Retrofitting and Rehabilitation of Civil Infrastructure
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Lecture 34
Design of Flexural Strengthening

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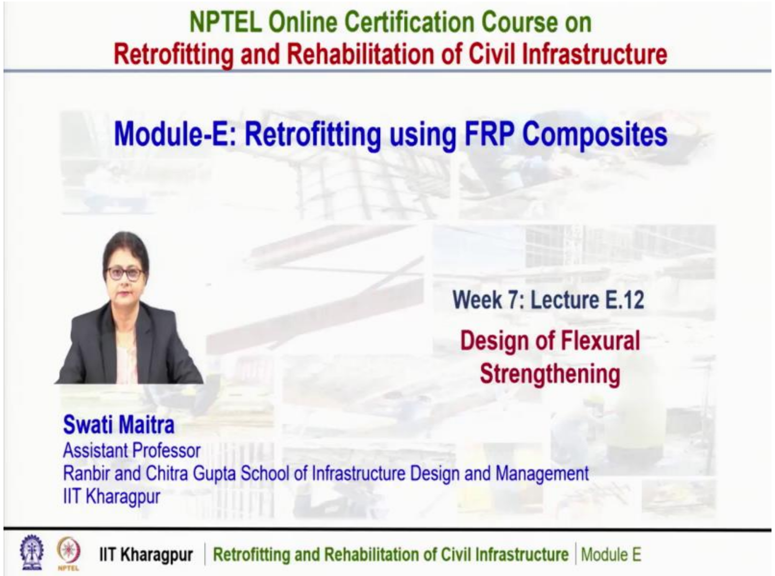

Hello friends, welcome to the NPTEL online certification course Retrofitting and Rehabilitation of Civil Infrastructure. Today, we will discuss module E, the topic for module E is retrofitting using fiber reinforced polymer composites.

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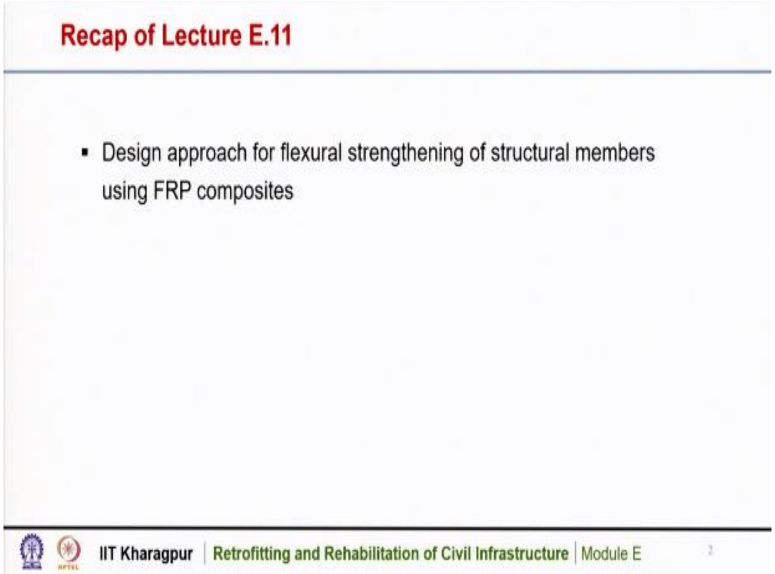

In the previous lecture we have discussed the design approach for flexural strengthening of structural members using fiber reinforced polymer composites. We have discussed the various approaches for flexural strengthening based on strain compatibility, force equilibrium and also considering the different failure modes.

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Today, we will discuss a design of flexural strengthening of structural members using fiber reinforced polymer composites.

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Based on the design approaches for flexural strengthening of structural members like beams or slabs using fiber reinforced polymer composites we can design a structural member. So, here is a design example and that we will discuss today. In this example a simply supported concrete beam has been considered, which is reinforced with 3 numbers of 28-millimeter diameter steel bars as longitudinal reinforcement and is located in a warehouse and is subjected to a 50 percent increase in its life load carrying requirements.

An analysis of the existing beam indicates that the beam still has sufficient shear strength to resist the new required shear strength and meets the deflection and crack control serviceability requirements. Its flexural strength however is inadequate to carry the increased life load. We have to design an FRP strengthening system for this beam.

So, in this design example we will design an FRP strengthening system for this existing beam, which has some longitudinal reinforcement and it is subjected to a increase in its live load carrying requirements.

So, the flexural capacity of the beam needs to be increased and its shear strength capacity however is adequate. So, we need to design this flexural strengthening system for the beam using FRP composite.

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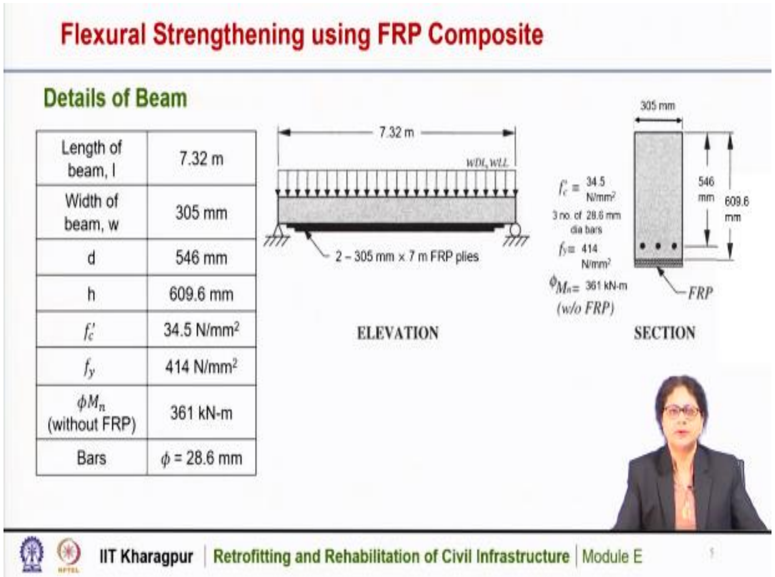

These are the details of the beam, here we can see that this is a simply supported beam and it is subjected to UDL. This is due to dead load as well as live load and this is the cross section of the beam. The dimensions and other details are shown here, the beam is of length 7.32-meter, width is 305-millimeter, depth is 546 millimeter that is the effective depth and the overall depth is 610 millimeters. The compressive strength for the concrete is 34.5 Newton per millimeter square and the steel has yield strength of 414 Newton per millimeter square.

The nominal moment for the beam is 361 kilo Newton meter and it has 3 bars 28-millimeter diameter and this is the section for the beam we can see here that this is a beam and it has 3 reinforcement bars which is subjected to a UDL and is simply supported. Now, for the increased live load we need to design the beam using FRP composite so that it can take the additional live load on it.

