Retrofitting and Rehabilitation of Civil Infrastructure
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Lecture 21
Micromechanics of Composites (Continued)

Hello friends, welcome to the NPTEL online certification course Retrofitting and Rehabilitation of Civil Infrastructure. Today we will discuss module D. The topic for module D is Fiber Reinforced Polymer Composites and its Characteristics.

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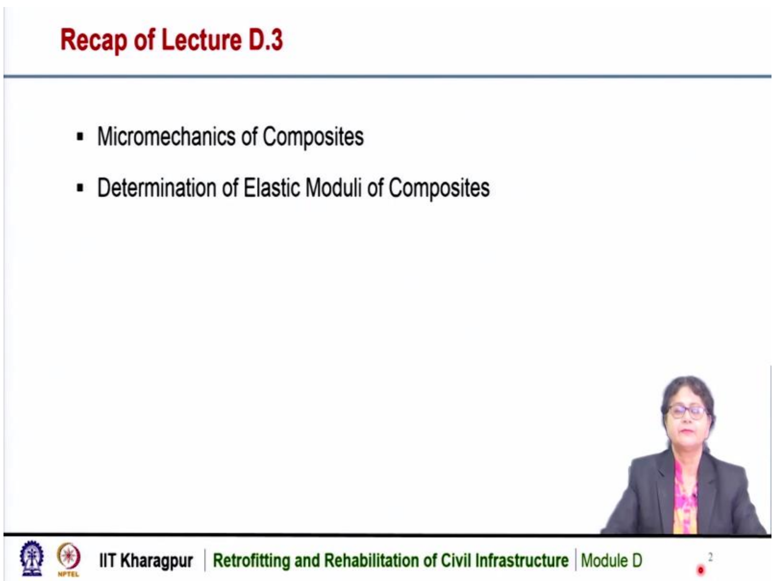

In the previous lecture, we have discussed Micromechanics of Composites. Composites have two major phases one is the off the composite at the fiber level. By knowing the properties of the fiber and the matrix within the composite and their relative proportions in terms of their volume and weight, we can determine the properties of the composite.

In the previous lecture, we have discussed the derivation of the elastic moduli of composites, by knowing the elastic properties of the fibers and the matrix and their relative proportions in terms of weight and volume within the composite.

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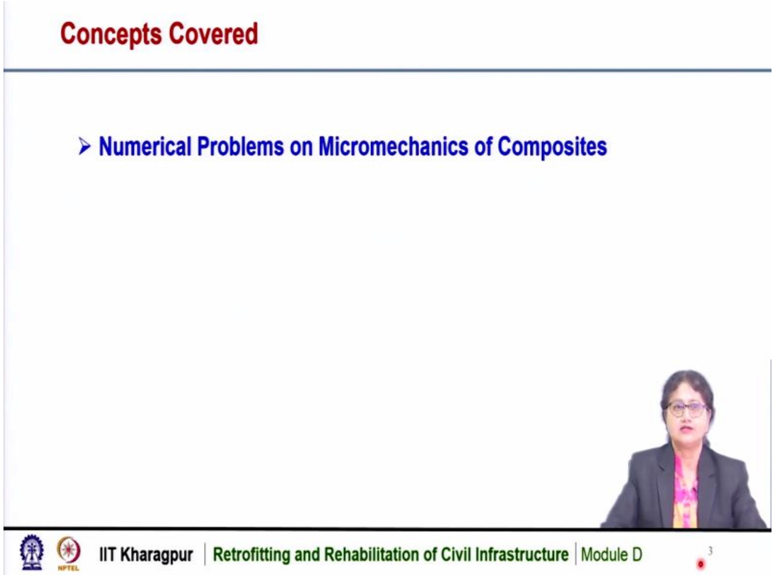

Today, we will discuss some of the numerical problems and their solutions on micromechanics of composite the concepts have been discussed in the previous lecture. And based on that, we will discuss some numerical problems and their solutions.

