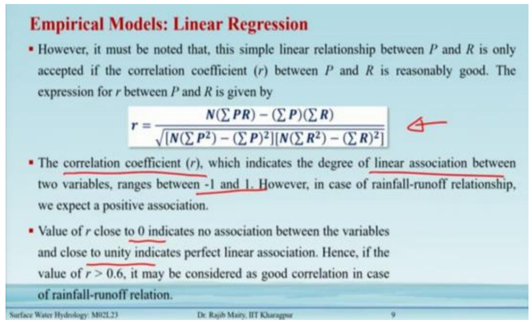


However, it must be noted that this simple linear relationship between P and R is only accepted if the correlation coefficient (r) between P and R is reasonably good. The expression for r between P and R is given by

$$r = \frac{N(\sum PR) - (\sum P)(\sum R)}{\sqrt{[N(\sum P^2) - (\sum P)^2][N(\sum R^2) - (\sum R)^2]}}$$

The correlation coefficient (r), which indicates the degree of linear association between two variables, ranges between -1 and 1. However, in the case of the rainfall-runoff relationship, we expect a positive association.

Value of r close to 0 indicates no association between the variables and close to unity indicates perfect linear association. Hence, if the value of r > 0.6, it may be considered a good correlation in the case of rainfall-runoff relation.

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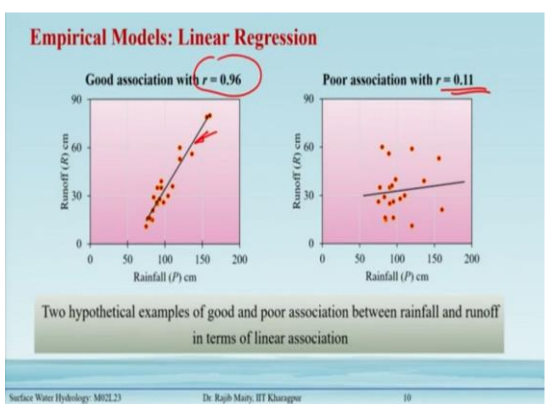

One pictorial example that you can see on the left-hand side of fig.1. These red dots are showing the pairs of rainfall and this runoff; and we fit the linear regression model, which is shown by this black straight line. And here this type of scattering may be an indication or maybe one just example that r equals 0.96, which is close to 1. Whereas, on the right-hand side of fig.1, r is very close to very less close to 0, which is 0.11; so, the scatter part you can just take. So, on the left-hand side whatever the model is there, that is more acceptable than what is there on the right-hand side.

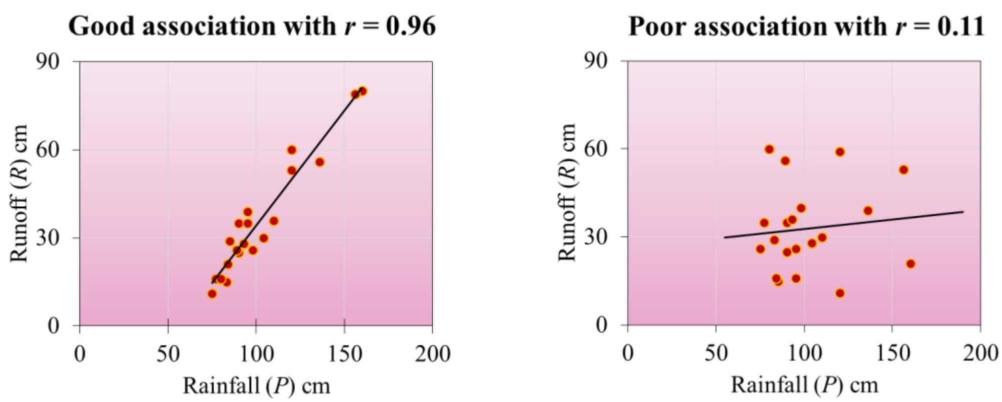


Fig.1 shows the two hypothetical examples of good and poor association between rainfall and runoff in terms of linear association

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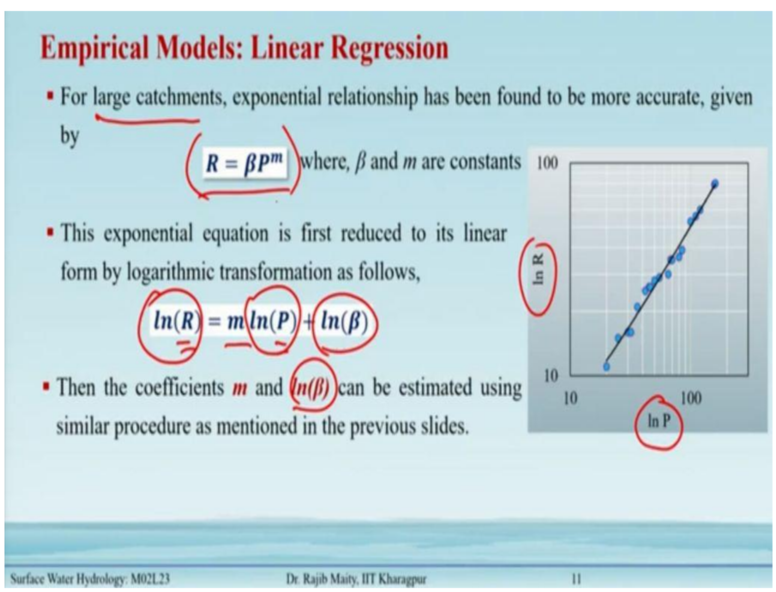

