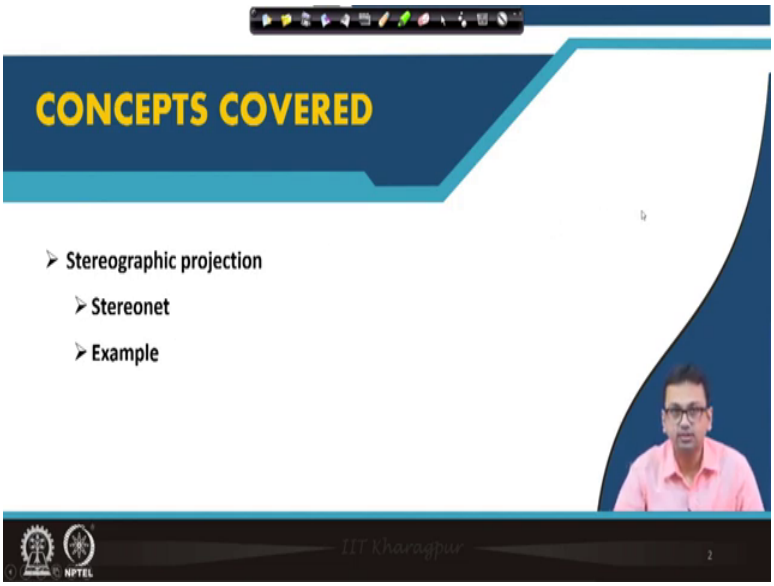
Rock Mechanics and Tunneling
Professor Debarghya Chakraborty
Department of Civil Engineering
Indian Institute of Technology Kharagpur
Lecture 05
Stereographic Projection

Hello everyone, I welcome all of you to the fifth lecture of this course Rock Mechanics and Tunneling. So in our previous lecture I have told you that we will learn in little more detail about the discontinuities in rock, means specifically the orientation of discontinuities in rock. So we will try to learn in detail about the orientation of discontinuities in rock.

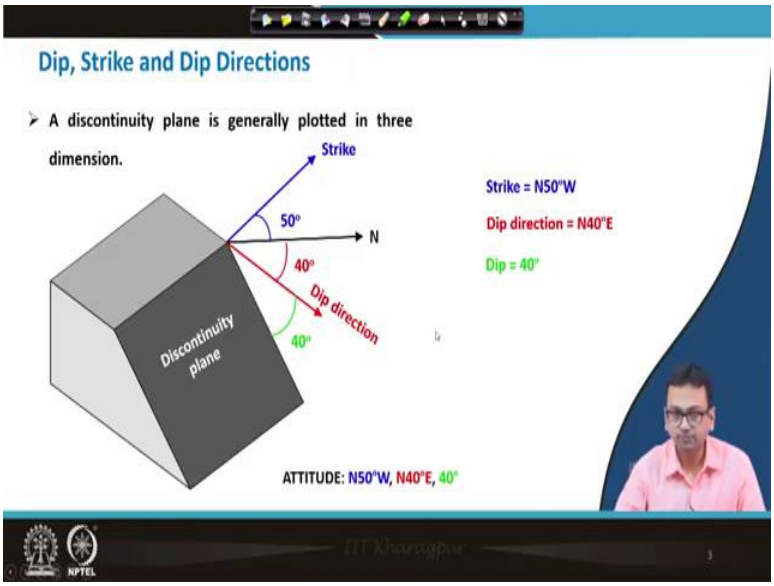
Because as we have seen identifying the orientations of these discontinuities, that is extremely important. So in this part of our lecture we will focus on the Stereographic Projection.

(Refer Slide Time: 01:18)



So what is this, why it is important, let us try to understand. So the stereographic projection, under that we will first discuss about the construction of stereonet and we will solve an example. Before that some of the things which we already know, we will just quickly revise and then our main content will start.

(Refer Slide Time: 01:40)



So, I have already told you about the Dip, Strike and Dip Directions. With the help of this Dip, Strike and Dip Direction we can tell about the orientation of different discontinuities that we have discussed in detail, but we will find in some cases, using only this idea is not good enough. So in those complicated situations what to do, that we will basically learn today.