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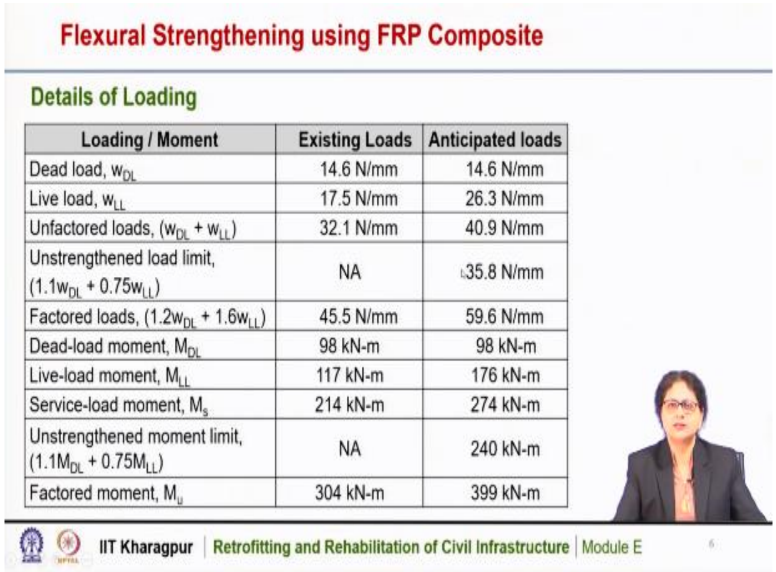

Loading / Moment	Existing Loads	Anticipated loads
Dead load, w_{DL}	14.6 N/mm	14.6 N/mm
Live load, w_{LL}	17.5 N/mm	26.3 N/mm
Unfactored loads, $(w_{DL} + w_{LL})$	32.1 N/mm	40.9 N/mm
Unstrengthened load limit, $(1.1w_{DL} + 0.75w_{LL})$	NA	35.8 N/mm
Factored loads, $(1.2w_{DL} + 1.6w_{LL})$	45.5 N/mm	59.6 N/mm
Dead-load moment, M_{DL}	98 kN-m	98 kN-m
Live-load moment, M_{LL}	117 kN-m	176 kN-m
Service-load moment, M_s	214 kN-m	274 kN-m
Unstrengthened moment limit, $(1.1M_{DL} + 0.75M_{LL})$	NA	240 kN-m
Factored moment, M_u	304 kN-m	399 kN-m

These are the details of loading on the beam. The existing loads are given and also the anticipated loads are given. So, the dead load has been given for the existing beam is 14.6 Newton per millimeter, it is the same because if we do the FRP composite strengthening so there is no practical change in the dead load.

The life load is given here and this has been increased to 50 percent of it so it becomes 26 Newton per millimeter. So, the unfactored loads are also shown here, this is for the existing condition and this is for the anticipated condition. So, we need to calculate the unstrengthen load limit that is $(1.1\ W_{DL} + 0.75\ W_{LL})$ and that we have discussed in the previous lecture that the existing member should have a sufficient strength to go for retrofitting, so that we need to check.

The factored loads are also given for the existing condition and also for the anticipated loads, the dead load moment, live load moment, service load moments are, all are given here and that can be calculated because we know the dead load and the dimensions of the beam. And this is the unstrengthen moment limit that is also calculated here and this is the factored moment.

So, for the existing load condition 304 kilo Newton meter is the factored moment and for the anticipated load it is 399 kilo Newton meter. So, these are the details of the load, the existing loads are given and also the anticipated loads. Now, we need to check whether the strengthening limits have been satisfied or not and if that is so then how we can retrofit it.

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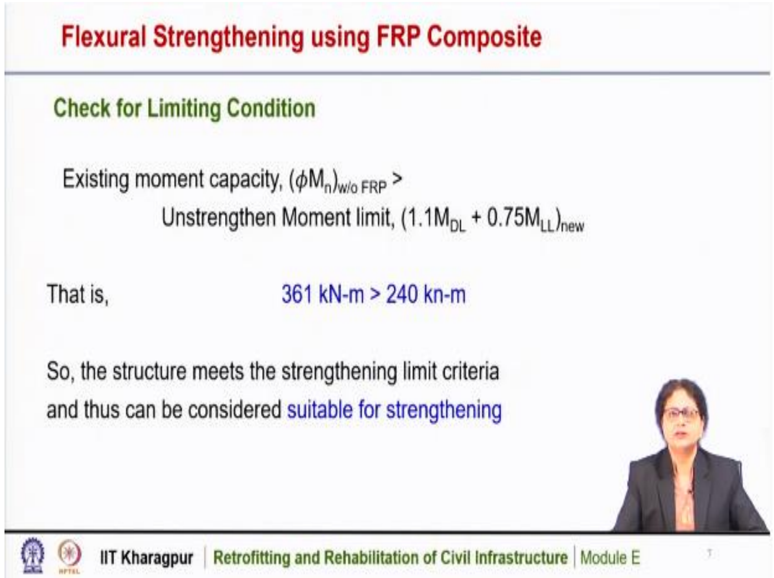

So, check for the limiting condition, this is the existing moment capacity should be more than the unstrengthen moment limit that is ($1.1\ M_{DL} + 0.75\ M_{LL}$) of the new structure. So, the existing moment capacity should be more than this much so that it can be considered for retrofitting. So, the existing moment capacity is 361 and that is more than 240 kilo Newton meter, which is the unstrengthen moment limit. So, the structure meets the strengthening limit criteria and thus can be considered as suitable for strengthening.

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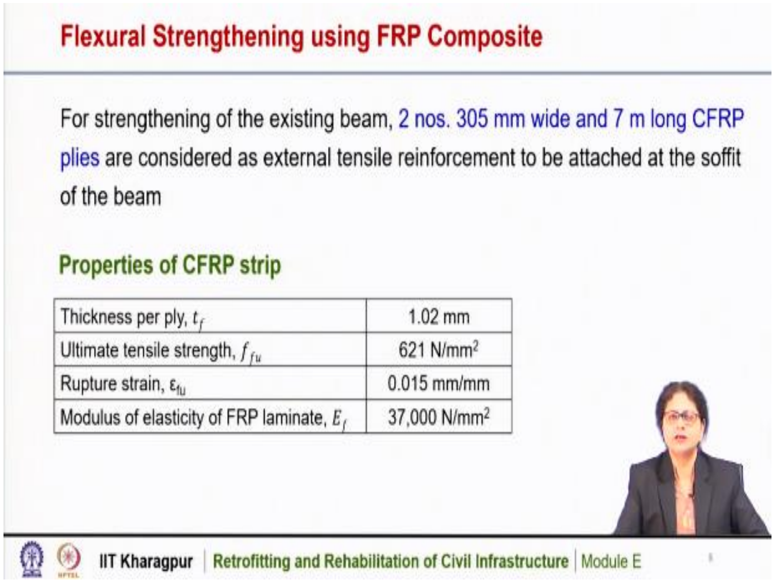

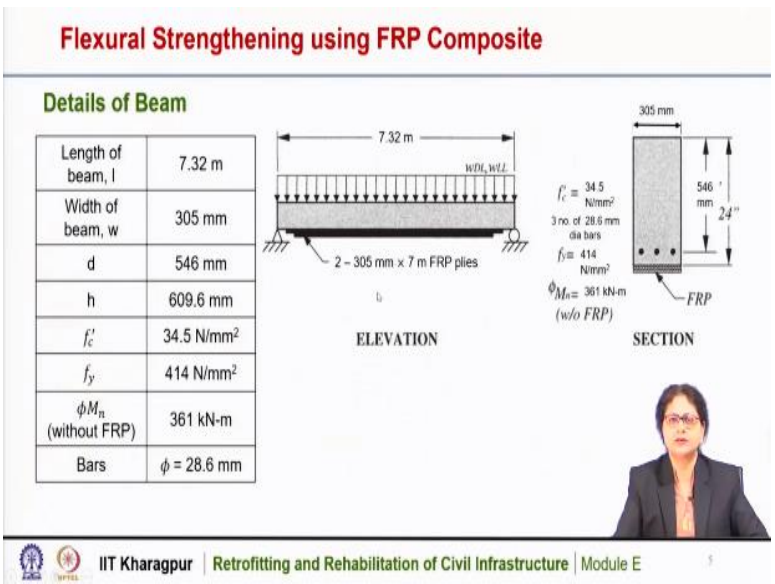

Now, we need to strengthen it with FRP composite for strengthening of the existing beam 2 numbers of 305 millimeter wide and 7-meter long CFRP sheets are considered as external tensile reinforcement and that is to be attached at the soffit of the beam. So, here in this diagram also it is shown that we are considering 2 FRP plies of 7-meter length and they are of full width. So, they are of width 305 millimeter and that needs to be attached at the bottom of the beam for flexural strengthening.

