Rock Mechanics and Tunneling
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Lecture 53
Methods to determine stresses around openings: Kirsch equation (contd.)

Hello everyone. I welcome all of you to the second lecture of module 11. We have started discussing about the analysis of stresses in tunneling which is very much important from the design perspective, or design point of view. So, today in our second lecture, we will continue our discussion related to the Kirsch equation. We are mainly focusing methods to determine stresses around openings. Therefore, by using Kirsch equation, we will solve one problem first.

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We will also discuss about the zone of influence which is very much important for constructing or designing tunnel. If there are some pre-existing structures nearby or you may have to construct any structure nearby with respect to existing tunnel, then this knowledge of zone of influence can be quite useful.

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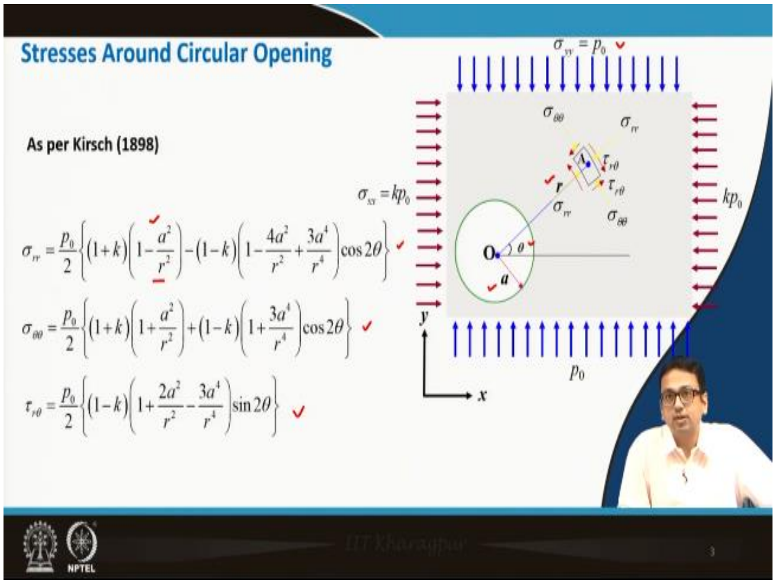

So, this diagram is nothing new, you already have seen in our last lecture and as per Kirsch (1898), the radial stress, tangential stress and shears stress can be obtained by using this equations.

Here, r is the radial distance of the point A from the center of the tunnel O; *a* is the radius of the tunnel and another parameter, θ is the angle of the point A with respect to horizontal. Other than that, P_0 is overburdened stress, so σ_{xx}, σ_{yy}, and k are discussed earlier. So, with this knowledge, let us first try to understand how to utilize these equations very clearly.

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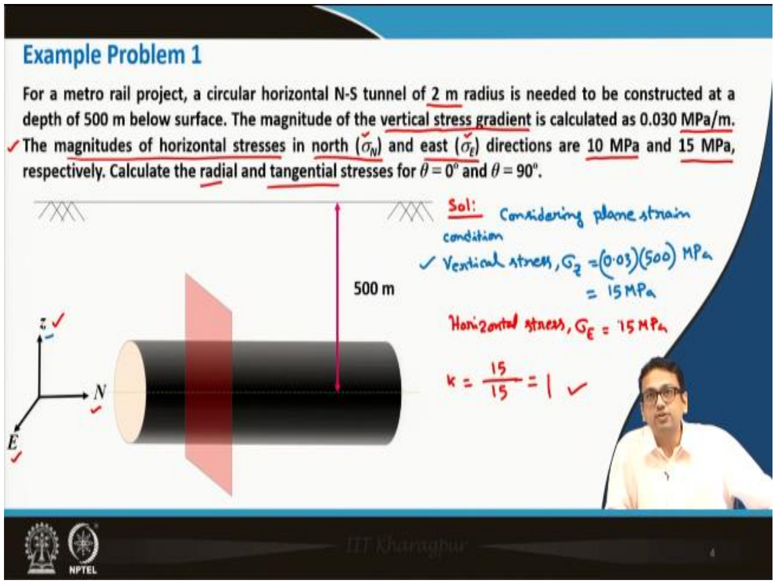