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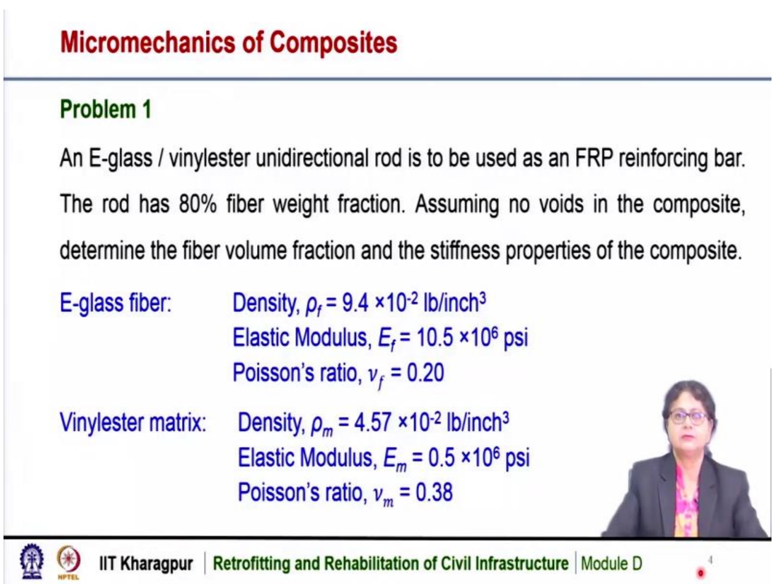

The problem 1 is that an E-glass vinylester unidirectional rod is to be used as an FRP reinforcing bar. The rod has 80 percent fiber weight fraction assuming no voids in the composite, determine the fiber volume fraction and the stiffness properties of the composite, the properties of the fiber and the matrix are given the fiber is the E-glass fiber having density ρ_f is equal to 9.4×10^{-2} pounds per inch cube, the elastic modulus is also given 10.5×10^{6} psi, pounds per square inch.

And the Poisson's ratio is 0.2. Similarly, for the vinyl for the vinylester matrix, the density elastic modulus and Poisson's ratios are given. In the problem the suffix f is used for fiber, suffix m is use is used for matrix and c is used for the composite.

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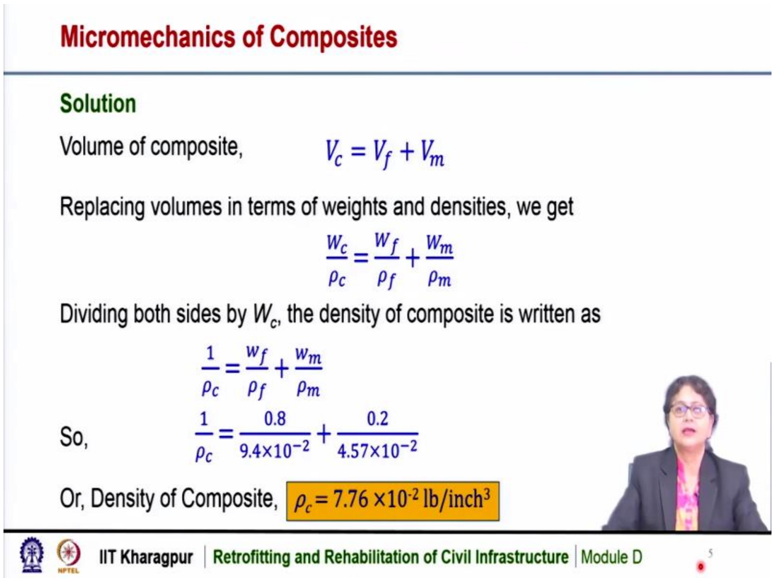

So, the solution is that the volume of composite can be written as the volume of the fiber plus the volume of the matrix that is $V_c = V_f + V_m$ replacing the volumes in terms of their weights and densities, we get the relationship as $W_c/\rho_c = W_f/\rho_f + W_m/\rho_m$. Now, dividing both sides by W_c that is the weight of the composite, the density of composite is written as $1/\rho_c = w_f/\rho_f + w_m/\rho_m$.

Now, W_f / W_c that is the weight of fiber by the weight of composite is nothing but the weight fraction of the fiber. Similarly, the weight of matrix by the weight of composite is the weight fraction of the matrix so, that is w_m.

Now, by putting these values in the equation we get the density of the composite as 7.76×10^{-2} pound per cubic inches. So, we can get the density of the composite by knowing the density of the fiber and the density of the matrix and they are weight fractions.