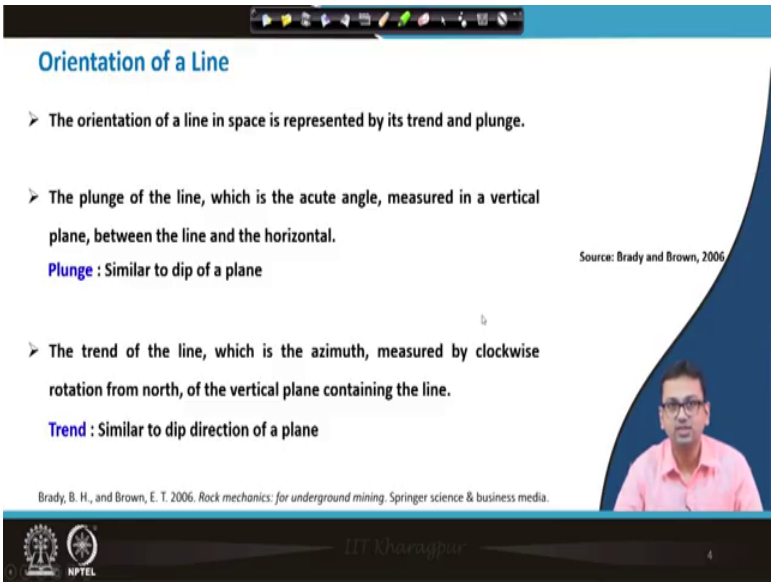
So essentially we will discuss about the orientations of the discontinuity planes or intersection line, but we will learn here a new topic that is the stereographic projection. So, we will see again just a small animation. So as I have told you that a discontinuity plane is generally plotted in three dimensions.

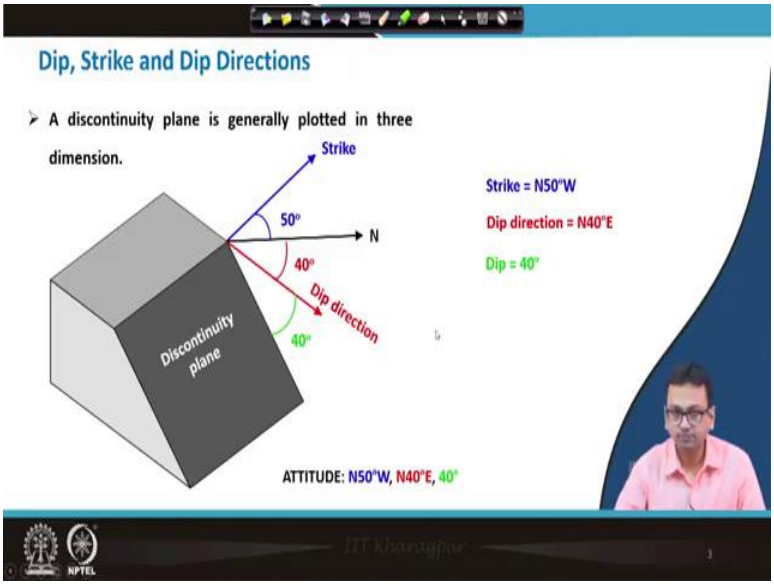
Now, if I consider this is a discontinuity plane then if I find that this is the north direction, this angle will give us the strike, why? Because as we know that, you see, this is the horizontal plane and this is the discontinuity plane. And this discontinuity plane and this horizontal plane are intersecting at this line. So, the orientation of this line will give us the strike. So you can see that I have randomly taken it as 50^{o}.

Now, we can write the strike as $N50^{o}W$. Now, as we know that the strike and the dip direction are 90^{o} apart. So we can very easily find out the orientation of this, which is 40^{o} and this is nothing but the dip direction. And now, you can see here another 40^{o} with green color, this is nothing but the vertical angle which we call the dip.

So, this red line is the dip direction which is 40^{o} and this 40^{o} is the dip. So finally we can say that the strike is $N50^{o}W$, dip direction is $N40^{o}E$ and dip is 40^{o}, and as we know that the attitude will consist of strike first, then dip direction and then dip. So with these three parameters we can define the orientation of a plane in three dimensions that is already known to us.