For large catchments, the exponential relationship is more accurate, given by

$$R = \beta P^{m}$$

where β and m are constants

This exponential equation is first reduced to its linear form by logarithmic transformation as follows,

$$ln(R) = m\ ln(P) + ln(\beta)$$

Then the coefficients ***m*** and ***ln(β)*** can be estimated using a similar procedure as mentioned in the previous slides.

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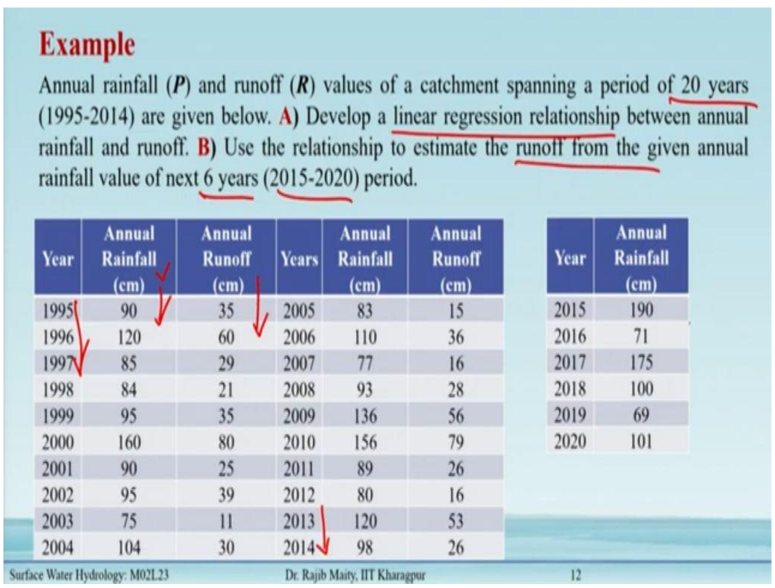


| Year | Annual Rainfall (cm) | Annual Runoff (cm) | Years | Annual Rainfall (cm) | Annual Runoff (cm) |
|---|---|---|---|---|---|
| 1995 | 90 | 35 | 2005 | 83 | 15 |
| 1996 | 120 | 60 | 2006 | 110 | 36 |
| 1997 | 85 | 29 | 2007 | 77 | 16 |
| 1998 | 84 | 21 | 2008 | 93 | 28 |
| 1999 | 95 | 35 | 2009 | 136 | 56 |
| 2000 | 160 | 80 | 2010 | 156 | 79 |
| 2001 | 90 | 25 | 2011 | 89 | 26 |
| 2002 | 95 | 39 | 2012 | 80 | 16 |
| 2003 | 75 | 11 | 2013 | 120 | 53 |
| 2004 | 104 | 30 | 2014 | 98 | 26 |

| Year | Annual Rainfall (cm) |
|---|---|
| 2015 | 190 |
| 2016 | 71 |
| 2017 | 175 |
| 2018 | 100 |
| 2019 | 69 |
| 2020 | 101 |

**Example**

Annual rainfall (***P***) and runoff (***R***) values of a catchment spanning a period of 20 years (1995-2014) are given below. **A)** Develop a linear regression relationship between annual rainfall and runoff. **B)** Use the relationship to estimate the runoff from the given annual rainfall value of the next 6 years (2015-2020) period.

| Year | Annual Rainfall (cm) | Annual Runoff (cm) | Years | Annual Rainfall (cm) | Annual Runoff (cm) |
|---|---|---|---|---|---|
| 1995 | 90 | 35 | 2005 | 83 | 15 |
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| Year | Annual Rainfall (cm) |
|---|---|
| 2015 | 190 |
| 2016 | 71 |
| 2017 | 175 |
| 2018 | 100 |
| 2019 | 69 |
| 2020 | 101 |