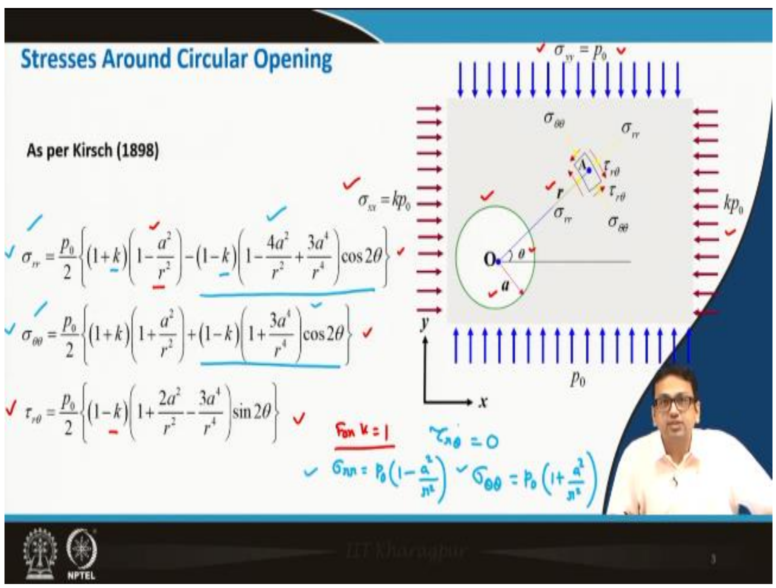

So, let us read it.

For a metro tunnel project, a circular horizontal north south tunnel of 2 meters radius is needed to be constructed at a depth of 500 meters below surface, as shown in picture. The magnitude of the vertical stress gradient is calculated as 0.03 MPa/m. The magnitude of horizontal stresses in σ_N and east σ_E directions are 10 MPa and 15 MPa respectively. So, by using the question, radial and tangential stresses for theta is equal 0º and θ is equal to 90º, so calculate the radial and tangential stresses.

So, by considering plain strain condition, the vertical stress will be

$$\sigma_z = 0.03 \times 500 \text{ MPa}$$

$$= 15 \text{ MPa}$$

Now, horizontal stress will be

$$\sigma_E = 15 \text{ MPa}$$

Now, we can very easily obtain the magnitude of *k*. So, *k* is the ratio between the horizontal stress by vertical stress, so horizontal stress is 15 MPa and vertical stress is 15 MPa. So,

$$k = \frac{\sigma_E}{\sigma_z} = \frac{15}{15}$$

= 1

Now, as per Kirsch (1898),

$$\sigma_{rr} = \frac{p_0}{2}\left\{(1+k)\left(1-\frac{a^2}{r^2}\right)-(1-k)\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\sigma_{\theta\theta} = \frac{p_0}{2}\left\{(1+k)\left(1+\frac{a^2}{r^2}\right)+(1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\tau_{r\theta} = \frac{p_0}{2}\left\{(1-k)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right\}$$

By replacing *k* =1 in those previous equation,

$$\sigma_{rr} = p_0\left(1-\frac{a^2}{r^2}\right)$$

$$\sigma_{\theta\theta} = p_0\left(1+\frac{a^2}{r^2}\right)$$

$$\tau_{r\theta} = 0$$

Hence, k has become 1. On the other hand, if suppose k value is not equal to 1, and if suppose you get it at suppose 0.7, then instead of these two special equations you have to utilize the previous complete equations.

So, we have to calculate the radial and tangential stresses for θ is equal to 0º and θ is equal to 90º. Now, let us further proceed.

