**Rock Mechanics and Tunneling**
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**Lecture 54**
**Methods to determine stresses around openings: Kirsch equation (contd.) and Greenspan's method**

Hello everyone, I welcome all of you to the third lecture of module eleven. So, in module 11 we are discussing about the analysis of stresses in tunneling. In lecture three we will discuss about the methods to determine stresses around opening, so basically we will continue our discussion on the Kirsch equation, also we will discuss about few other things like the Greenspan's method.

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So, we will take a problem make our concepts clear. I will also very briefly discuss about the Solution of Terzaghi and Richart and the Greenspan's method. These two are quite old methods.

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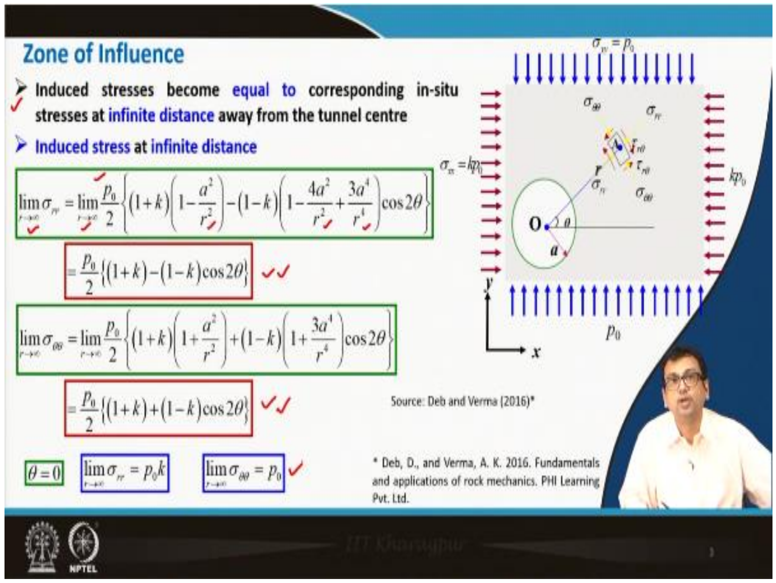


Now, just before starting the problem let us revise the last topic. So, in our last lecture we discussed about the zone of influence. I stated that the induced stresses become equal to corresponding in-situ stresses at infinite distance away from the tunnel center, which means at about an infinite distance, the effect of tunnel will not be there. That is why at that location it will be nothing but the in-situ stress.

The induce stress at infinite distance is given by

$$\lim_{r\to\infty}\sigma_{rr} = \lim_{r\to\infty}\frac{p_o}{2}\left\{(1+k)\left(1-\frac{a^2}{r^2}\right)-(1-k)\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$= \frac{p_o}{2}\left\{(1+k)-(1-k)\cos 2\theta\right\}$$

$$\lim_{r\to\infty}\sigma_{\theta\theta} = \lim_{r\to\infty}\frac{p_o}{2}\left\{(1+k)\left(1+\frac{a^2}{r^2}\right)+(1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$= \frac{p_o}{2}\left\{(1+k)+(1-k)\cos 2\theta\right\}$$

This is nothing but the well-known Kirsch equation.

Now for $\theta$=0

$$\lim_{r \to \infty} \sigma_{rr} = p_o k \qquad \lim_{r \to \infty} \sigma_{\theta\theta} = p_o$$