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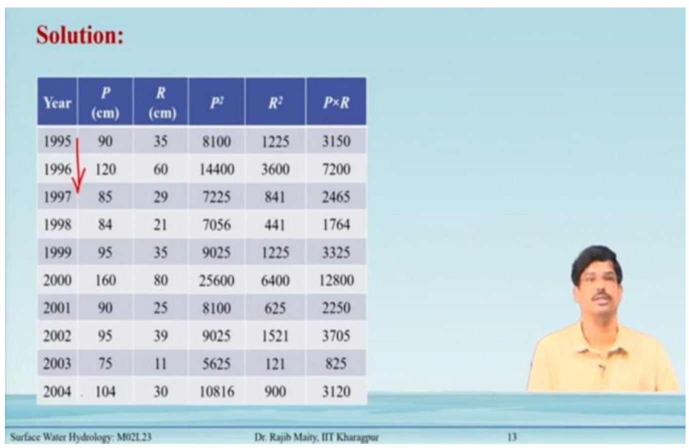

**Solution:**

| Year | $P$ (cm) | $R$ (cm) | $P^2$ | $R^2$ | $P \times R$ |
|---|---|---|---|---|---|
| 1995 | 90 | 35 | 8100 | 1225 | 3150 |
| 1996 | 120 | 60 | 14400 | 3600 | 7200 |
| 1997 | 85 | 29 | 7225 | 841 | 2465 |
| 1998 | 84 | 21 | 7056 | 441 | 1764 |
| 1999 | 95 | 35 | 9025 | 1225 | 3325 |
| 2000 | 160 | 80 | 25600 | 6400 | 12800 |
| 2001 | 90 | 25 | 8100 | 625 | 2250 |
| 2002 | 95 | 39 | 9025 | 1521 | 3705 |
| 2003 | 75 | 11 | 5625 | 121 | 825 |
| 2004 | 104 | 30 | 10816 | 900 | 3120 |

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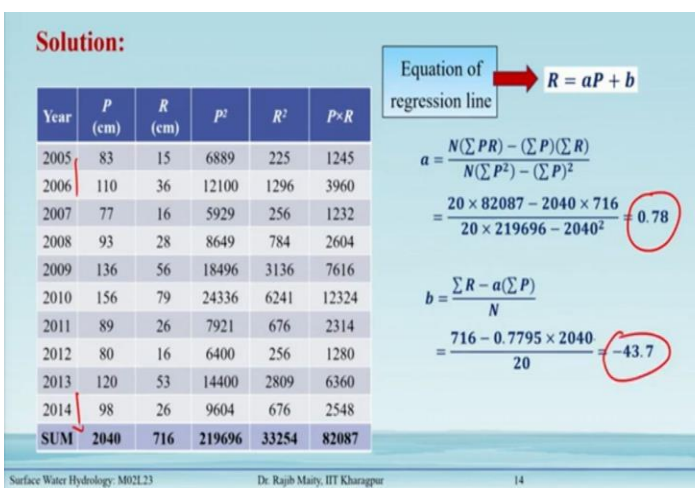

**Solution:**

| Year | $P$ (cm) | $R$ (cm) | $P^2$ | $R^2$ | $P \times R$ |
|---|---|---|---|---|---|
| 2005 | 83 | 15 | 6889 | 225 | 1245 |
| 2006 | 110 | 36 | 12100 | 1296 | 3960 |
| 2007 | 77 | 16 | 5929 | 256 | 1232 |
| 2008 | 93 | 28 | 8649 | 784 | 2604 |
| 2009 | 136 | 56 | 18496 | 3136 | 7616 |
| 2010 | 156 | 79 | 24336 | 6241 | 12324 |
| 2011 | 89 | 26 | 7921 | 676 | 2314 |
| 2012 | 80 | 16 | 6400 | 256 | 1280 |
| 2013 | 120 | 53 | 14400 | 2809 | 6360 |
| 2014 | 98 | 26 | 9604 | 676 | 2548 |
| **SUM** | **2040** | **716** | **219696** | **33254** | **82087** |

Equation of regression line → $R = aP + b$

$$a = \frac{N(\sum PR) - (\sum P)(\sum R)}{N(\sum P^2) - (\sum P)^2} = \frac{20 \times 82087 - 2040 \times 716}{20 \times 219696 - 2040^2} = 0.78$$

$$b = \frac{\sum R - a(\sum P)}{N} = \frac{716 - 0.7795 \times 2040}{20} = -43.7$$

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**Solution:**

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|---|---|---|---|---|---|
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| **SUM** | **2040** | **716** | **219696** | **33254** | **82087** |