The properties of the CFRP sheets are given here the thickness per ply is 1.02 millimeter and the ultimate strength is 621 Newton per millimeter square, the rupture strain is also given 0.015 and also the modulus of elasticity of the FRP. So, these are the properties of the CFRP strip that has been considered for strengthening of the existing beam member.

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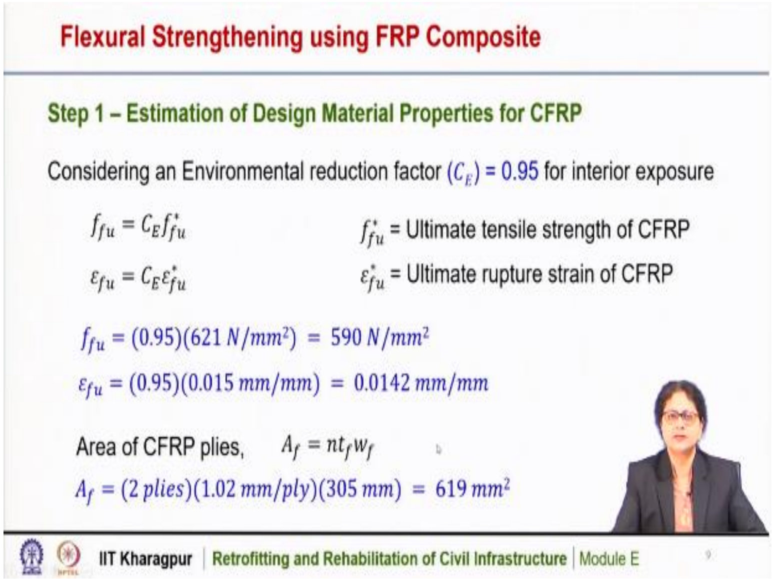

Now, for the design we have to follow certain steps and that we have discussed in detail in the previous lecture. The major steps and the approaches of design have been discussed. So, here in this example first we need to estimate the design material properties for the CFRP, the beam is situated in an interior condition.

So, the interior exposure condition needs to be considered for estimating the environmental reduction factor. So, based on the interior exposure the environmental reduction factor that is CE has been taken as 0.95 and this needs to be multiplied with the ultimate stress and strain to get the design material properties of the CFRP.

So, the design material properties of the CFRP this is the ultimate tensile strength that needs to be considered that is $f_{fu} = (CE) f_{fu}{}^*$ this is the ultimate tensile strength of the CFRP as obtained from the manufacturer's data or we can test it also. And the strain also similarly to be multiplied with the CE factor so that we can get the design strain and these have been calculated here.

So, f_{fu} is 0.95 × 621 that is the ultimate strength of the CFRP and the ultimate strain is 0.015 so that needs to be multiplied with the environmental correction factor to get the design strength values and the design strain values. So, these are calculated here and since we have used two plies so the area of CFRP plies is calculated using this equation $A_f = nt_f w_f$.

So, considering these things we can find out that the area of the CFRP plies are 619 millimeter square. So, this is the area of the CFRP plies that is used for this strengthening purpose.

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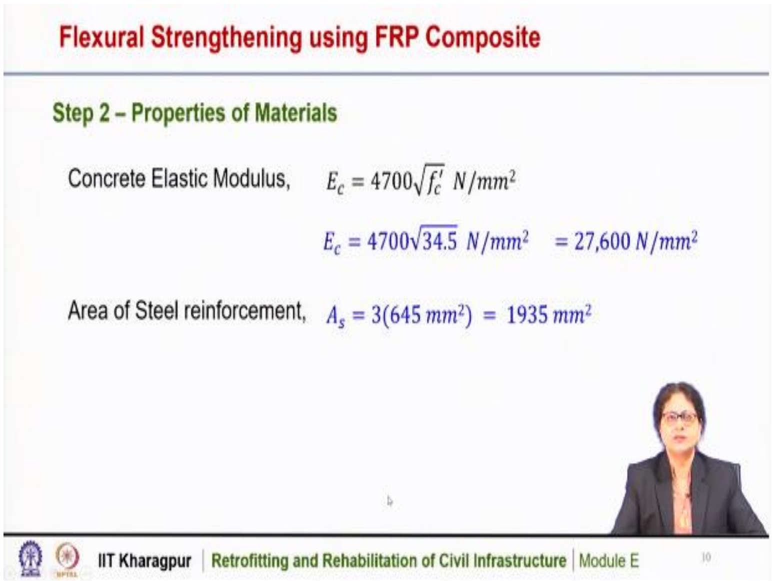

Next, we need to estimate the properties of steel and concrete. So, the concrete elastic modulus is estimated using this equation this is as per American Concrete Institute it may be little different in Indian guidelines. So, we are following this as per the ACI guidelines. So, elastic modulus of concrete is $E_c = 4700\sqrt{f_c'}\ N/mm^2$, that is the compressive strength, so we can get the modulus value as 27600 Newton per millimeter square.

The area of steel reinforcement has been given 3 no 28 millimeter diameter bars have been used, so the total area of steel reinforcement is coming out as 1935 millimeter square. So, these are the properties of the steel, concrete and FRP and also their areas.

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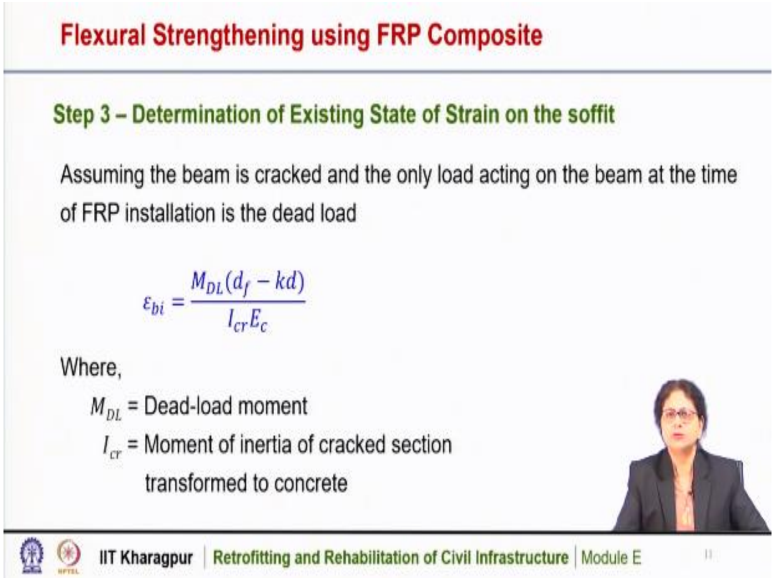

Now, we need to determine the existing state of strain on the soffit of the beam. Assuming the beam is cracked and the only load acting on the beam at the time of FRP installation is the dead load. We have mentioned in the previous lecture that we need to estimate also the epsilon bi, which is the strain of the concrete at the soffit during the time of FRP installation.

So, this can be estimated using this equation $\varepsilon_{bi} = [M_{DL}(d_f - kd)]/(I_{cr}E_c)$, M_{DL} is the dead load moment, d_f is the effective depth, kd is the depth of the neutral axis, I_{cr} is the moment of inertia of the cracked section transformed to concrete and E_c is the elastic modulus of the concrete.