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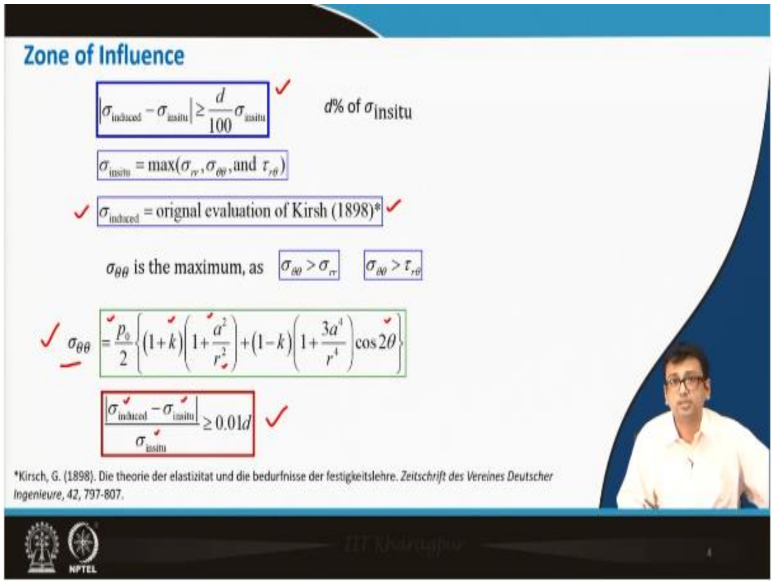
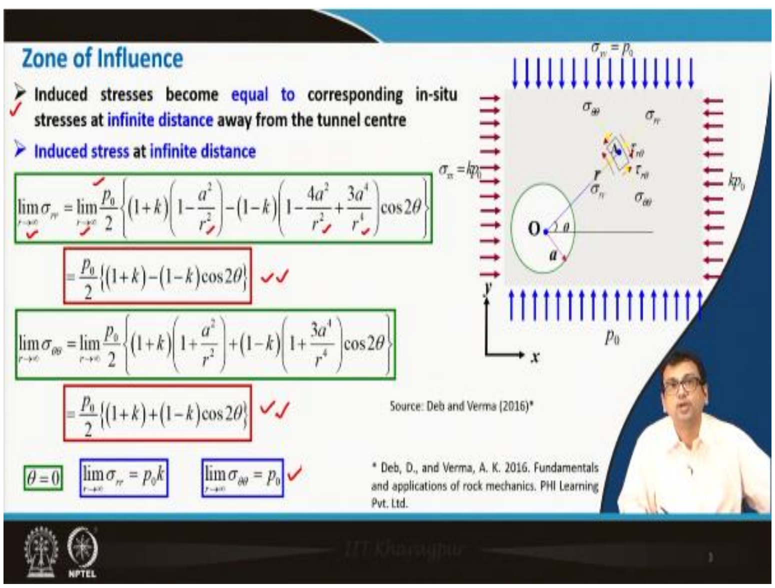



Topic zone of influence is presented using equation,

$$|\sigma_{induced} - \sigma_{insitu}| \geq \frac{d}{100}\sigma_{insitu} \qquad d\% \text{ of } \sigma_{\text{insitu}}$$

So we are interested in finding out that at a certain distance how much percentage of this sigma in-situ is present.

Now, it may happen that may be your tunnel is at central location where may be you can allow suppose 5 % of the in-situ stress so then accordingly you have to find out at what distance you should place your tunnel. We can get $\sigma_{\text{insitu}}$ as follows:

$$\sigma_{\text{insitu}} = \max(\sigma_{rr}, \sigma_{\theta\theta}, \text{ and } \tau_{r\theta})$$

$\sigma_{\text{induced}} =$ original evaluation of Kirsch (1989)*

$\sigma_{\theta\theta}$ is maximum as $\sigma_{\theta\theta} > \sigma_{rr} \quad \sigma_{\theta\theta} > \tau_{r\theta}$

So for $\sigma_{\text{induced}}$ we use equation

$$\lim_{r\to\infty} \sigma_{\theta\theta} = \lim_{r\to\infty} \frac{p_o}{2}\left\{(1+k)\left(1+\frac{a^2}{r^2}\right)+(1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