Equation of regression line

$$\boldsymbol{R = aP + b}$$

$$\boldsymbol{a = \frac{N(\sum PR) - (\sum P)(\sum R)}{N(\sum P^2) - (\sum P)^2}}$$

$$\boldsymbol{= \frac{20 \times 82087 - 2040 \times 716}{20 \times 219696 - 2040^2} = 0.78}$$

$$\boldsymbol{b = \frac{\sum R - a(\sum P)}{N}}$$

$$\boldsymbol{= \frac{716 - 0.7795 \times 2040}{20} = -43.7}$$

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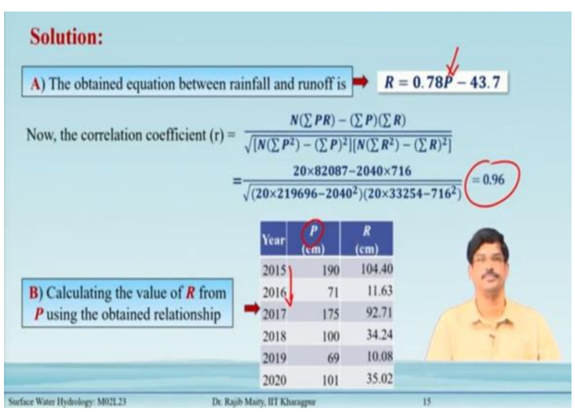

**A**) The obtained equation between rainfall and runoff is

$$R = 0.78P - 43.7$$

Now, the correlation coefficient (r)

$$\frac{N(\sum PR) - (\sum P)(\sum R)}{\sqrt{[N(\sum P^2) - (\sum P)^2][N(\sum R^2) - (\sum R)^2]}}$$

$$= \frac{20\times82087-2040\times716}{\sqrt{(20\times219696-2040^2)(20\times33254-716^2)}} = 0.96$$

**B**) Calculating the value of ***R*** from ***P*** using the obtained relationship

| Year | *P* (cm) | *R* (cm) |
|---|---|---|
| 2015 | 190 | 104.40 |
| 2016 | 71 | 11.63 |
| 2017 | 175 | 92.71 |
| 2018 | 100 | 34.24 |
| 2019 | 69 | 10.08 |
| 2020 | 101 | 35.02 |

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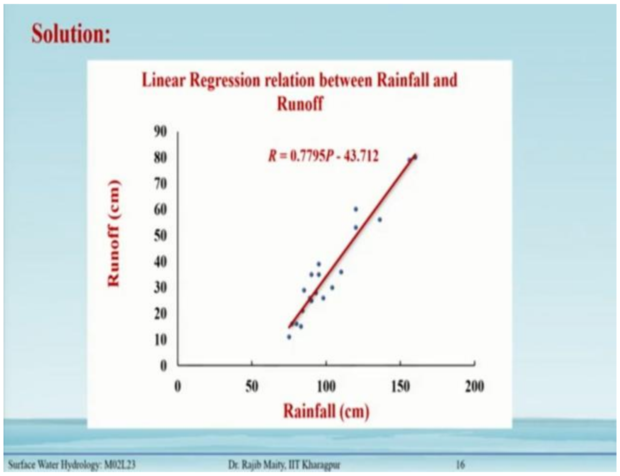

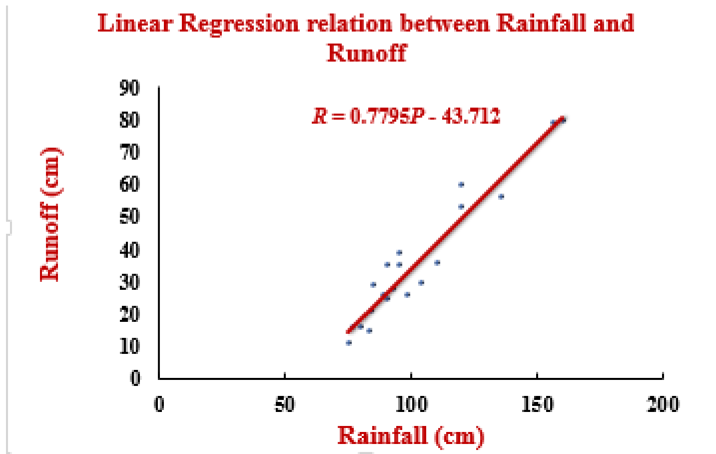


Fig. 2 shows the Linear Regression relation between Rainfall and Runoff

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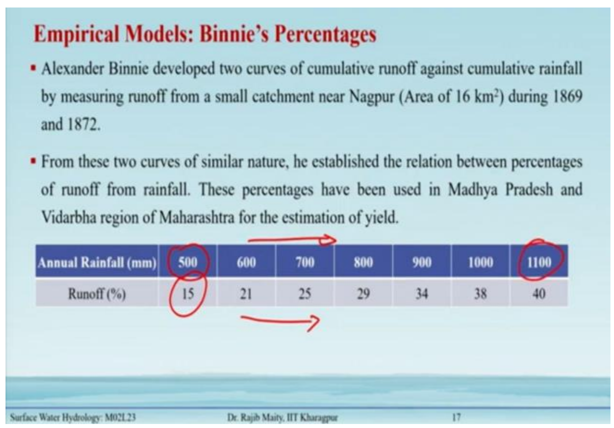


**Empirical Models: Binnie's Percentages**

Alexander Binnie developed two curves of a cumulative runoff against cumulative rainfall by measuring runoff from a small catchment near Nagpur (Area of 16 km2) during 1869 and 1872.