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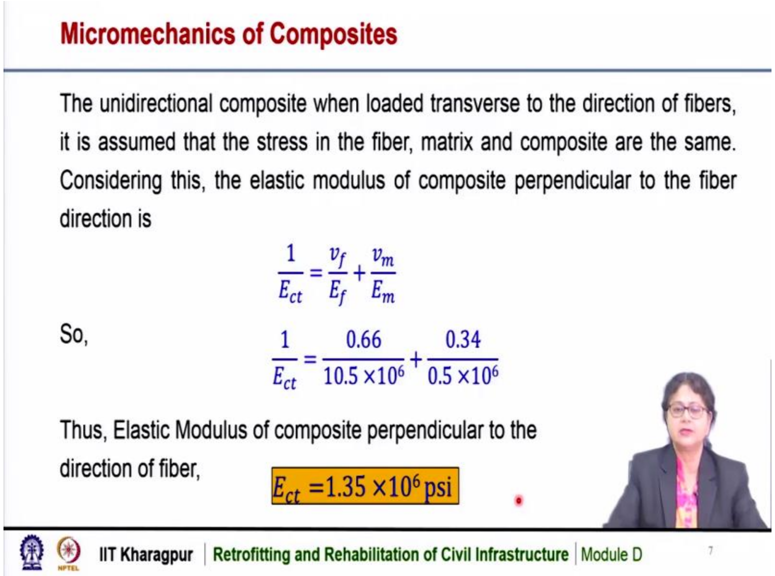

Now, using this relationship, we can also find the fiber volume fraction. The fiber volume fraction is given by v_f is equal to $w_f \times \rho_c / \rho_f$ that we have derived in the previous lecture.

So, we get the fiber volume fraction v_f is equal to 0.66. So, this is the fiber volume fraction in the composite that is coming out as 0.66, since, there is no void so, the other part is the matrix. So, the matrix volume fraction is $1\text{-}v_f$ which is 0.34.

Now, we need to determine the properties of the composite, the unidirectional composite when it is loaded along the direction of the fibers, it is assumed that the strain in the fiber matrix and the composite are the same considering this the elastic modulus of the composite along the fiber direction is E_{cl}, that is the elastic modulus of the composite along the fiber direction is equal to E_f into the volume fraction of fiber plus E_m into the volume fraction of the matrix this we have derived in the previous lecture.

So, by putting the values of the elastic modulus of fiber and the matrix and their volume fractions that we have obtained here, we get E_{cl} value is equal to 7.1×10^6 psi. So, this is the elastic modulus of the composite along the fiber direction from the rule of mixture we can get the equation E_{cl} and the value is coming as 7.1×10^6 psi.

Now, we need to determine the elastic modulus of the composite transverse to the direction of the fiber, direction of the fibers it is assumed that the stress in the fiber, the stress in the matrix and the stress in the composite are the same. Considering this the elastic modulus of composites perpendicular to the fiber direction is given by this equation this also we have derived in the previous lecture 1/ E_{ct} that is the elastic modulus of the composite perpendicular to the direction of the fibers is equal to $v_f/E_f + v_m/E_m$.

Now, by putting the values of E_f, E_m and the volume fractions of fibers and matrix, we get the elastic modulus of the composite perpendicular to the direction of the fiber as 1.35×10^6 psi. So, we get the elastic modulus of the composite along the direction of the fiber as well as perpendicular to the direction of the fiber by knowing the properties of the fiber and matrix and their fractions relative weights and volume fractions.