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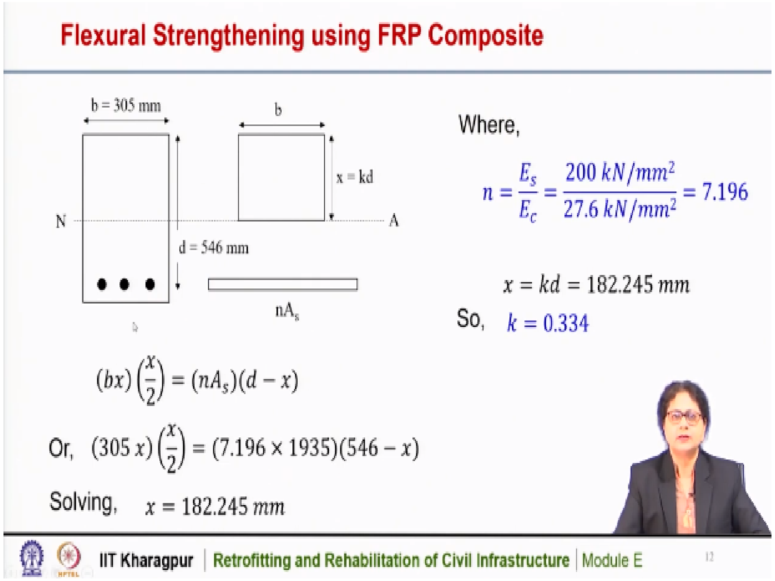

So, considering the cracked section as we can see here this is the beam section and considering the cracked section, we can find out that this is the neutral axis and by considering the moment equilibrium we can write this equation and the modular ratio that is the ratio of the elastic modulus of steel and concrete that is n is calculated here as 7.196.

Now, using this equation and we know the value of As, d we can solve x. So, once we know x we can find out the k because x is kd, so from that we can find out the value of k.

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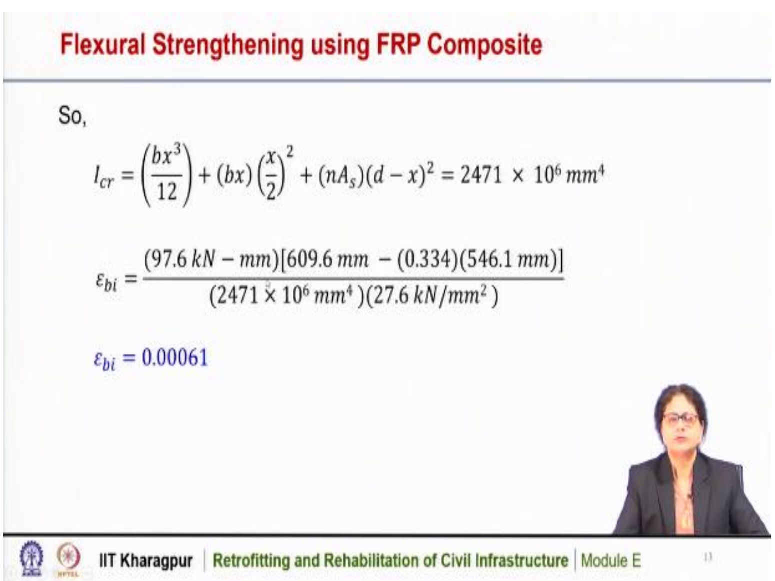

Now, this crack moment of inertia, we can calculate using this relationship and that is coming out as 2471×10^6 millimeter^4. So, we can get I_{cr} and from that now we can get the strain in the concrete soffit at the time of FRP installation using this equation ε_{bi}. So, knowing the dead load moment and the depth and the kd and the I_{cr} also the E_c we can find out ε_{bi}, which is coming out as 0.00061.

So, this is the strain in the concrete at the time of FRP installation. So here you can remember the internal stress strain distribution of the concrete and then we can correlate this to that diagram.

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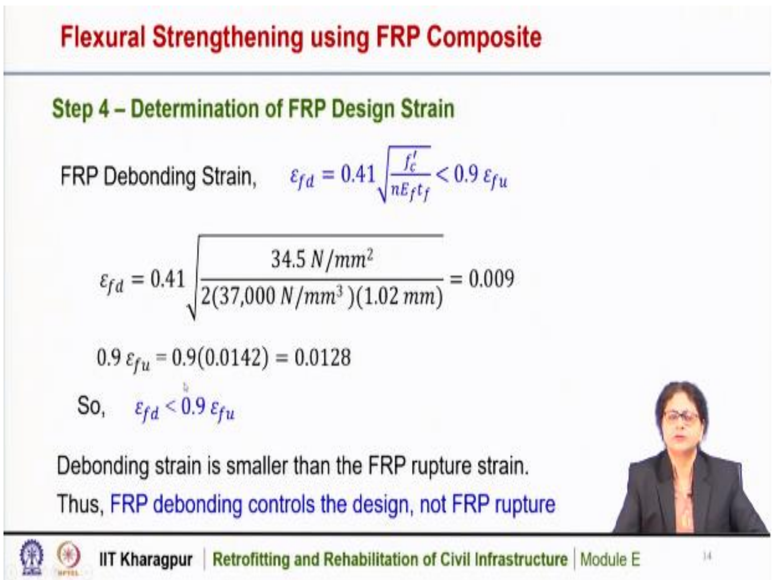

Now, after knowing the strain at the bottom of the concrete at the time of FRP installation we need to determine the FRP designs strain. Now, the FRP design strain we have to calculate first the FRP debonding strain and this debonding strain can be obtained from this equation, we discussed this that there may be different types of failure for an FRP strengthened member, it could be due to concrete crushing or it could be due to rupture of FRP or it could be due to debonding.

So, there are limiting conditions for each of this, the concrete may crush when the ultimate strain in the concrete when it reaches 0.003, then there will be crushing of concrete. When the strain in the FRP reaches its ultimate strain then there may be a rupture of FRP and if the strain in the FRP reaches its debonding strain limit then there will be debonding of the FRP from the concrete.

So, this is the relationship for FRP debonding strain and that should be less than 0.9 times the ultimate strain of the FRP. $\varepsilon_{fd} = 0.41\sqrt{[f_c'/(nE_f t_f)]} < 0.9\varepsilon_{fu}$, n is the 2 number of plies and t_f is the thickness of each ply, so we can get ε_{fd} as 0.009.

Now, we will calculate $0.9\varepsilon_{fu}$ and we obtained as this is 0.0128, so this ε_{fd} is less than 0.9 ε_{fu}. So, the debonding strain is smaller than the FRP rupture strain, thus FRP debonding controls the design not FRP rupture.

So, we have calculated the FRP debonding strain and we found that the FRP debonding strain is less than the rupture strain. So, in the design the FRP debonding actually controls not the FRP rupture.

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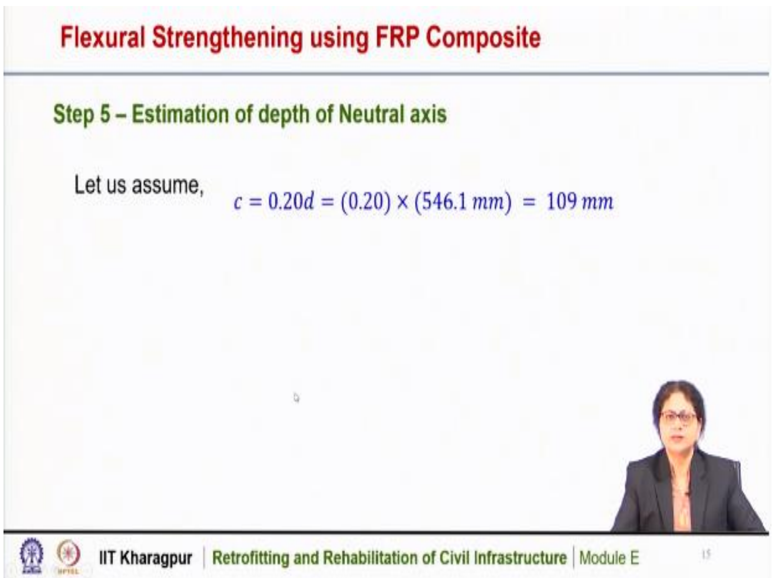

Now, we need to estimate the depth of the neutral axis because in the previous lecture we have discussed that we have to consider the force equilibrium, also the strain compatibility. So, here in this case while estimating the depth of neutral axis we will use the force equilibrium and for that an iterative procedure need to be carried out.