(Refer Slide Time: 04:44)





Now, what about the orientation of a line? So, for that let us introduce first two important terms and then I will discuss why I am doing this. So, first let us see the two terms about the orientation

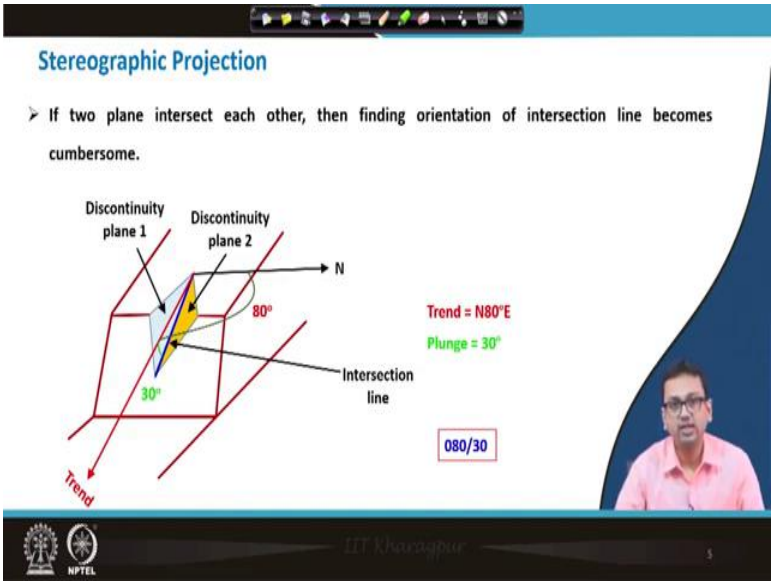
of a line. For the orientation of a plane we need strike, dip direction and dip; likewise for that the orientation of a line in space, we need the trend and the plunge.

Now, what is plunge? The plunge of the line, which is the acute angle measured in a vertical plane between the line and the horizontal. So what comes to your mind, what we can understand, that plunge is, to some extent similar to the dip of a plane. If you go back here as we know the angle is 40^{o}, that was the dip, so likewise what was the dip, it was the vertical angle.

Similarly in case of the orientation of line, two terms, trend and plunge, these two terms are important. So plunge is what? The plunge of the line which is the acute angle measured between the line and the horizontal in a vertical plane. So what we can say that plunge is similar to dip of a plane. Similarly what is trend?

The trend of line which is the azimuth measured by clockwise rotation from the North, of the vertical plane containing the line. So it is again to some extent, we can say that the trend is similar to the dip direction of a plane. So, for the orientation of plane we will use strike, dip direction, and dip and for the orientation of a line we will use plunge and trend.

(Refer Slide Time: 07:37)



Now you will see why it is important. Why we need to learn this stereographic projection actually, that is what I am writing here. So if two planes intersect each other, then finding the

orientation of intersection line becomes cumbersome, obviously if you try to think, that is obviously going to be cumbersome.

For example, let us consider this is a slope and there is some wedge failure has occurred. So we will learn about wedge failure in detail obviously, when we will learn about the slope stability, but here we are just considering that this is a slope where this block has actually came out. So some failure has occurred there. And what we can notice here that this is the discontinuity plane 1 and this is the discontinuity plane 2.

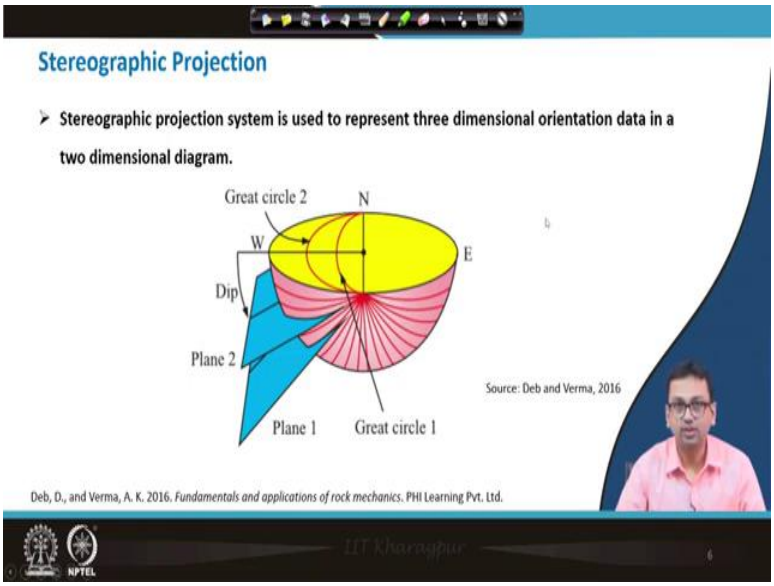
So this line is nothing but the intersection line between discontinuity plane 1 and discontinuity plane 2. Now if I want to present the orientation of this line, with the help of dip, strike and dip direction, we can tell about the orientation of the planes but what about the orientation of this intersection line?