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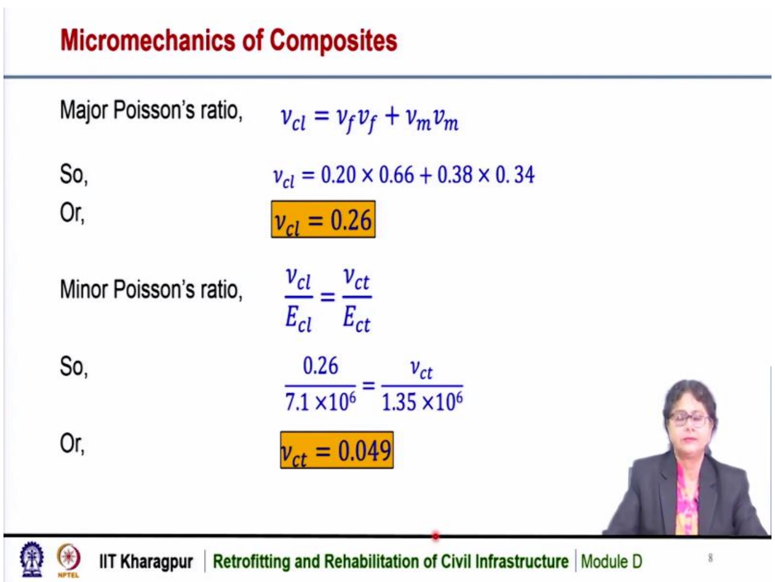

Now, we will determine the Poisson's ratio, it has two Poisson's ratio. Major Poisson's ratio is given by this equation $\nu_{cl} = v_f \nu_f + v_m \nu_m$ that is the individual Poisson's ratio of fiber and matrix. So, those are given and the volume fractions have already been determined. So, we can get the major Poisson's ratio of the composite ν_{cl} is equal to 0.26.

Similarly, the minor Poisson's ratio of the composite is given by this equation and we have explained this in the previous lecture that is $\nu_{cl}/E_{cl} = \nu_{ct}/E_{ct}$. Putting the values of the elastic modulus of the composite and the major Poisson's ratio what has been obtained here, we get the minor Poisson's ratio as 0.049.

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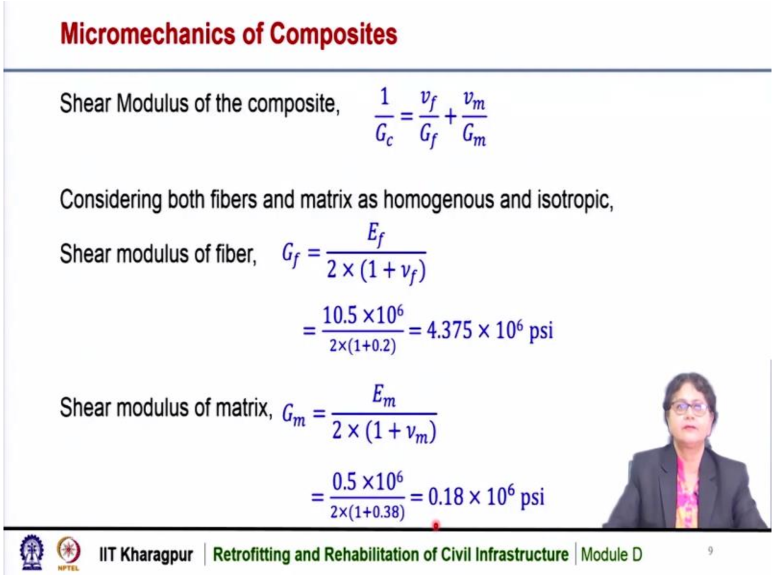

Now, we will determine the shear modulus of the composite the shear modulus is expressed by this equation $1/G_c = v_f/G_f + v_m/G_m$ that is the shear modulus of the individual fiber and the matrix and their volume fractions. Considering both fibers and matrix as homogeneous and isotropic, we can find out the shear modulus of fiber and the matrix using this equation.

The composite is not homogeneous and isotropic, but individual fibers and matrix can be assumed as homogeneous and isotropic. So, we can find out the shear modulus of fiber by knowing the elastic modulus and the Poisson's ratio. So, G_f can be found out with this equation $G_f = E_f/[2\times(1+\nu_f)]$ and this is equal to 4.375×10^6 psi.

Similarly, the shear modulus of matrix is estimated as $G_m = E_m/[2\times(1+\nu_m)]$. And what is the shear mod the matrix as 0.18×10^6 psi. Now, we will put the shear modulus of fibers and matrix into this equation to determine the shear modulus of the composite.