From these two curves of similar nature, he established the relation between percentages of runoff from rainfall. These percentages have been used in the Madhya Pradesh and Vidarbha regions of Maharashtra for the estimation of yield.

| Annual Rainfall (mm) | 500 | 600 | 700 | 800 | 900 | 1000 | 1100 |
|---|---|---|---|---|---|---|---|
| Runoff (%) | 15 | 21 | 25 | 29 | 34 | 38 | 40 |

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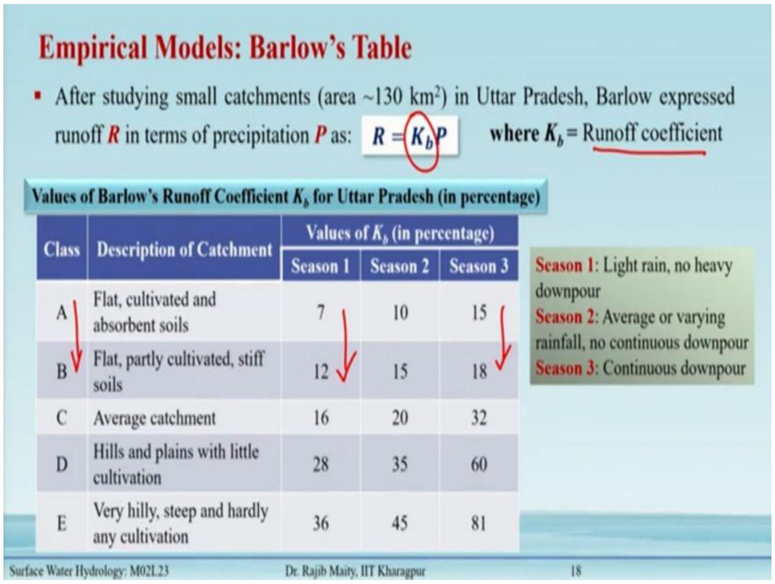


**Empirical Models: Barlow's Table**

After studying small catchments (area ~130 $km^2$) in Uttar Pradesh, Barlow expressed runoff $\boldsymbol{R}$ in terms of precipitation $\boldsymbol{P}$ as:

$$\boldsymbol{R = K_b P}$$

where $K_b$ = Runoff coefficient

**Values of Barlow's Runoff Coefficient $K_b$ for Uttar Pradesh (in percentage)**

| Class | Description of Catchment | Values of $K_b$ (in percentage) | | |
|---|---|---|---|---|
| | | Season 1 | Season 2 | Season 3 |
| A | Flat, cultivated and absorbent soils | 7 | 10 | 15 |
| B | Flat, partly cultivated, stiff soils | 12 | 15 | 18 |
| C | Average catchment | 16 | 20 | 32 |
| D | Hills and plains with little cultivation | 28 | 35 | 60 |
| E | Very hilly, steep and hardly any cultivation | 36 | 45 | 81 |

**Season 1**: Light rain, no heavy downpour

**Season 2**: Average or varying rainfall, no continuous downpour

**Season 3**: Continuous downpour

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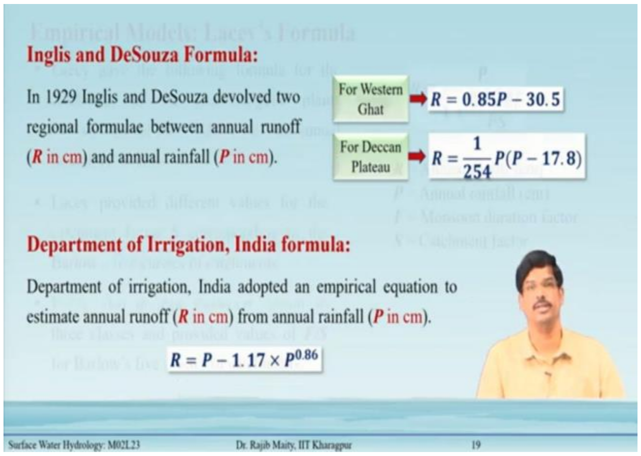


**Inglis and DeSouza Formula:**

In 1929 Inglis and DeSouza devolved two regional formulae between annual runoff (***R*** in cm) and annual rainfall (***P*** in cm).