So, for that we have to assume value of neutral axis and then we will check whether the force equilibrium has been achieved or not. So, let us assume the depth of neutral axis c as 0.2 times the effective depth, so it is 0.2 × 546.1 millimeter, so the first trial is c as 109 millimeter.

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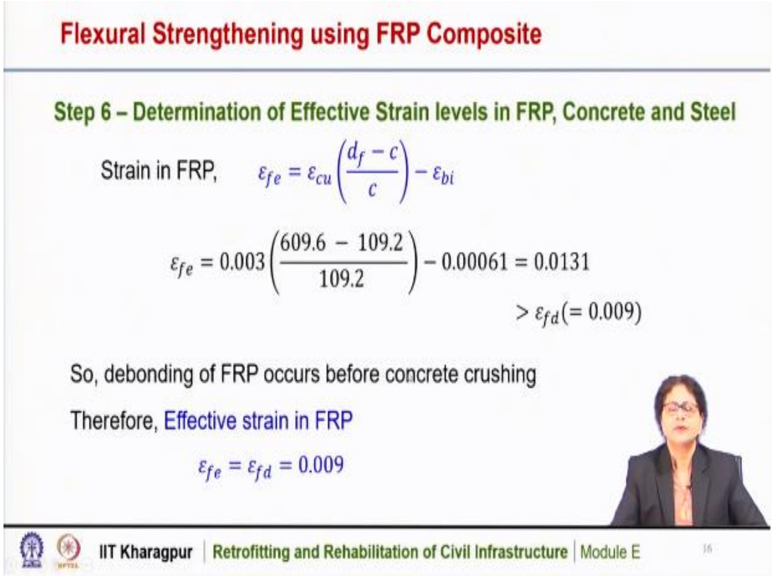

Now, we will determine the effective strain level in the FRP, concrete and steel. So, we have discussed this, the internal stress strain distribution in the beam. So, from that we have discussed that the strain in the FRP can be written as $\varepsilon_{fe} = \varepsilon_{cu}[(d_f - c)/c] - \varepsilon_{bi}$.

So, you must be remembering this, so ε_{cu} is the ultimate strain in concrete, d_f is the depth of the beam, c is the depth of the neutral axis and ε_{bi} is the strain in the concrete at the time of FRP installation that we have just found out. So, by putting these values we get ε_{fe} that is the strain in the FRP is equal to 0.0131 and this value is more than the ε_{fd} what we have obtained in the previous step.

So, the strain in the FRP is more than the debonding strain which is equal to 0.009, so debonding of FRP occurs before concrete crushing, because this relationship uses the ultimate strain in concrete from the force equilibrium. So, here, so this equation uses the ultimate strain in concrete that is epsilon cu however it shows that when the concrete is at its ultimate strain, the strain in the FRP is 0.0131, which is much more than the debonding strain.

So, this strain will not be achieved by the FRP because before that debonding will occur. So, debonding of the FRP occurs before concrete crushing. So, we can write the effective strain in FRP that is ε_{fe} is equal to ε_{fd} which is 0.009.

Since there is debonding of the FRP before concrete crushing, so the strain in the concrete is less than its ultimate strain at debonding because the concrete will not reach to its ultimate strain value and before that there will be debonding. So, the strain in the concrete will also be less than its ultimate strain. So, from that similar triangle only we can find out the effective strain in concrete using this relationship by rearranging the terms.

So, ε_c that is the effective strain in concrete is equal to $\varepsilon_c = (\varepsilon_{fe}+\varepsilon_{bi})\ [c/(d_f-c)]$. So, putting the values we get ε_c is 0.0021, so this is the effective strain in concrete that will be there 0.0021.

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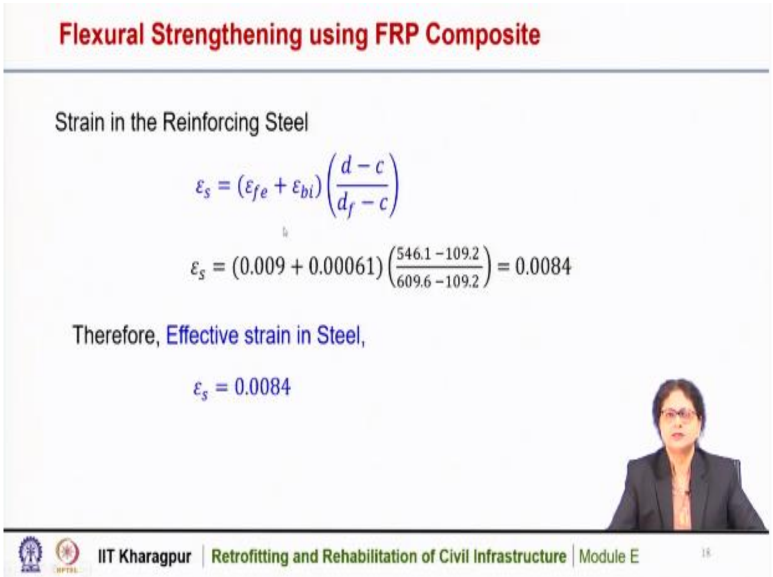

Now, we will find out the strain in the reinforcing steel. So similarly, we can get this equation for the strain in the reinforcing steel that is ε_s, which is written as $\varepsilon_s = (\varepsilon_{fe}+\varepsilon_{bi})$ $[(d-c)/(d_f-c)]$. So, by putting the values we get ε_s that is the effective strain in steel is equal to 0.0084. So, this is the effective strain in steel, so here in this step we have determined the effective strain in FRP, effective strain in concrete and effective strain in steel.

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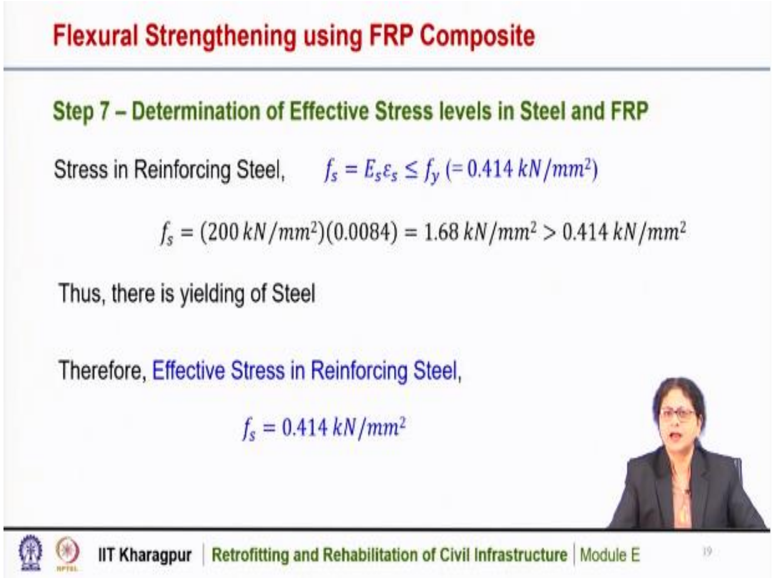

Now, in the next step we will determine the effective stress levels in steel and in FRP. Stress in the reinforcing steel can be written as $f_s = E_s\varepsilon_s$ that is the elastic modulus of steel into the strain in steel and that should be less than equal to f_y that is the yield strength of steel.

So, E_s is given and ε_s we have obtained, so f_s that is the stress in reinforcing steel is coming out as 1.68 kilo Newton per millimeter square and this is more than 0.0414 kilo Newton per millimeter square which is f_y. So, this indicates that there will be yielding of steel. So, the effective stress in the reinforcing steel will be its yield stress that is 0.414 kilo Newton per millimeter square. However, the strengthened beam will be still taking the load and that load will be taken by the FRP.