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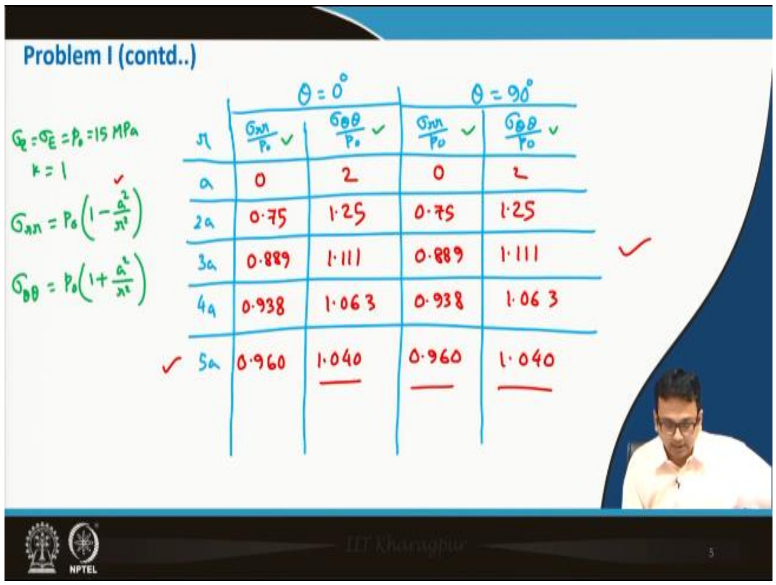

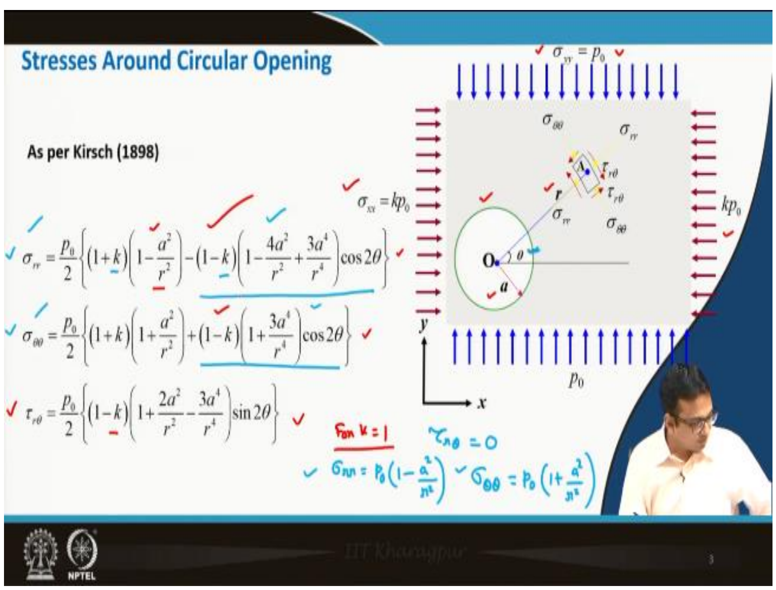

Now, we have to create a table.

r	$\theta = 0°$		$\theta = 90°$	
	σ_{rr}/p	$\sigma_{\theta\theta}/p$	σ_{rr}/p	$\sigma_{\theta\theta}/p$
1a	0	2	0	2
2a	0.75	1.25	0.75	1.25
3a	0.889	1.111	0.889	1.111
4a	0.938	1.063	0.938	1.063
5a	0.960	1.040	0.960	1.040

At a radial distance of 5a, the radial stress will be actually the 96 percent of the in-situ stress and whereas the tangential stress is 4 percent, more than the in-situ stress. So, this is a nice observation. In case beyond 6a, 7a, the quantity will converge towards to 1 clearly that you can understand.