For Western Ghat

$$\boldsymbol{R = 0.85P - 30.5}$$

For Deccan Plateau

$$\boldsymbol{R = \frac{1}{254}P(P - 17.8)}$$

**Department of Irrigation, India formula:**

Department of irrigation, India adopted an empirical equation to estimate annual runoff (***R*** in cm) from annual rainfall (***P*** in cm).

$$\boldsymbol{R = P - 1.17 \times P^{0.86}}$$

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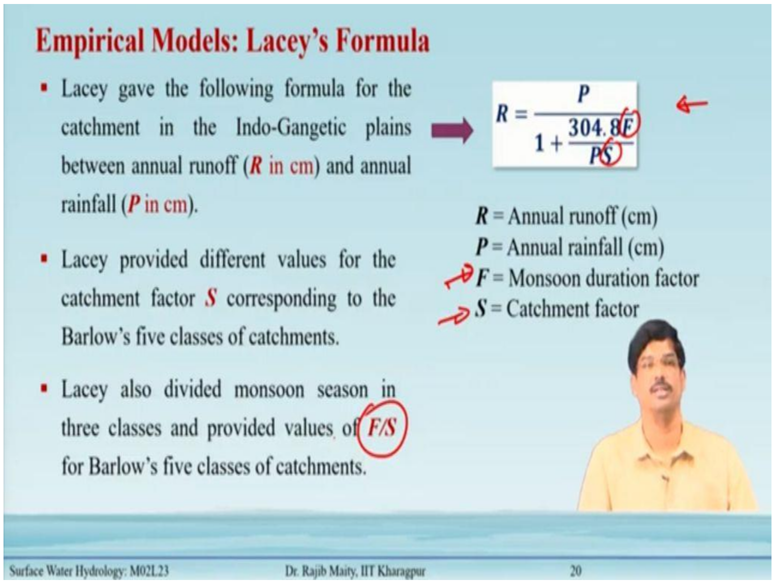


Lacey gave the following formula for the catchment in the Indo-Gangetic plains between annual runoff (R in cm) and annual rainfall (P in cm).

$$\boldsymbol{R = \frac{P}{1 + \frac{304.8F}{PS}}}$$

Where, R = Annual runoff (cm)

P = Annual rainfall (cm)

F = Monsoon duration factor

S = Catchment factor

Lacey provided different values for the catchment factor S corresponding to Barlow's five classes of catchments. Lacey also divided monsoon season into three classes and provided values of F/S for Barlow's five classes of catchments.

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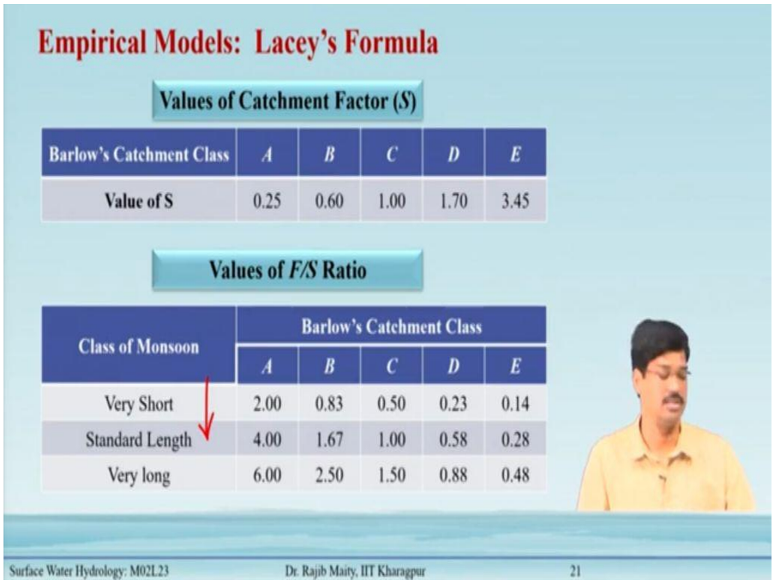


**Empirical Models: Lacey's Formula**

**Values of Catchment Factor (*S*)**

| Barlow's Catchment Class | *A* | *B* | *C* | *D* | *E* |
|---|---|---|---|---|---|
| **Value of S** | 0.25 | 0.60 | 1.00 | 1.70 | 3.45 |

**Values of *F/S* Ratio**

| Class of Monsoon | Barlow's Catchment Class | | | | |
|---|---|---|---|---|---|
| | *A* | *B* | *C* | *D* | *E* |
| Very Short | 2.00 | 0.83 | 0.50 | 0.23 | 0.14 |
| Standard Length | 4.00 | 1.67 | 1.00 | 0.58 | 0.28 |
| Very long | 6.00 | 2.50 | 1.50 | 0.88 | 0.48 |

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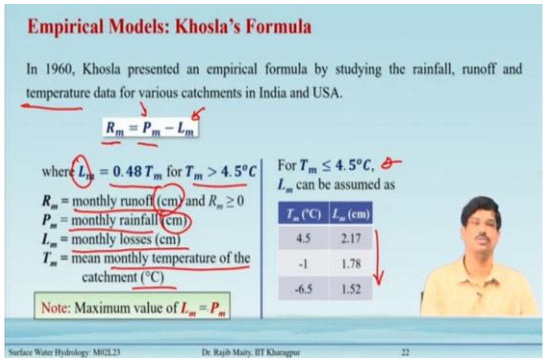