For that we need plunge and trend, so what we will be the plunge and trend here? So if this is the North, then this angle which is arbitrarily taken as 80^{o}, is nothing but the trend, and the vertical angle is arbitrarily taken as 30^{o}, this vertical angle is nothing but the plunge. Now trend and plunge, either you can write it in this form $N80^{o}E$ and the plunge is 30^{o}, or you can simply write it in this form.

First the trend, then the plunge, so trend/plunge. Now trend, it should be a three digit number, so that you can easily identify that this is the trend and this is the plunge. So, though it was 80^{o}, so you should write it as 080/30. So this is another way of representing the orientation of a line in three dimensions.

So now you see as I have stated, if two planes intersect each other, then finding the orientation of the intersection line becomes cumbersome and at that time we take the help of stereographic projection and we present this geological data on a stereonet.

(Refer Slide Time: 10:35)



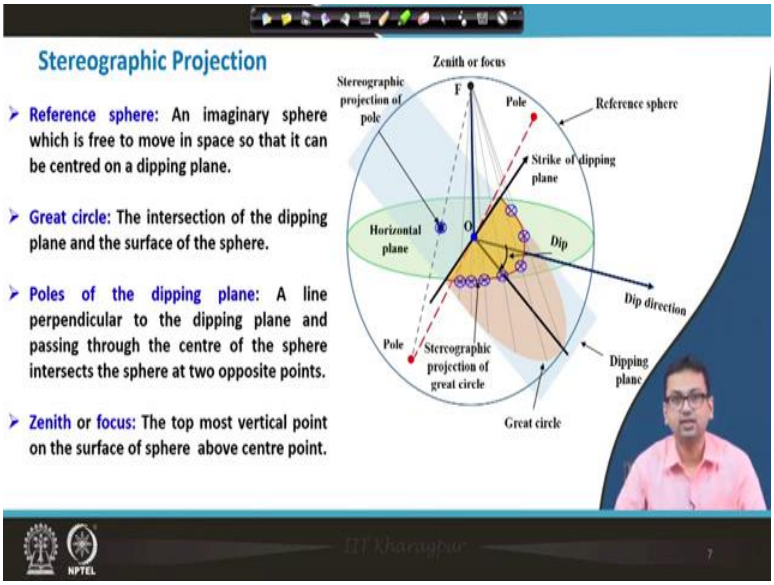
So, we have to understand, what is stereonet? How it is prepared and how it can be used for plotting the geological data on the stereonet. So, let us first understand the stereographic projection. Stereographic projection system is used to represent three dimensional orientation data in a two dimensional diagram. So if we can present the three dimensional orientation in a two dimensional diagram then our life becomes relatively easier.