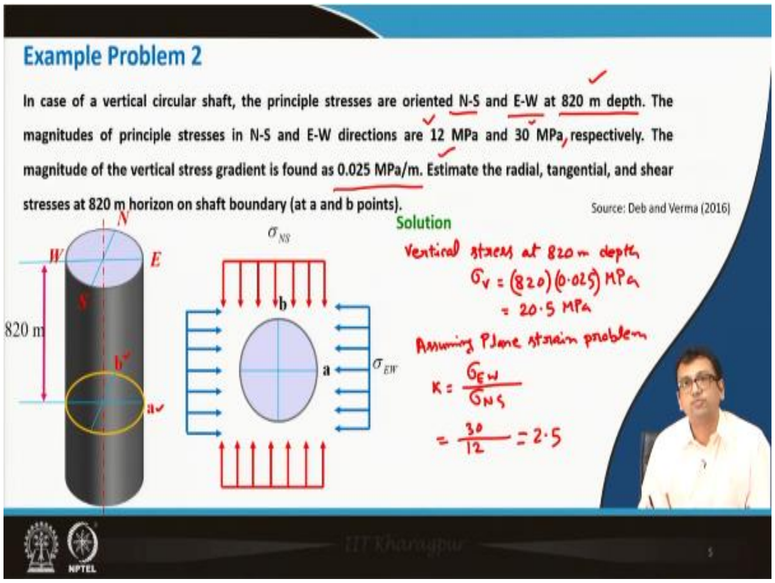
And for $\sigma_{\text{insitu}}$

$$\sigma_{\text{insitu}} = \frac{p_o}{2}\left\{(1+k)+(1-k)\cos 2\theta\right\}$$

Now putting above values in the equation below, we can get the solution

$$\frac{\left|\sigma_{induced} - \sigma_{insitu}\right|}{\sigma_{insitu}} \geq 0.01d$$

So for given values of $p_o$, a, k or r value, we have to find out what percentage of in-situ stress is allowed or vice versa. (Refer Slide Time: 8:20)




Now, we will take problem 2 in this module,

**Q2.** In case of a vertical circular shaft the principle stresses are oriented like N-S and E-W at 820 meter depth. The magnitudes of principal stresses in N-S and E-W are 12 MPa and 30 MPa respectively. The magnitude of vertical stress gradient is found as 0.025 MPa/m.Estimate the radial, tangential and shear stresses at 820 meter horizon on shaft boundary (at a and b points).

**Solution:**

Vertical stress at 820 m depth, $\sigma_v = 820(0.025)$ MPa=20.5 MPa

Assuming plane strain problem, $k = \dfrac{\sigma_{EW}}{\sigma_{NS}} = \dfrac{30}{12} = 2.5$

$$\sigma_{rr} = \frac{p_o}{2}\left\{(1+k)\left(1-\frac{a^2}{r^2}\right)-(1-k)\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\sigma_{\theta\theta} = \frac{p_o}{2}\left\{(1+k)\left(1+\frac{a^2}{r^2}\right)+(1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\tau_{r\theta} = \frac{p_o}{2}\left\{(1-k)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right\}$$

For boundary, $r = a$

$$\sigma_{rr} = 0$$

$$\sigma_{\theta\theta} = p_o\left\{(1+k)+2(1-k)\cos 2\theta\right\}$$

$$\tau_{r\theta} = 0$$

For point a, $\theta = 0, p_o = 12$ MPa, $k = 2.5$

$$\sigma_{\theta\theta}^{a} = p_o\left(1+k+2-2k\right) = p_o(3-k) = 12(3-2.5) = 6 \text{ MPa}$$