**Empirical Models: Khosla's Formula**

In 1960, Khosla presented an empirical formula by studying the rainfall, runoff, and temperature data for various catchments in India and USA.

$$R_m = P_m - L_m$$

where $L_m = 0.48\, T_m$ for $T_m > 4.5^oC$

$R_m$ = monthly runoff (cm) and $R_m \geq 0$

$P_m$ = monthly rainfall (cm)

$L_m$ = monthly losses (cm)

$T_m$ = mean monthly temperature of the catchment (°C)

For $T_m \leq 4.5^oC$,
$L_m$ can be assumed as

| $T_m$ (ºC) | $L_m$ (cm) |
|---|---|
| 4.5 | 2.17 |
| -1 | 1.78 |
| -6.5 | 1.52 |

It may be noted that the Maximum value of $L_m = P_m$

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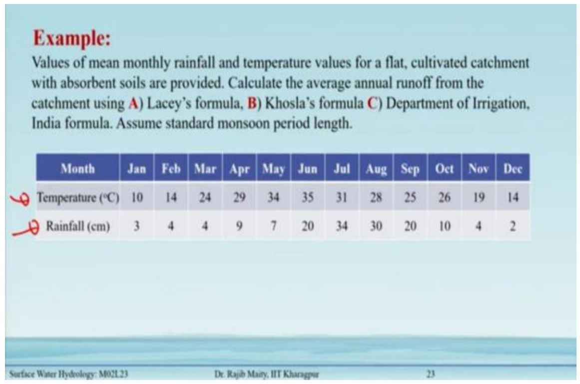
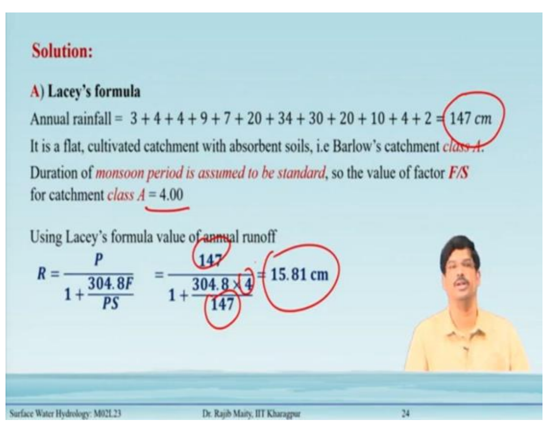



**Example:**

Values of mean monthly rainfall and temperature values for a flat, cultivated catchment with absorbent soils are provided. Calculate the average annual runoff from the catchment using **A**) Lacey's formula, **B**) Khosla's formula **C**) Department of Irrigation, India formula. Assume standard monsoon period length.

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temperature (°C) | 10 | 14 | 24 | 29 | 34 | 35 | 31 | 28 | 25 | 26 | 19 | 14 |
| Rainfall (cm) | 3 | 4 | 4 | 9 | 7 | 20 | 34 | 30 | 20 | 10 | 4 | 2 |

(Refer Slide Time: 25:10)



**Solution:**

A) **Lacey's formula**

Annual rainfall = 3+4+4+9+7+20+34+30+20+10+4+2=147 $cm$

It is a flat, cultivated catchment with absorbent soils, i.e Barlow's catchment class A.

Duration of monsoon period is assumed to be standard, so the value of factor F/S for catchment class A = 4.00

Using Lacey's formula value of annual runoff

$$\boldsymbol{R = \frac{P}{1 + \frac{304.8F}{PS}} \quad = \frac{147}{1 + \frac{304.8 \times 4}{147}} = 15.81 \text{ cm}}$$

So the value of annual runoff from the catchment = 15.81 cm

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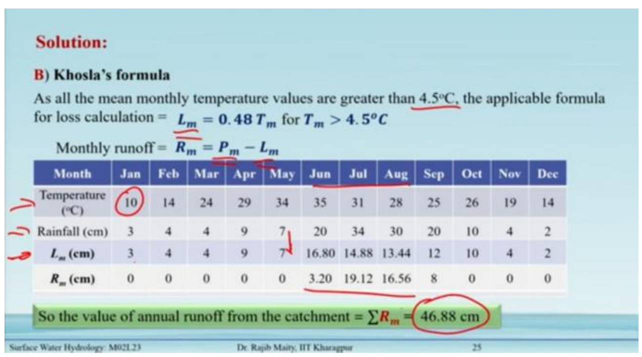