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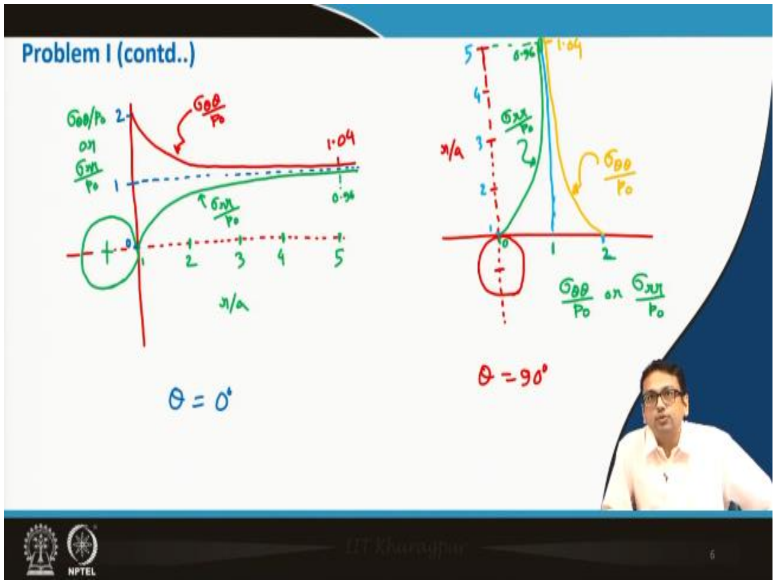

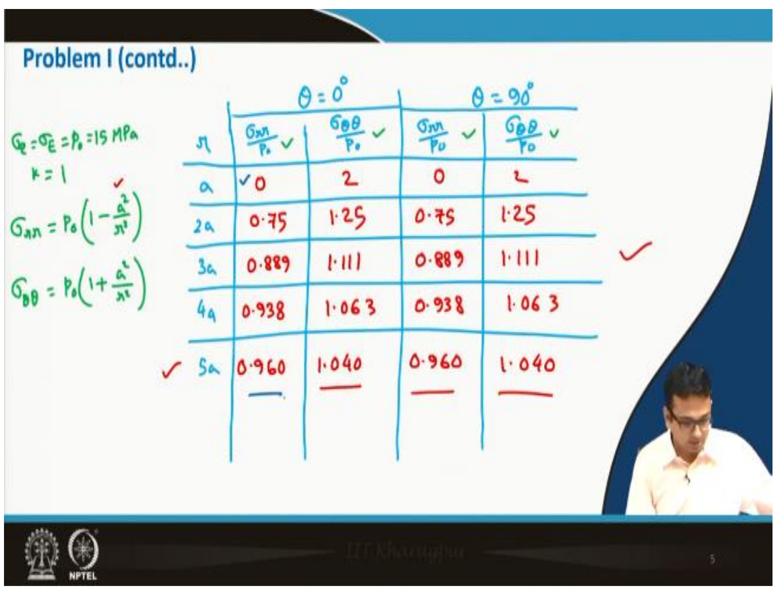

So, to clear the idea, we will showcase a simple plot actually, so that will clear your concept. How the stresses are distributed near opening?

One axis is considered to be $\sigma_{\theta\theta}/P_0$ that is also σ_{rr}/P_0. We will take it as 1, here suppose 2, so this one is suppose 0. Other axis is mentioned as *r/a*.

Now, θ is equal to 0ºcase. So, now we know that if we first start with the σ_{rr}/P_0, how the diagram will look like? We can very easily understand. It will start from 0 and gradually it will converge to 0.96, means close to 1. So, actually if you plot these value on a graph paper you will find that this is suppose let me mark this one also because this is $\sigma_{rr}/P_0 = 1$. The final figure is shown in the slide.

Now, σ_{rr}/P_0 magnitude is 0.96 at 5a. Similarly, if we draw it for the $\sigma_{\theta\theta}/P_0$, how will it look? It will be the mirror image only, which is corresponding to 5a is 1.04.

So, you can clearly understand beyond 5a, I mean five times the radius of the tunnel, the influence of the tunnel is quite less. Now this was for $\theta = 0^o$ case, now same thing we can just draw it for our other case that is $\theta = 90^o$. So, how it will look? So maybe let me same thing, this way we will do it, so here suppose the tunnel is like this and this axis is *r/a*, what will happen is suppose it is my 1, so it will be like 2, 3, 4 and 5 just approximately, presented with different color.

So, just we can again cross check, you see from 2 it will start and it will go towards 1 and it is happening like this, so this is very important in both $\theta = 0^o$ and $\theta = 90^o$ direction in this case about 5a, the effect will be very less 4 percent.