For point b, $\theta = 90, p_o = 12$ MPa, $k = 2.5$

$$\sigma_{\theta\theta}^{a} = p_o(3k-1) = 12(3\times 2.5-1) = 78 \text{ MPa}$$

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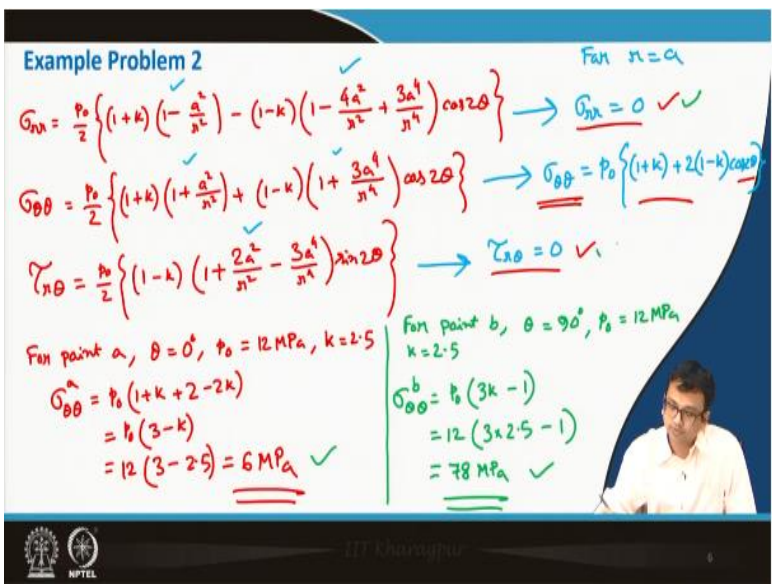

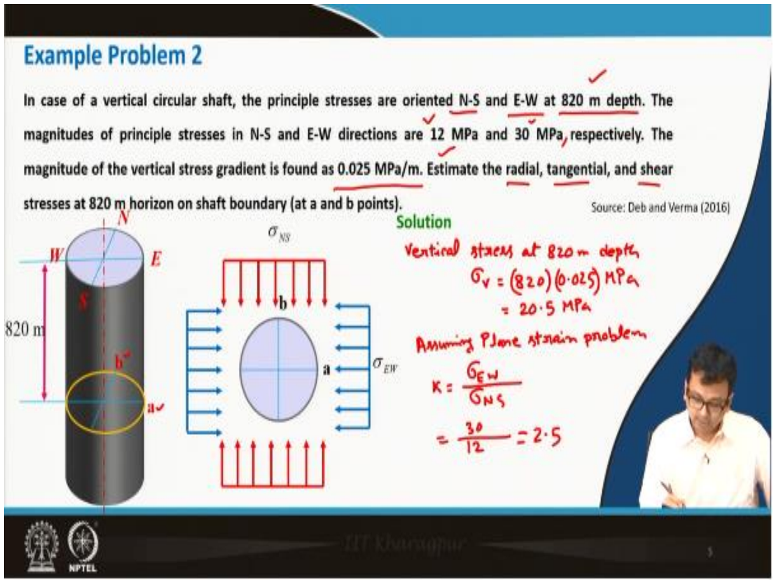


So, these are the tangential and radial stresses are at a and b both are equal to 0 and shear stresses at a and b both are equal to 0 and if you consider vertical stress which we have obtained here that is though it is not asked but just we have obtained it is 20.5 mega Pascal, so in this way we can design the vertical shafts also using this Kirsch situation.

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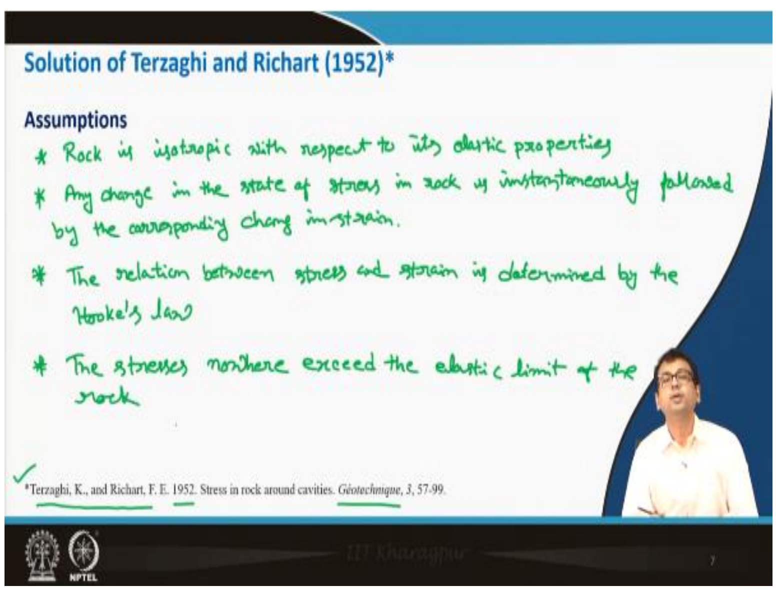