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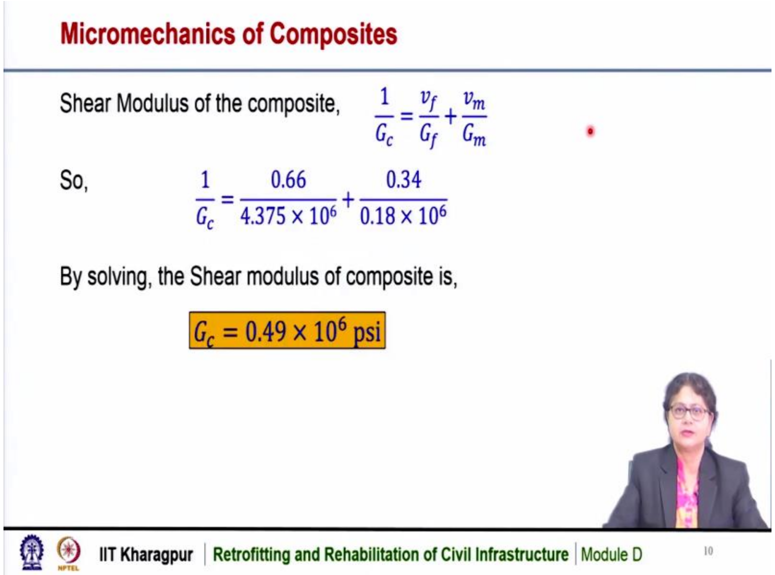

So, by putting these values and the falling fractions of fibers and matrix, we get the shear modulus of the composite is 0.49×10^6 psi. So, we can get all the elastic modulus of the composite. The elastic modulus along the direction of the fiber and transfers to the direction of the fiber, the shear modulus and the Poisson's ratios values, we can obtain from the elastic properties of the fiber and matrix and their volume and weight fractions.

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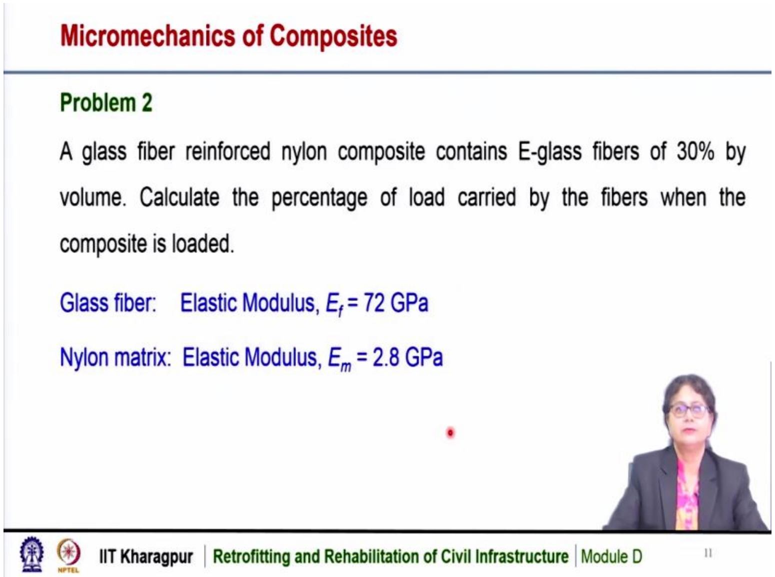

Now, we will have another problem. Problem 2 says that a glass fiber reinforced nylon composite contains E glass fibers of 30 percent by volume, calculate the percentage of load carried by the fibers when the composite is loaded? The properties of the glass fiber and the nylon matrix are given.

The elastic modulus of glass fiber is given us 72 GPa and the elastic modulus of the nylon matrix is given us 2.8 GPa. Their volume fractions are given, the volume fraction is 30 percent for the fiber, rest is for the matrix. And here also it is assumed that there is no void in the composite.