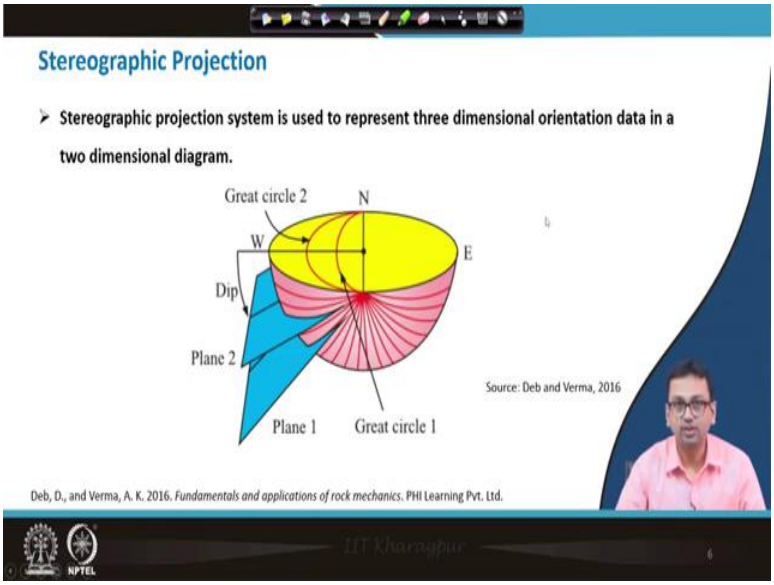
So, assume there is some discontinuity present over here. The orientation is given like this, so the dip is this. Now this discontinuity plane is intersecting this, this is the lower hemisphere if you consider, it is a half sphere you can consider, so lower half of the sphere, so this hemisphere if you consider, so here you can see the intersection plane is this and this is intersecting with the surface of this hemisphere.

So how it will look? This will look like this. If you project this one over this horizontal one, this will look like this kind of a circle or arch. So this is called a great circle, now here two planes are shown over here. So this is the great circle 1 and this is the great circle 2, but definitely it is not clear to you now, but in next slide when I will explain how it is constructed, your concept will be 100% clear.

So this is just to show you the pictorial representation of this 3D discontinuity plane on a 2D equatorial plane. So, I am showing this in this diagram, but the next slide will clear all your doubts.

(Refer Slide Time: 13:05)





Here, step by step, I will explain how to obtain these great circles. So, side by side, we will see some of the definitions also. Suppose, this is a discontinuity plane or intersection plane, or let us consider this is a dipping plane. So, now if I have the dipping plane, then I need to choose a reference sphere like this.

You just imagine a sphere over here. Now what is the condition or what is the reference sphere? It is an imaginary sphere which is free to move in space so that it can be centered on a dipping plane. So this is, O is the center of the imaginary sphere and this center is lying over the dipping plane or intersection plane.

And the imaginary sphere which is free to move in space so that it can be centered on a dipping plane. So you imagine a sphere and there is a dipping plane or intersection plane and center of the sphere is located over this plane. Now, this is called the reference sphere.

Now, this is the horizontal plane. It is, like if you consider sphere and if you consider equator, so it is just the half of the sphere, top half, bottom half, so this is the horizontal plane what we are also imagining. Now what is the great circle? The intersection of the dipping plane and the surface of the reference sphere, intersection of the dipping plane and the surface of the sphere will give us the great circle,

So it will look something like this. If you imagine the sphere and this is the dipping plane. Now, the dipping plane will intersect the sphere at these points and its shape will be circular so the intersection of the dipping plane and the surface of the sphere will give us the great circle. So it is called a great circle.

Now see, this great circle if I go back, you see this part was there. Now, I am just removing this part, because the orientation of the great circle, whether it is, if you consider this as the lower hemisphere and this as the upper hemisphere, they are not going to give us any extra information basically, right?

So, this is the dip, whatever the angle is developing here we will see on the upper part, here also the same angle. So basically this is not giving us any extra information. So, this great circle representing the plane appears on both the upper and lower hemisphere, so the same thing is appearing. So there is no need for considering the upper part.

Now onwards we can concentrate on only the lower part of the hemisphere. So that is why the upper part, what was there, I have just removed and let us focus on the lower part of the hemisphere. Now, so let us consider this is the strike of the dipping plane, so this is the dip direction and this is the dip. So these things are known to us, right?

Now another term is called pole. What is pole? Pole of a dipping plane; so, this is the dipping plane, then what is the pole of a dipping plane? Let us see the line perpendicular to the dipping plane. So it is perpendicular to this dipping plane, so imagine that, you have to little bit imagine here. This is called dipping plane we can see, now a line perpendicular to this dipping plane and passing through O i.e. it is the center of the sphere.