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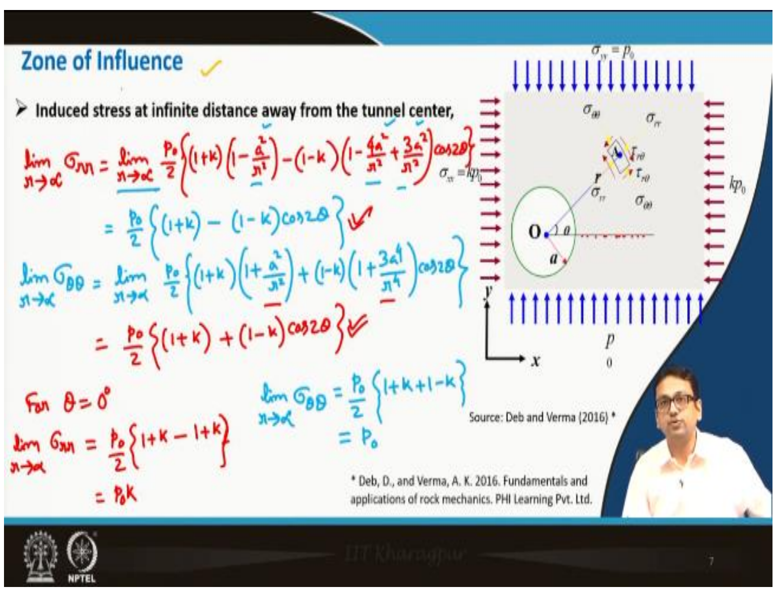

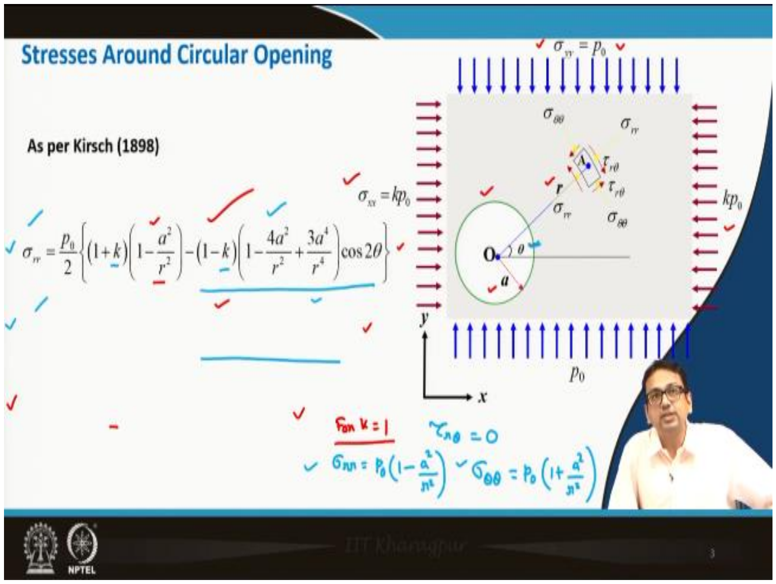

Now, with relation to this only one very important thing we should learn that is the zone of influence. When we construct any structure, suppose a tunnel, then we have to new tunnel then if we find a building is nearby present, so building foundation whether that is within the zone of influence of tunnel or not that we should check otherwise the structure may fail.

Now, or on the other hand a tunnel is already present there and we have to suppose construct a building over there or any other structure or some other utility pipeline you need to lay maybe, so in that case also you should check whether the tunnel is influencing that one or not and according to that you need to take the measures.

Anyway so now the idea is very simple, already you know the equations, the Kirsch equation is known to you, now what we will do is? So, we can write one thing that induced stress at infinite distance away from the center of the tunnel. The things what I am teaching you, these are mainly from the Deb and Verma's book.

So, how to induced stresses at infinite distance away from the center of the tunnel can be obtained? So, here if you consider infinite distance from your center of the tunnel, so how will you simulate, means mathematically how will you present that, you will just consider limit r tends to infinity, so then only we will be able to find out the expression for that induced stress at an infinite distance.