Now, we will briefly discuss about another theory or work by Terzaghi, Karl Terzaghi who is considered as the father of soil mechanics, has also worked in the area of rock mechanics. So, this paper Terzaghi and Richart published in geotechnique in the year 1952.

So, one can read this paper for better understanding. In this paper of Terzaghi and Richart they have further explored the Kirsch equations provided some arguments for stress analysis surrounding the opening.

So, let me first mention the assumptions of their consideration of this work,

1. Rock mass is isotropic with respect to its elastic properties.
2. Any change in the the state of stress in the rock is instantaneously followed by the corresponding change in the strain.
3. The relation between stress and strain is determined by Hooke's law.
4. The stresses nowhere exceeds the limit of the rock. These are the some of the assumptions.

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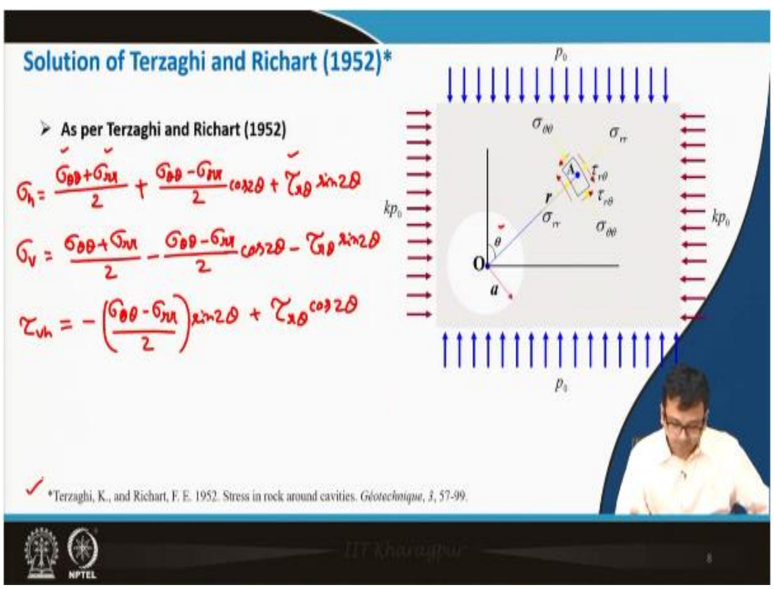