So a line perpendicular to the dipping plane and passing through the center of the sphere intersects the sphere at two, diametrically opposite locations. So you see the diametrically opposite locations, it is intersecting the sphere. So let us read it once again. A line perpendicular to the dipping plane and passing through the center of the sphere intersect the sphere at two opposite points.

So they both are called pole, so again, this pole and this pole, just this is on the upper hemisphere, this is on the lower hemisphere. So we can only, this is not giving any extra information. So we can only, as we have taken only this part of your great circle, likewise we will consider this pole only for the remaining discussion of this slide.

Then let us identify, I mean learn about another term that is zenith or focus, what is this? The top most vertical point on the surface of the sphere above the center point is called the zenith or focus. So, top most vertical point on the surface of the sphere is the zenith or focus. Now, we will do the stereographic projection.

So first, let us give the name of this as F. Now, first we will project and we will try to obtain the projection of this pole. So, they are connected with the help of a dotted line. Now, you see, where this line is intersecting with this horizontal plane or equatorial plane, that point we are marking. So, this point is nothing but the stereographic projection of the pole.

Now, the great circle and the pole are the 3D information and in order to represent these terms, we need 3D diagram, but we want to project everything on a 2D diagram; so, we want to project everything on this horizontal plane only. So, I have projected the pole over here, similarly we can project this great circle on the horizontal plane.

How? You see this line; the points over here will be connected with this focus. So, now where it is intersecting the horizontal plane, I have marked it with this cross. Similarly, another line has

been marked with another cross, and so on. Crosses are identified and now there is another cross and now I will join these points. So we will get the projection of this great circle on this horizontal plane. So, this is nothing but the stereographic projection of the great circle.

So I hope the idea is clear to you, just quickly, I will run the animation once again. So, first what we are doing? We are identifying the dipping plane, then we drew the reference sphere, we are ensuring that the center of this sphere will be over this dipping plane. Now, we are identifying the horizontal plane, and then we will identify the great circle. That is nothing but intersection of the dipping plane and the surface of the sphere.

Then, this is the great circle and we have taken the lower half because similar information we will get from the upper hemisphere. So only lower hemisphere is good enough, then we have identified this at the extra information which you know already, identify the orientation of this plane in 3D, we can use strike, dip direction and dip, fine.

But again with this stereographic projection, what we did, we have identified the pole, so a line perpendicular to the dipping plane and passing through the center of the sphere. Then what we did, again two poles, upper pole will not give any extra information so we can only use the lower pole.

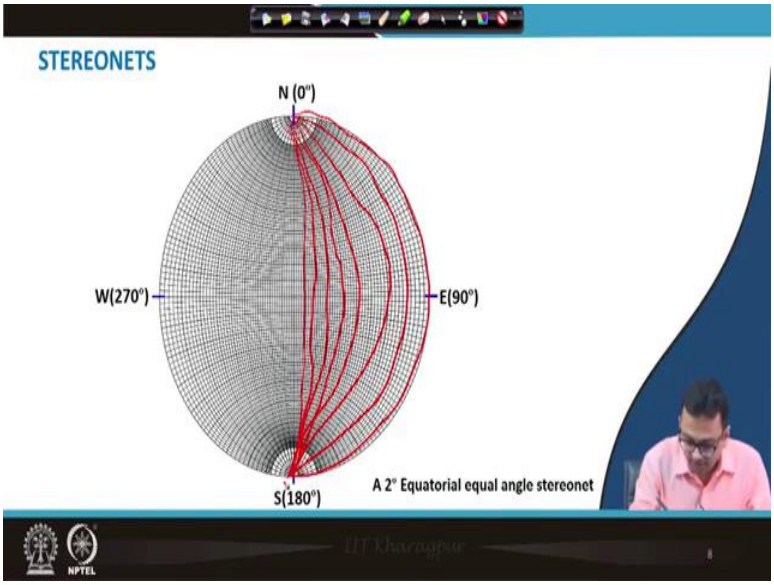
Now, we have identified zenith or focus, this is the top most vertical point on the surface of the sphere, then we have given a name F and then connected these two, pole and F and we have identified the point where it is intersecting this horizontal plane which is nothing but the stereographic projection of the pole, similarly we have the 3D data that we have plotted on the 2D plane. Now again the great circle that we will plot on this horizontal plane with the help of the projection.