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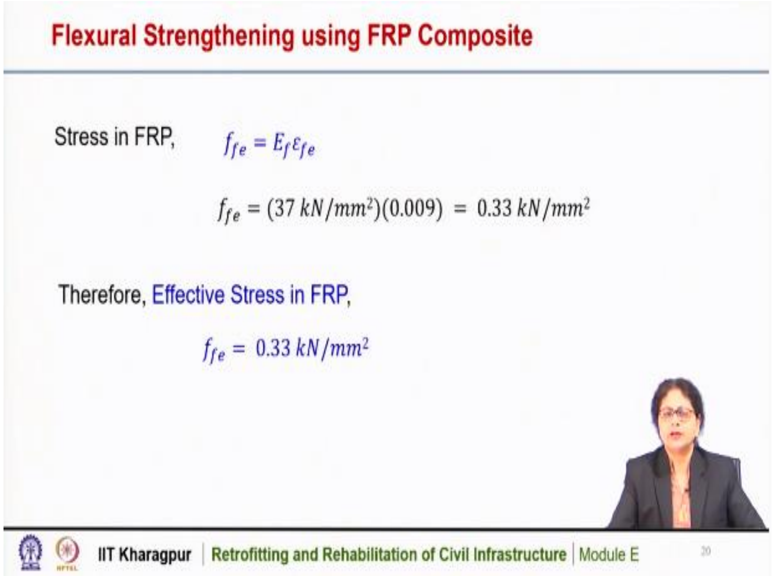

Now, we will find out the stress in the FRP. So, similarly, we can find out the stress in the FRP, $f_{fe} = E_f \varepsilon_{fe}$ and that we can write down like this E_f is we have obtained it using the environmental correction factor. So, by putting the values of E_f and ε_{fe} that is the strain in the FRP we can get the stress in the FRP as 0.33 kilo Newton per millimeter square.

So, the effective stress in FRP is 0.33 kilo Newton per millimeter square. So, in this step we have determined the effective stress in steel and effective stress in FRP.

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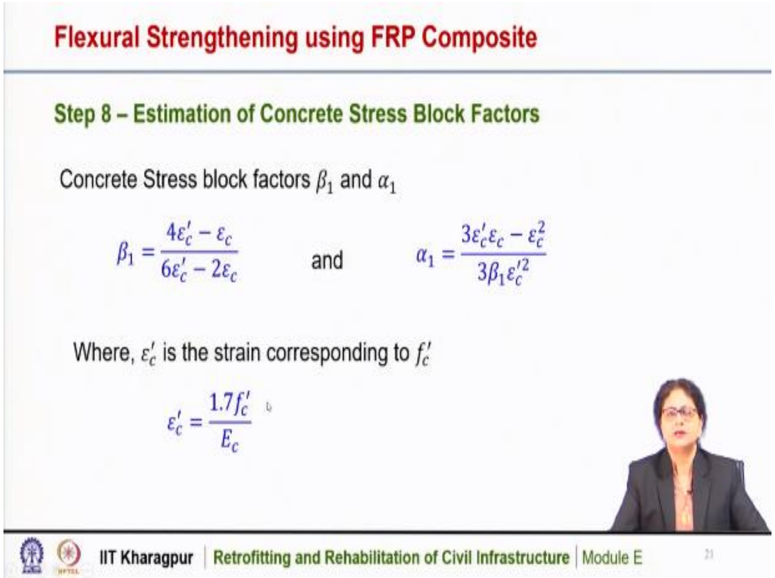

Now, we will estimate the concrete stress block factors, you may be remembering that the stress distribution in concrete is non-linear. Now, we will make it for an equivalent rectangular section and for that there are two factors, two stress block factors, β_1 and α_1. Now, β_1 and α_1 can be obtained using these relationships as are shown here, these are the relationship for β_1 and α_1 for the concrete stress block factors and here ε_c' is the strain corresponding to f_c'.

So, ε_c' can be obtained using this relationship $\varepsilon_c' = 1.7f_c'/E_c$, but f_c' is the compressive strength of concrete and E_c is the elastic modulus.

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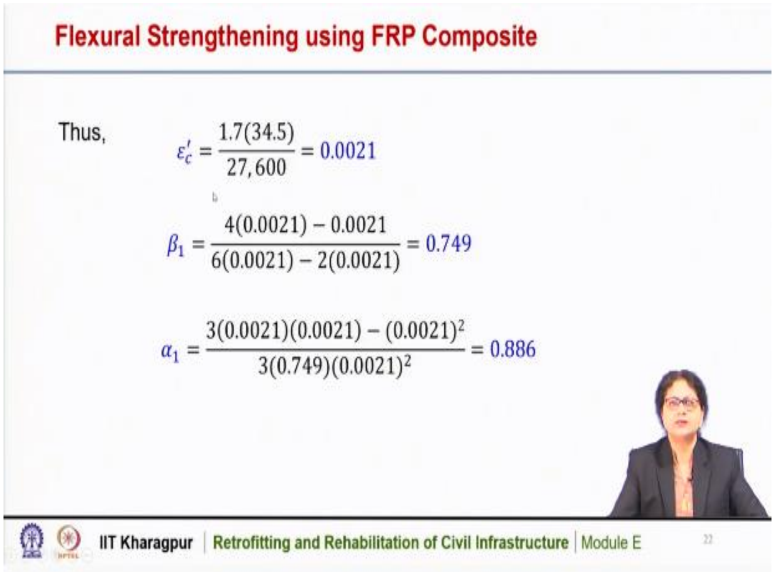

So, we can get the $\varepsilon_c{}'$ as 0.0021 and by using the $\varepsilon_c{}'$ value we can find out β_1 as 0.749 and α_1 as 0.886. So, the stress block factors are obtained.

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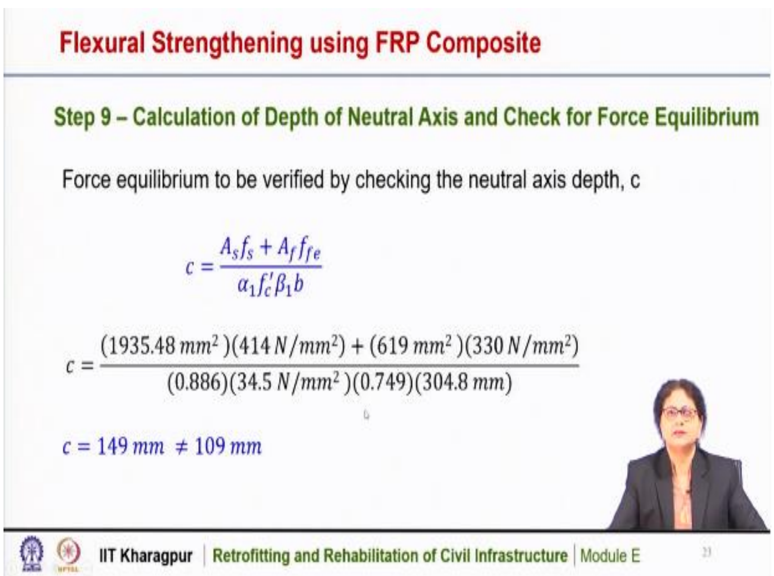

Now, we have to calculate the depth of neutral axis and check for the force equilibrium. So, this is the relationship for the neutral axis, so the force equilibrium needs to be verified by checking the neutral axis depth c. So, this is the expression for the neutral axis from the force equilibrium. So, $c = (A_s f_s + A_f f_{fe})/(\alpha_1 f_c{}' \beta_1 b)$

So, by putting these values we get that c is equal to 149 millimeter and this is not equal to what we have assumed, we have assumed 109 millimeter. So, it is not equal to our assumed value, so we have to go for another trial.