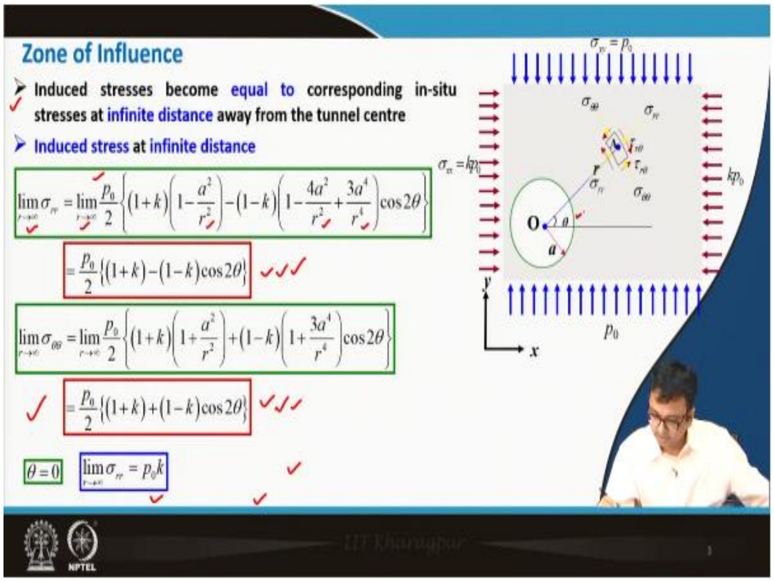


Now, based on these, Terzaghi has given some of the equations,

$$\sigma_h = \frac{\sigma_{\theta\theta} + \sigma_{rr}}{2} + \frac{\sigma_{\theta\theta} - \sigma_{rr}}{2}\cos 2\theta + \tau_{r\theta}\sin 2\theta$$

$$\sigma_v = \frac{\sigma_{\theta\theta} + \sigma_{rr}}{2} - \frac{\sigma_{\theta\theta} - \sigma_{rr}}{2}\cos 2 - \tau_{r\theta}\sin 2\theta$$

$$\tau_{vh} = -\left(\frac{\sigma_{\theta\theta} - \sigma_{rr}}{2}\right)\sin 2 + \tau_{r\theta}\cos 2\theta$$

So, basically these equations were obtained by the stress transformation and the tangential stress, radial stress and the shear stress obtained in Kirsch equation remains same.

So, these were minor modification you can notice. Few other more important things were also there for which you can refer this paper.

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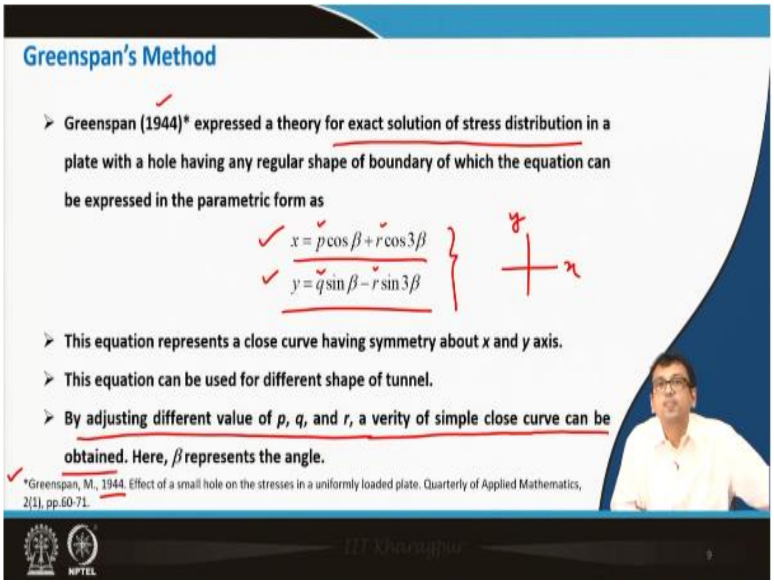


Now, another topic is Greenspan's method, details can be obtained in this paper, Greenspan's M 1944.

This is an excellent and important method for solving analytically a problem related to tunnel design. Greenspan's method is very important and when in the 1944 there was no computer, super computers available we had to remember things earlier; these methods are strongly based on mathematics.

Greenspan (1944)* expressed theory for exact solution, this is the most important exact solution of stress distribution in a plate with a hole having any regular shape of boundary of which the equation can be expressed in the parametric form as

$$x = p\cos\beta + r\cos 3\beta$$

$$y = q\sin\beta - r\sin 3\beta$$

This equation represents a close curve having symmetry about x and y axis. This equation can be used for different shape of tunnel. By adjusting different values of *p*, *q* and *r*, a variety of simple close curve can be obtained. Here $\beta$ represents the angle.