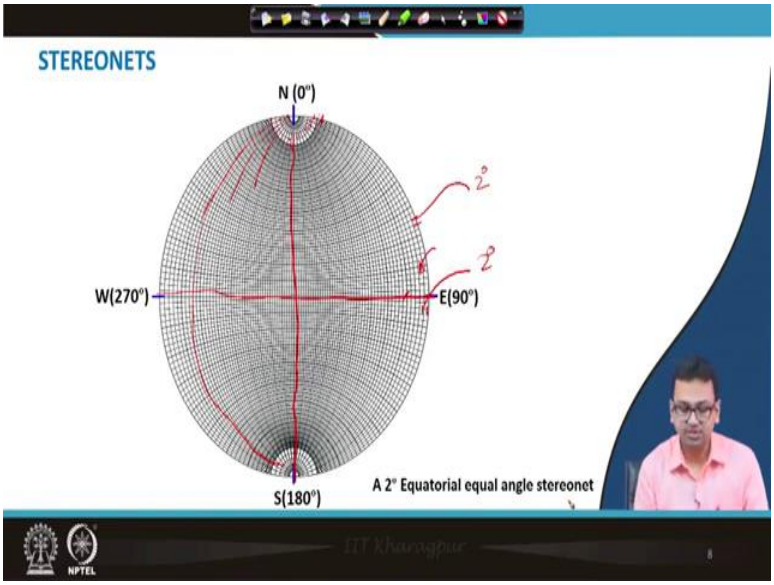
So, we are doing the projection, you see I am marking the points where it is intersecting the horizontal plane. Again, last one, and then yes, I am marking it. So this is nothing but the great circle and the stereographic projection of this great circle.

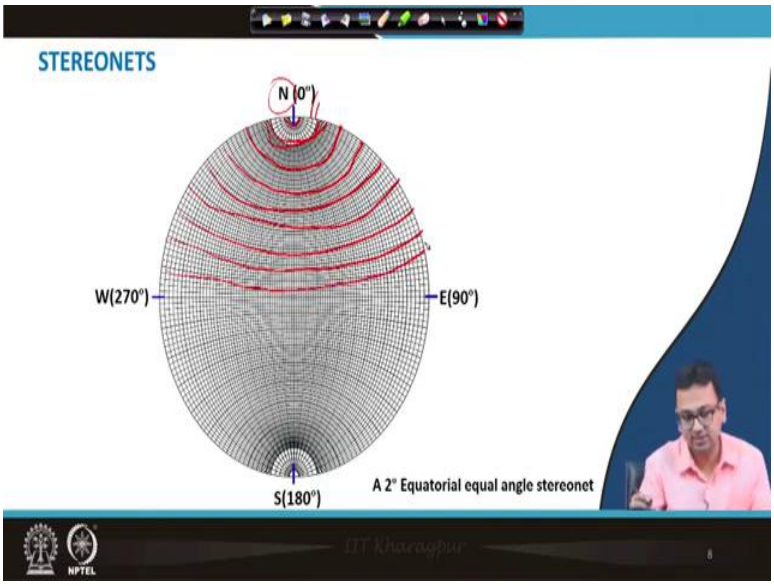
So now quickly, I will just show you this previous diagram. You see, this is nothing but the stereographic projection of these great circles on this horizontal plane, okay? So, now I am

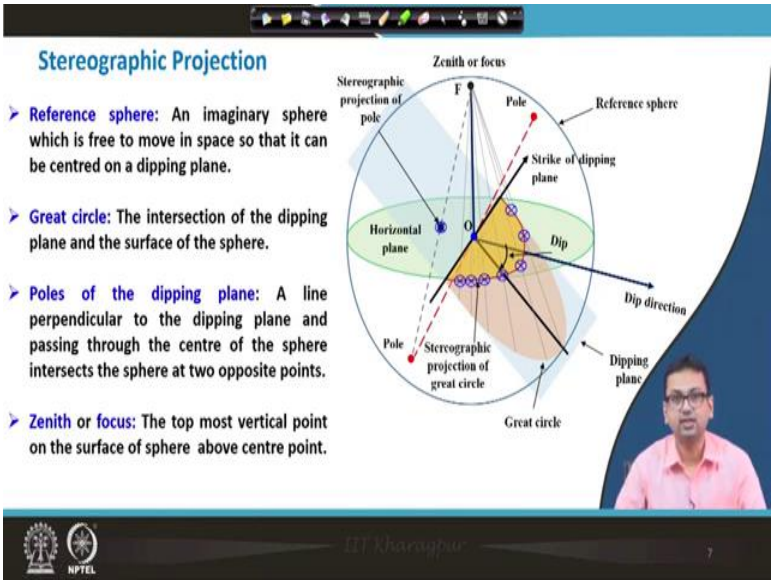
moving to the next slide. So, we have understood this stereographic projection and we are getting the great circles with the help of this. Now, how can we use this for our work?