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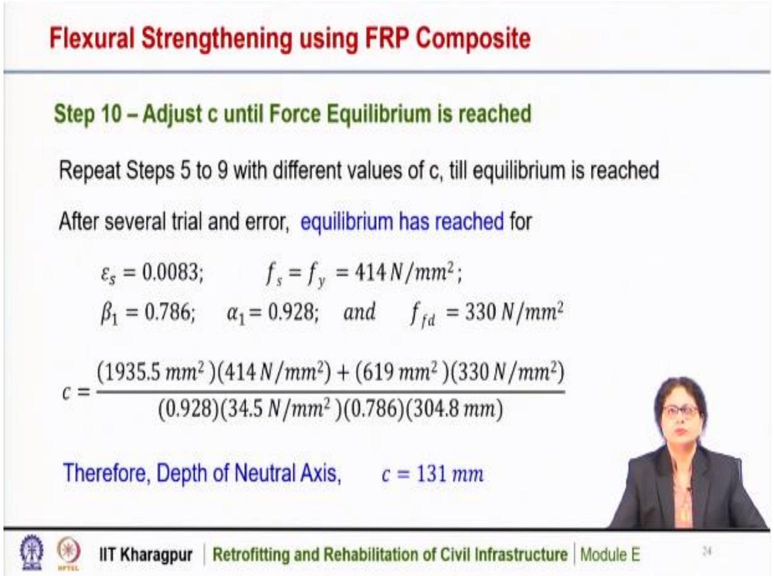

Now, we can assume another value of c and can repeat the steps 5 to 9 with different values of c until the equilibrium is reached. So, after carrying out several trial and error here in this problem the equilibrium has been reached for these values of beta 1, alpha 1. So, these are the values we have obtained for the value of neutral axis and with this assumed value of c the equilibrium is reached.

So, the c is coming out as 131 millimeter. So, the depth of neutral axis is obtained as 131 millimeter, where the force equilibrium is maintained. So, this value of neutral axis depth we will consider.

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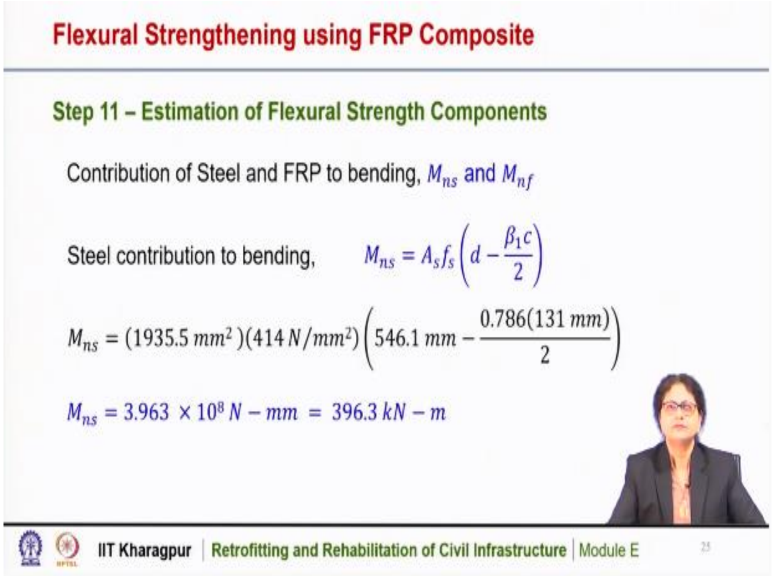

Now, we have to estimate the flexural strength components, the strength will be taken by the steel as well as the FRP. So, the contribution of steel and FRP to bending we have to calculate. So, the contribution of steel is written as M_{ns} and the contribution of FRP is M_{nf}.

So, the steel contribution to bending is $M_{ns} = A_s f_s[d-(\beta_1 c/2)]$, that we have discussed in the previous lecture. So, by putting these values we can get the value of M_{ns} that is the contribution of steel to bending. So, this is coming out as 396.3 kilo Newton meter. So, this is the moment carried by the steel reinforcement.

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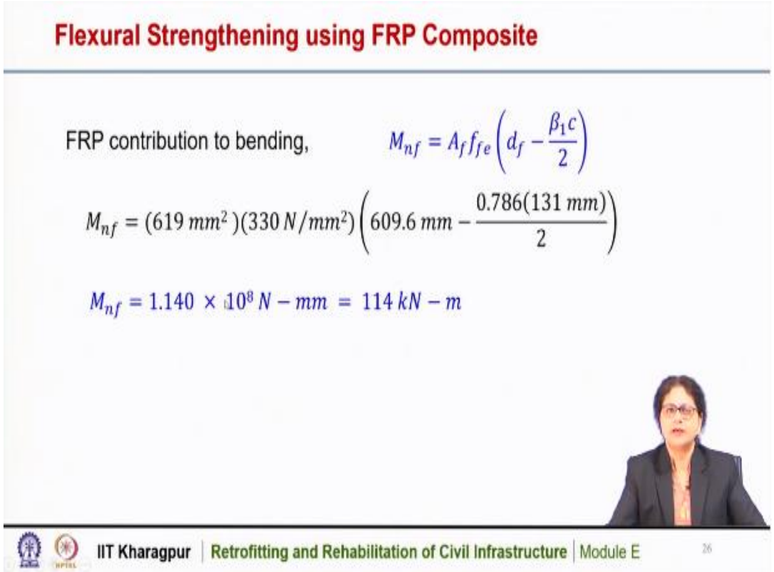

And this is the moment carried by the FRP. So, FRP contribution to bending is given by this equation $M_{nf} = A_f f_{fe}[d_f-(\beta_1 c/2)]$. So, by putting these values we get the moment contribution of the FRP, which is coming out as 114 kilo Newton meter.

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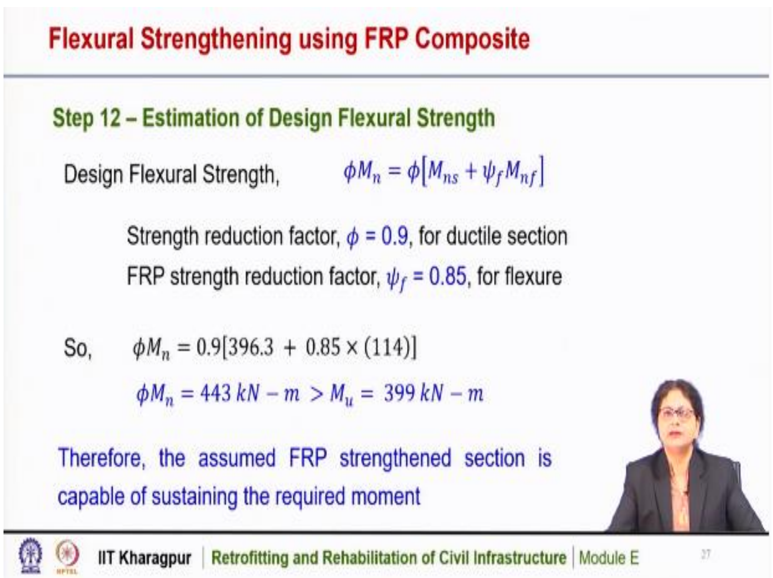

Now, we have to estimate the design flexural strength. So, the design flexural strength is given by this equation and this also was mentioned in the previous lecture. So, $\phi M_n = \phi M_{ns}+\psi_f M_{nf}$, $\boldsymbol{\phi}$ is the strength reduction factor and for ductile section the value is 0.9 as given in the ACI guideline. And for the FRP we need to use another strength reduction factor to take care of the uncertainties and the non-uniformity of the properties and that value is 0.85.

So, we can find out the design flexural strain that is ϕM_n = 0.9 [396.3 + 0.85× (114)] = 443 kN-m.

So, it is coming out as 443 kilo Newton meter, which is more than the required moment that is 399 kilo Newton meter. So, the assumed FRP strengthen section is capable of sustaining the required moment. So, here in this calculation we have estimated the design flexional strength, the total moment shared by the steel and the FRP is coming out as 443 kilo Newton meter with 2 plies of FRP of carbon fiber composites plus the existing steel reinforcement and that is more than the required moment that is 399 kilo Newton meter. So, our assumed FRP section is safe and the FRP strengthened beam is now capable of taking the additional load or moment due to the increased life load.