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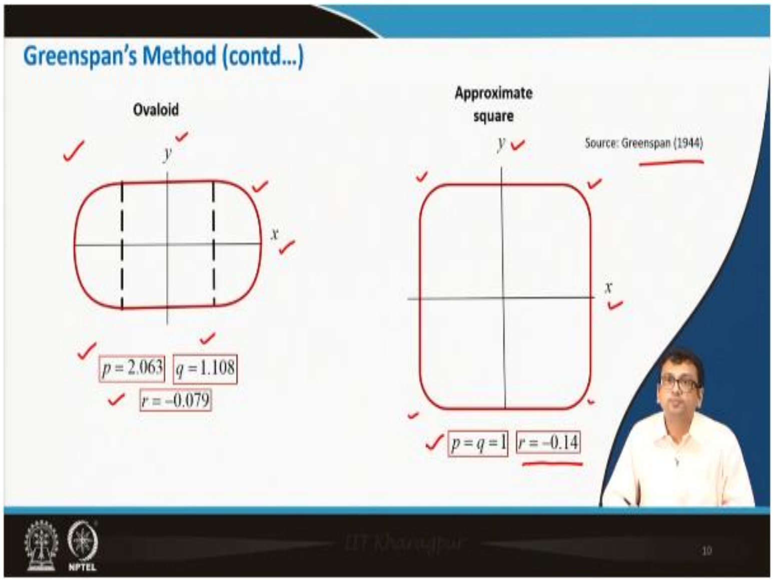


Now, for example take an ovaloid in the figure. If *p*=2.063, *q*=1.108 and *r*=-0.079 then we will get an ovaloid of this shape and then using this p, q and r value, expressions for stresses have been provided and you have to utilize these values to actually ultimately get the exact stress distribution in around and over opening of the shape.

Similarly, just to record an approximate square with curvature where β will come into picture. For this type of shape p=q=1 and r=- 0.14. Other expression for stresses are given by Greenspan so there we can give these values as input and obtain the stress distribution around an approximate square shaped hole.

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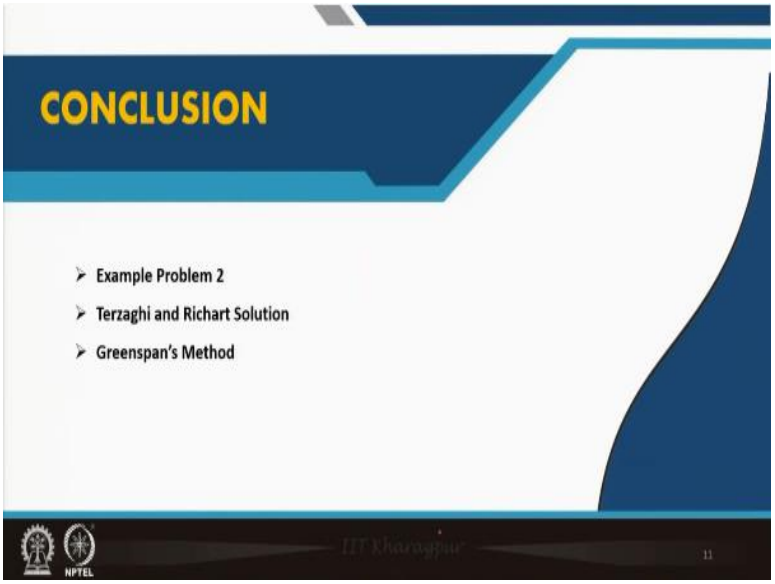


So, yes, I am not going into further details of this Greenspan's method but if you wish you can read that paper, at least you could able to learn about some method which came in the year 1944 and which is actually strongly based on mathematics and also gives the exact solution, thank you so with this I will conclude my today's lecture.