Induced stress at infinite distance away from the tunnel center,

$$\lim_{r\to\infty}\sigma_{rr} = \lim_{r\to\infty}\frac{p_0}{2}\left\{(1+k)\left(1-\frac{a^2}{r^2}\right)-(1-k)\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$=\frac{p_0}{2}\left\{(1+k)-(1-k)\cos 2\theta\right\}$$

$$\lim_{r\to\infty}\sigma_{\theta\theta} = \lim_{r\to\infty}\frac{p_0}{2}\left\{(1+k)\left(1+\frac{a^2}{r^2}\right)+(1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$=\frac{p_0}{2}\left\{(1+k)+(1-k)\cos 2\theta\right\}$$

For $\theta = 0^o$ case,

$$\lim_{r\to\infty}\sigma_{rr} = p_0 k$$

$$\lim_{r\to\infty}\sigma_{\theta\theta} = p_0$$

This one important equation where we are trying to find out the induced stress at infinite distance away from the tunnel center.

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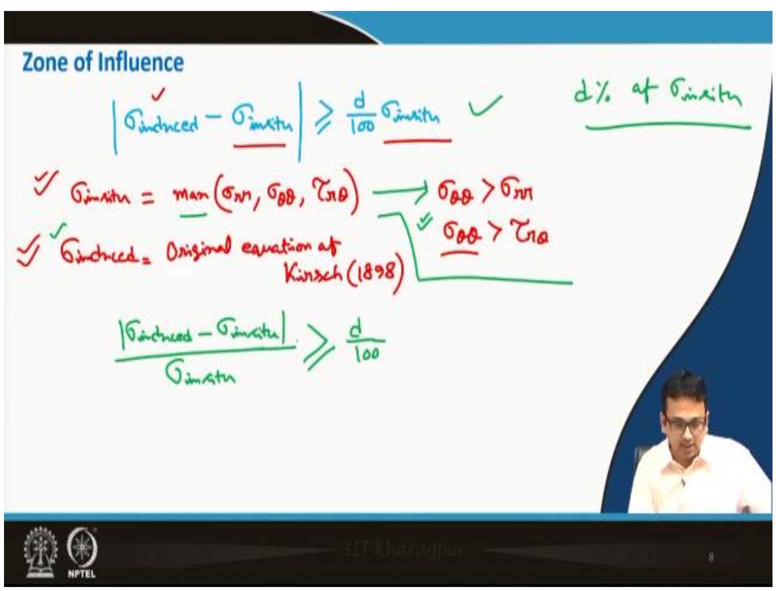

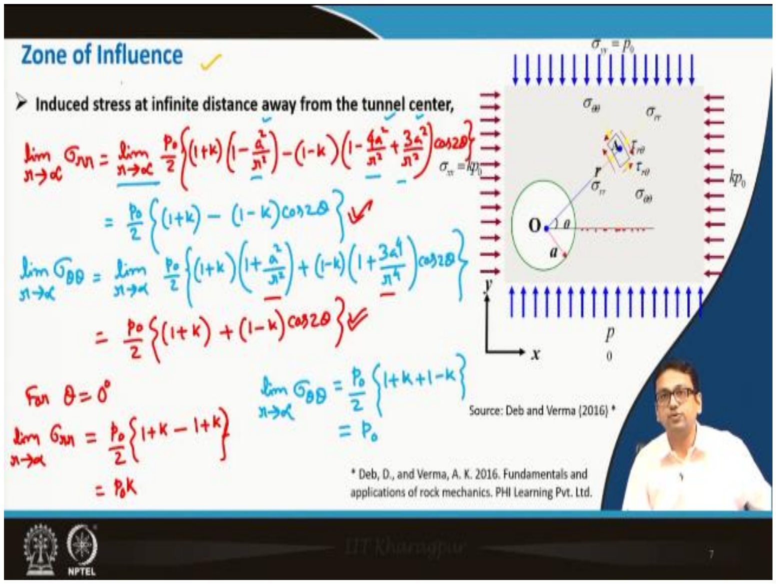

Now, our main objective is to get the zone of influence, up to which zone how much stress will induced that we are trying to find out. So, now we will further proceed with this idea only, you see we can write it as