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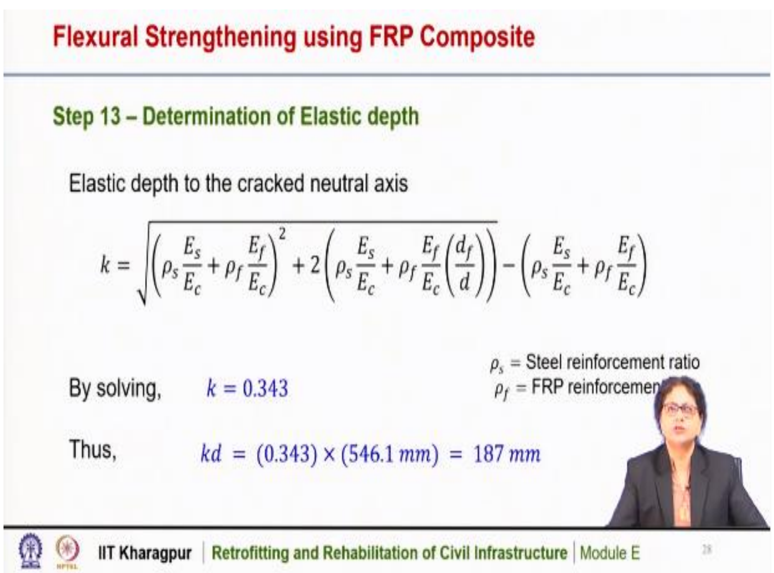

Now, we need to check the serviceability criteria and for that we need to determine the elastic depth. So, elastic depth to the cracked neutral axis has been given by this equation and we have to calculate the depth of neutral axis for the elastic condition and that is kd. Here ρ_s is the steel reinforcement ratio and ρ_f is the FRP reinforcement ratio. So, k can be estimated using this relationship, so by solving we get k is equal to 0.343 thus kd is equal to 187 millimeter.

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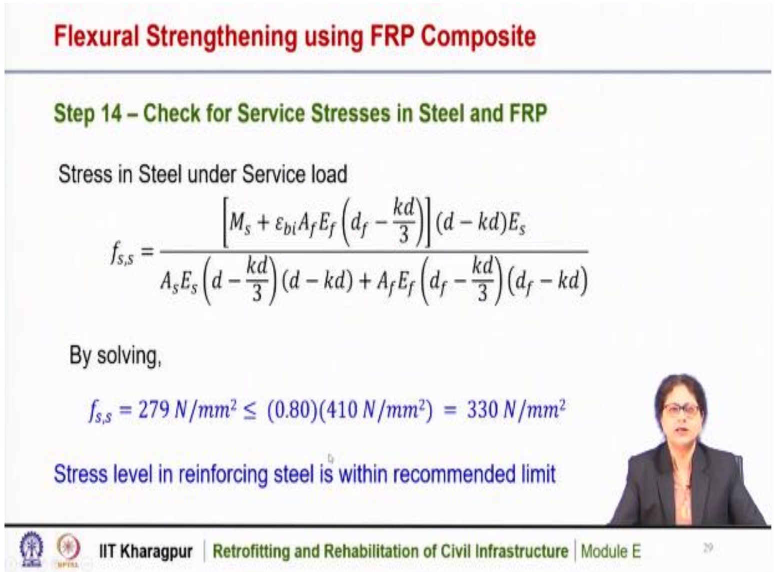

So, we can find out the service stresses in steel and FRP by knowing the value of kd. So, the stress in the steel under the service load condition is given by this equation, this also we have discussed in the previous lecture. So, this is the equation for estimating the stress in steel under the service load condition so that is coming out as 279 by putting the values.

So, we are getting the stress in steel under service load condition is 279 Newton per millimeter square, which is less than its limiting value which is 0.8 f_y, that is 410, so it is 330 Newton per millimeter square, so it is less than that. So, the stress level in the reinforcing steel is within the recommended limit.

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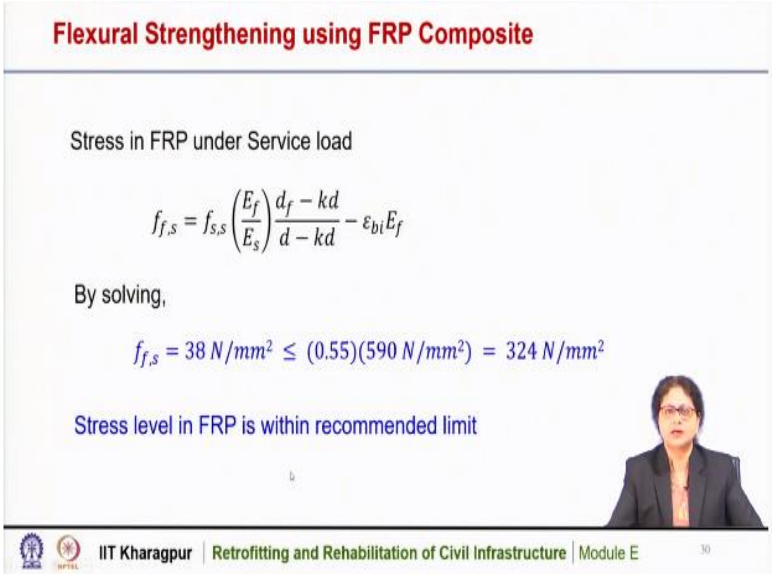

Now, we will check the stress in the FRP under service load condition and for that this is the relationship for estimating the stress in the FRP under service load condition. So, by putting the values we can get the value of stress in FRP that is $f_{f,s}$ and that is coming out as 38 Newton per millimeter square, which is less than 0.55 × 590 that is the limiting criteria which is equal to 324 Newton per mm square.

So, the stress level in the FRP is also within the recommended limit. So, from the serviceability condition also the stresses in the steel and in the FRP are within limits.

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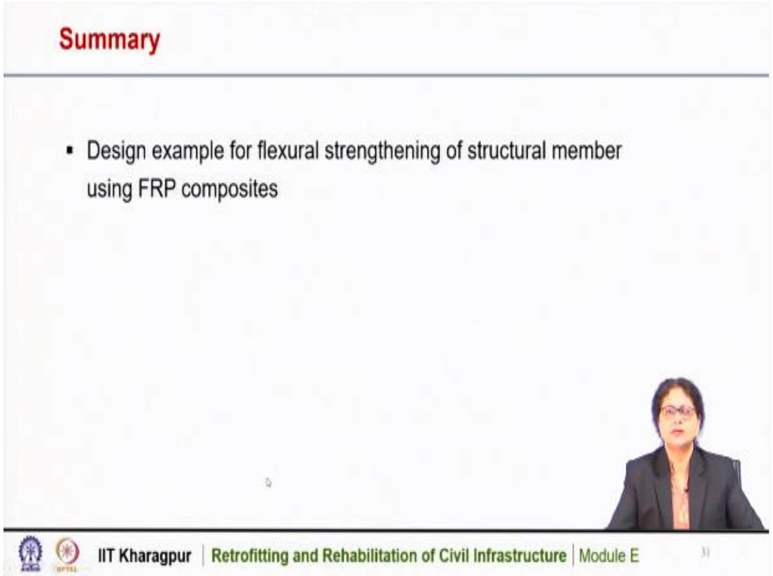

So, to summarize we have discussed design example for flexural strengthening of structural member using fiber reinforced polymer composite. We have discussed that a simply supported beam it is subjected to dead load and life load and if it is subjected to an increased loading by putting FRP we can improve its flexural capacity. So, this design example is for flexural strengthening of structural member using fiber reinforced polymer composites. Thank you.