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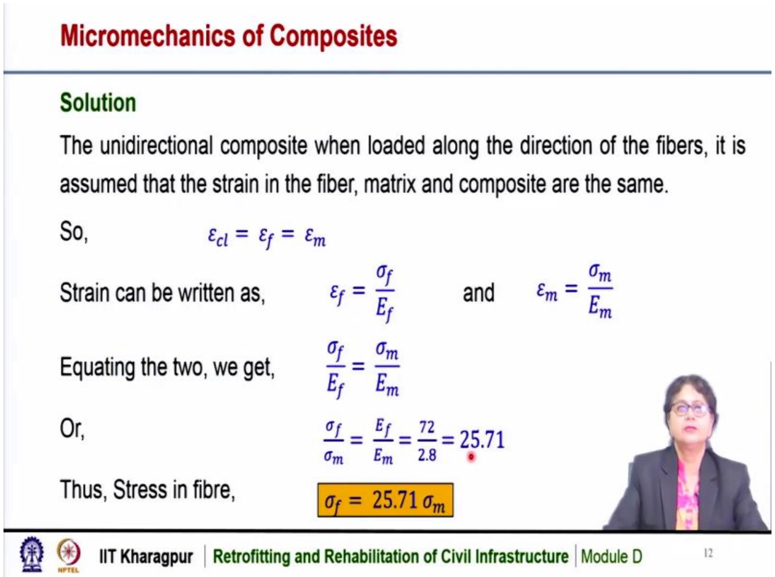

So, the solution is a unidirectional composite when loaded along the direction of the fibers, it is assumed that the strain in the fiber matrix and composite are the same that we have already discussed in the previous lecture. So, we can write that the $\varepsilon_{cl} = \varepsilon_f = \varepsilon_m$, ε is the strain and cl is for composite, f is for fiber and m is for the matrix. So, the strain can be written in terms of stress and elastic modulus that is $\varepsilon_f = \sigma_f / E_f$.

Similarly, for the matrix ε_m since the strains are same, so, we can equate and get this equation $\sigma_f/E_f = \sigma_m/E_m$ and thus σ_f / σ_m that is the stress in fiber divided by the stress in the matrix is equal to the ratio of their elastic modulus, so is equal to E_f / E_m and these values are given. So, we get the ratio as 25.71. So, σ_f that is the stress in the fiber can be written as $25.71 \times \sigma_m$.

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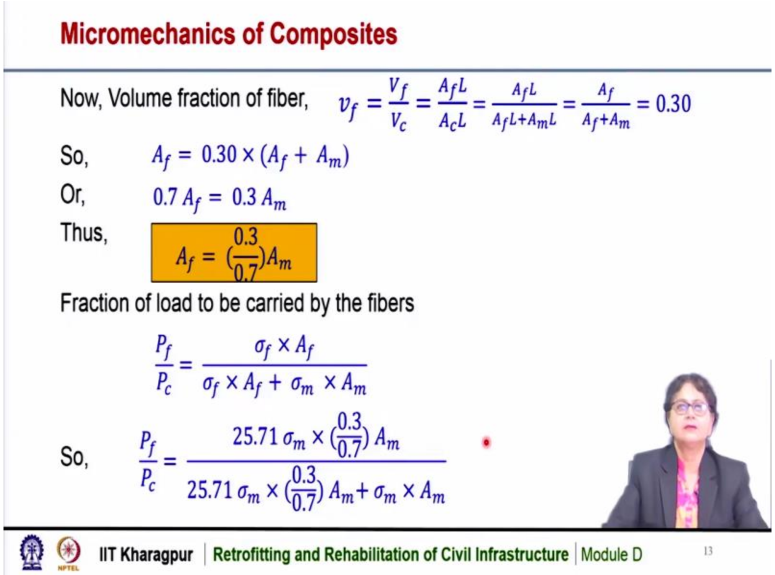

Now, the volume fraction of fiber can be written as this is the volume fraction v_f is equal to the volume of fiber divided by the volume of composite. So, this can also be written as, volume can be written as A_f into L is area into the length, cross sectional area of the fiber into the length and, for the composite is A_c into L, the length is the same for the fibers and the composite.

So, this can be written as $v_f = V_f/V_c = A_f L/A_c L = A_f L/(A_f L+A_m L) = A_f/(A_f+A_m) = 0.30$.

So, we can write $A_f = 0.30\times(A_f+A_m)$. So, $0.7A_f = 0.3A_m$, $A_f = (0.3/0.7)\ A_m$. Now, we need to determine the fraction of load to be carried by the fibers. So, P_f / P_c, we have to find out this ratio. So, P_f is the load carried by the fibers and P_c is the load current by the composite.