$$\left|\sigma_{\text{induced}} - \sigma_{\text{insitu}}\right| \geq \frac{d}{100}\sigma_{\text{insitu}}$$

$$\sigma_{\text{insitu}} = \max(\sigma_{rr}, \sigma_{\theta\theta}, \text{and } \tau_{r\theta})$$

$$\sigma_{\text{induced}} = \text{orignal evaluation of Kirsh (1898)}$$

So, any stress there or not that we are trying to find out, so we have to obviously take consider instead of taking all the things we can just do it for the maximum amount of it because we are interested in to see whether any of this radial or tangential or shear stress is influencing or not, so that we have to do.

So, σ_{induced} is nothing but the original equation of Kirsch so that will be obtained from the, original equation of Kirsch (1898).

Now, it is indicating here it is nothing but the d % of the $\sigma_{\text{in-situ}}$, this is indicating, so, this is what we are getting.

So, it can be rewritten by dividing $\sigma_{\text{in-situ}}$,

$$\frac{|\sigma_{\text{induced}} - \sigma_{\text{insitu}}|}{\sigma_{\text{insitu}}} \geq 0.01d$$

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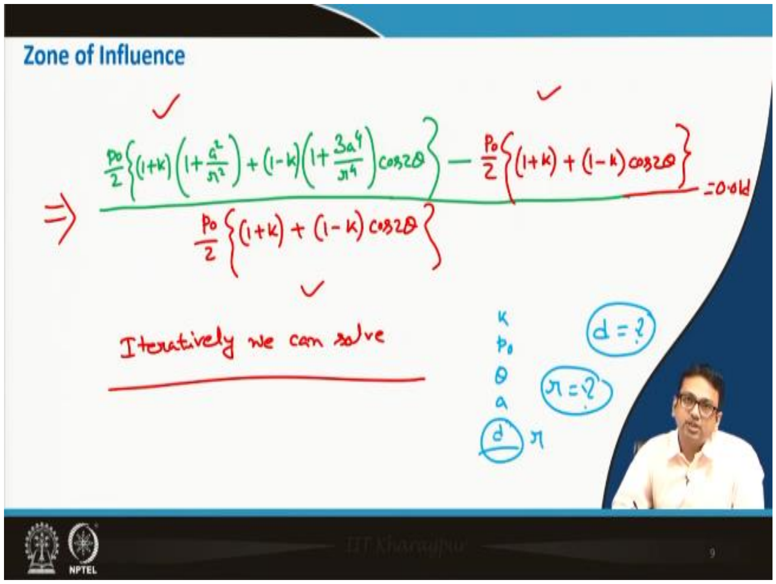

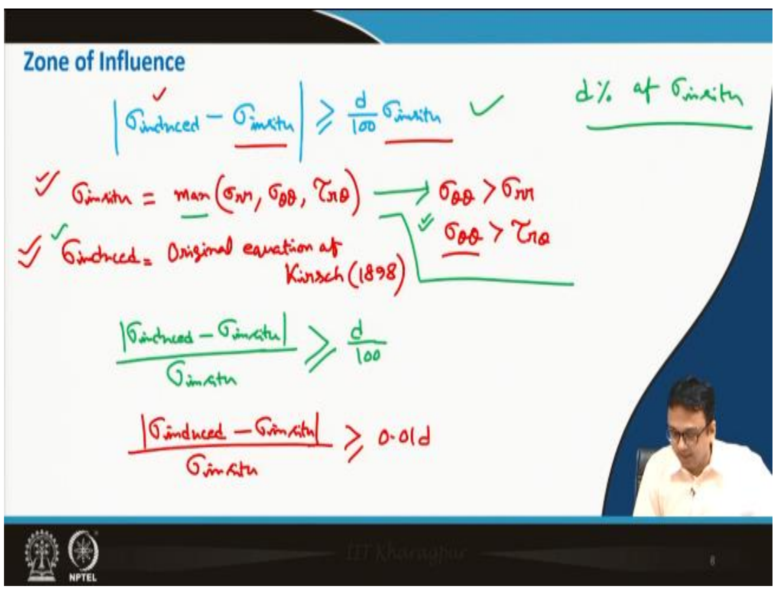

So, let us just go to the next page and finish this. By placing the original equation of Kirsch (1898)