B) **Khosla's formula**

As all the mean monthly temperature values are greater than 4.5$^o$C, the applicable formula for loss calculation =

$$\boldsymbol{L_m = 0.48\, T_m} \text{ for } \boldsymbol{T_m > 4.5^oC}$$

Monthly runoff = $R_m$ = $P_m$ –$L_m$

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Temperature ($^oC$) | 10 | 14 | 24 | 29 | 34 | 35 | 31 | 28 | 25 | 26 | 19 | 14 |
| Rainfall (cm) | 3 | 4 | 4 | 9 | 7 | 20 | 34 | 30 | 20 | 10 | 4 | 2 |
| **$L_m$ (cm)** | 3 | 4 | 4 | 9 | 7 | 16.80 | 14.88 | 13.44 | 12 | 10 | 4 | 2 |
| **$R_m$ (cm)** | 0 | 0 | 0 | 0 | 0 | 3.20 | 19.12 | 16.56 | 8 | 0 | 0 | 0 |

So the value of annual runoff from the catchment = $\sum \boldsymbol{R_m}$ = 46.88 cm

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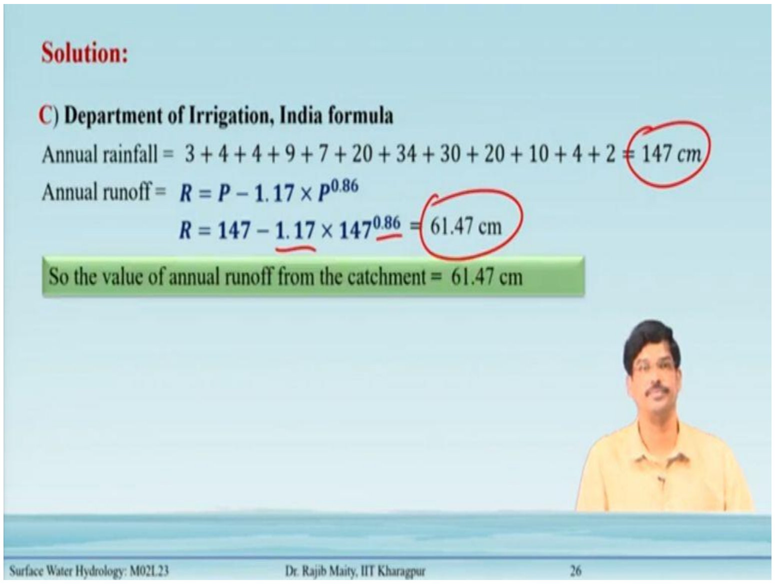



**C) Department of Irrigation, India formula**

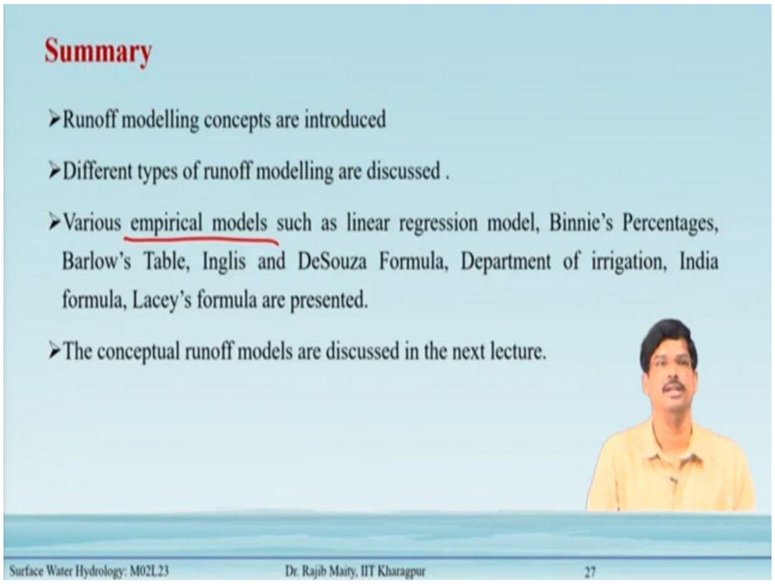
Annual rainfall = 3+4+4+9+7+20+34+30+20+10+4+2=147 $cm$

Annual runoff =

$$\boldsymbol{R = P - 1.17 \times P^{0.86}}$$
$$\boldsymbol{R = 147 - 1.17 \times 147^{0.86}} = 61.47 \text{ cm}$$

So the value of annual runoff from the catchment = 61.47 cm

(Refer Slide Time: 28:15)




**Summary**

In summary, we learned the following points from this lecture:

- Runoff modeling concepts are introduced
- Types of runoff modeling i.e., empirical and conceptual modeling are discussed.
- Various empirical models such as linear regression model, Binnie's Percentages, Barlow's Table, Inglis and DeSouza Formula, Department of irrigation, India formula, Lacey's formula are presented.
- The conceptual runoff models are discussed in the next lecture.