So, P_f that is the load current by the fiber can be written as the stress into the cross-sectional area. Similarly, for the composite, it is the stress in the fiber into the cross-sectional area of fiber plus the stress in the matrix into the cross-sectional area of the matrix. So, $P_f/P_c = (\sigma_f\times A_f)/(\sigma_f\times A_f+\sigma_m\times A_m) = 0.9168 = 91.68\%$.

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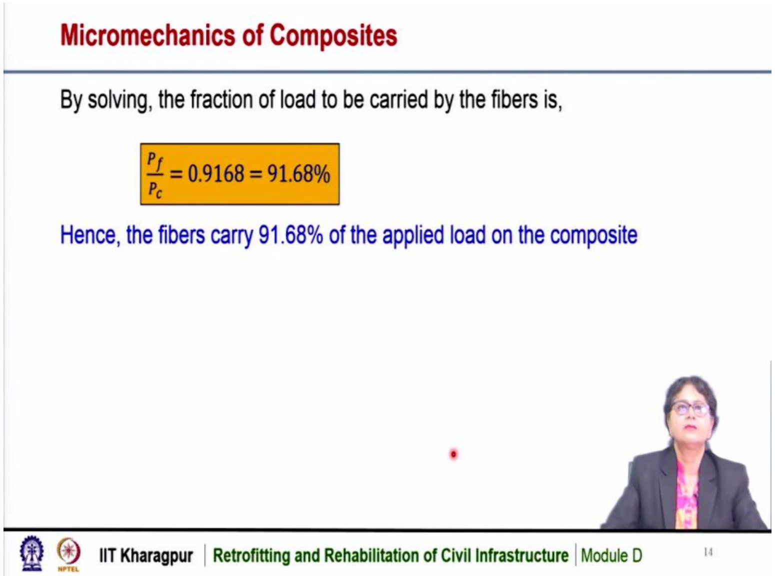

Now, by solving this and eliminating sigma m and Am from the numerator and the denominator, we can get that P_f / P_c is equal to 0.9168 that is 91.68 percent. So, the fibers carry 91.68 percent of the applied load on the composite. So, by knowing the volume fraction of the fiber and the matrix and their individual properties, we can find out that the percentage of load shared by the fibers.

So, we can see here that fibers carry a significant amount of the total load on the composite. So, for this case it is 91.68 percent of the total load is taken by the fiber. The fiber volume fraction is only 30 percent, but, it has significantly higher strength as compared to the matrix. So, the majority of the load is shared by the fibers.

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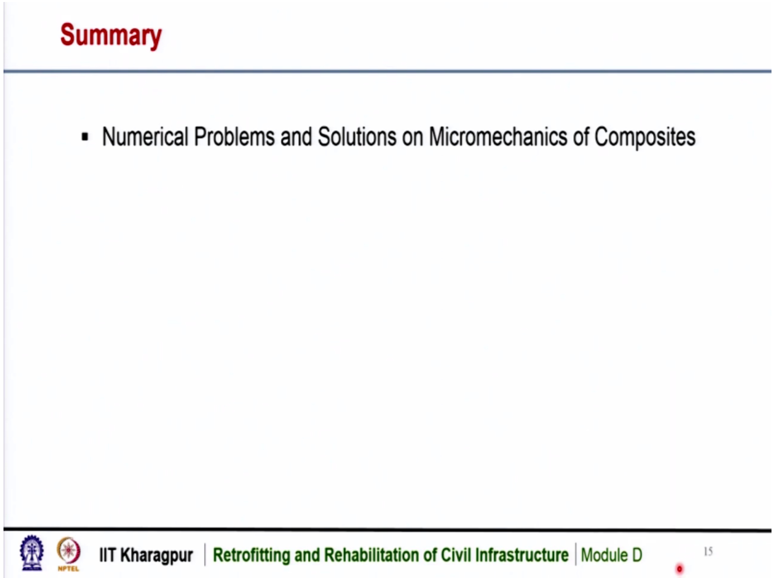

So, to summarize, we have discussed few numerical problems and their solutions on micromechanics of composite by knowing the properties of the fibers and the matrix and their relative proportions in terms of volume and weight, we can find out the elastic modulus of composites and that has been discussed using numerical problems. Thank you.