$$\frac{\frac{p_0}{2}\left\{(1+k)\left(1+\frac{a^2}{r^2}\right)+(1-k)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right\}-\frac{p_0}{2}\left\{(1+k)+(1-k)\cos 2\theta\right\}}{\frac{p_0}{2}\left\{(1+k)+(1-k)\cos 2\theta\right\}}=0.01d$$

So, we can write,

$$(1+k)\left(\frac{a^2}{r^2}-0.01d\right)+(1-k)\cos 2\theta\left(\frac{3a^4}{r^4}-0.01d\right)=0$$

So, σ_{induced} and $\sigma_{\text{in-situ}}$ are over here and is written the equation. Now, this equation can further simplify and actually, for you can solve it by iteratively you can solve it, so we can solve it iteratively. We can able to design the tunnel based on values of k and d. If you know suppose the k value, P_0 value, the orientation, and a value, you can find out the influence.

Then obviously you will be able to find out that suppose that whether and if you also given with the d value also, you will be to find out at what r you will get this d percent influence or on the other hand maybe it may happen the instead of this one maybe the r is given in that case instead of this one, if r is given then what you can find out from this equation at suppose 6a distance, r is equal to 6a, then at 6a distance what will be the influence that is the how much percentage of our $\sigma_{\text{in-situ}}$ will be present there probably that we will be able to find out, how much percentage of that.

So, this way you can utilize this equation, so this is something what we have obtained from the fundamental. By putting the given input value, you can anyway solve a problem given to you or in your practical life. When you will go for designing, you will be able to find out the distance at which how much percentage of in-situ stress or maybe if that is fixed then, suppose it may happen, suppose a tolerance of maybe 1 percent is there, where you will construct a new structure.

So then, you have to find out what distance you have to construct that with respect to existing tunnel or if you are constructing a new tunnel at what distance the tunnel should be place that you will be able to find out using this equation.

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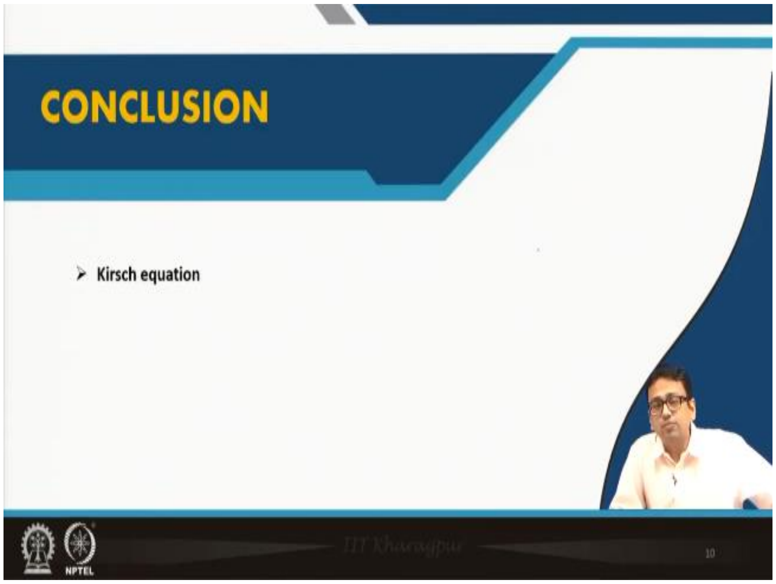

So, the idea is now clear to you. Today we have explored the Kirsch equation further and we have tried to understand very important thing that is zone of influence. So, thank you with this I am concluding our today's lecture, thank you.