(Refer Slide Time: 24:43)









So, for that purpose, these stereonets are available in the market. Now what is this? As you can see, it is written over here 'A 2^{o} Equatorial equal angle stereonet'. Now this is what? This is basically nothing but the horizontal plane what we have identified over here. And these curve lines are nothing but the great circles, you can imagine that is, all this line, so this line, you can consider as one great circle, this line, this line, likewise it is going.

So there are total nine should be there, you can clearly notice over here, probably I have drawn less. So, 10, 20, 30, 40, I mean it is also there. Total nine should be there. So it is actually give, these points are how, just, you see, this is 0, then next dip line or here you can identify them

better, so this is one line, this is the second line, third, fourth, fifth, sixth, seventh, eighth, ninth, and this is the vertical line.

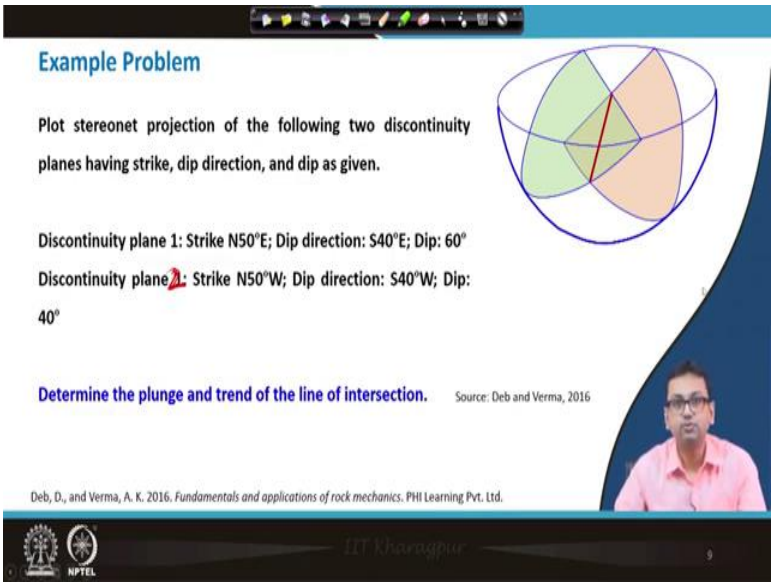
So, we can imagine when we will use it. Next, we can think them as the great circles. On this horizontal plane, that we have seen previously, similarly this side also, likewise, things are there. So great circles, there we can imagine them as the great circles like this. Other than that you can see there is a horizontal line and there is a vertical line and each of these divisions is nothing but 2^o. This, this; this is 2^o.

Similarly this, this, this or maybe let me show over here, this, this; this is also 2^o. So that is why it is a 2^o equatorial angle stereonet. So this type of thing is already available in the market. Now how this is called a stereonet? How to use it? The stereonets are being used to plot this geological data and to identify the orientation that we will just learn.

So first what we will do, we will just mark over here, this North, and also one thing, this is starting from 0, 90, 180, 270, and basically total 360^o over here, and each these, as I have stated these divisions are 2^o so likewise it is presented and you can see over here, this type of small circles are also there.

Here to here, it is starting 1, 2, like that it is going. So like that, you see, they are going, they are the small circles and how we will use this, we will see just now about this also. So now first after getting this one, what we will do, you just imagine this is north, this is east, this is south, this is west and then youros are given over here. Now what to do?

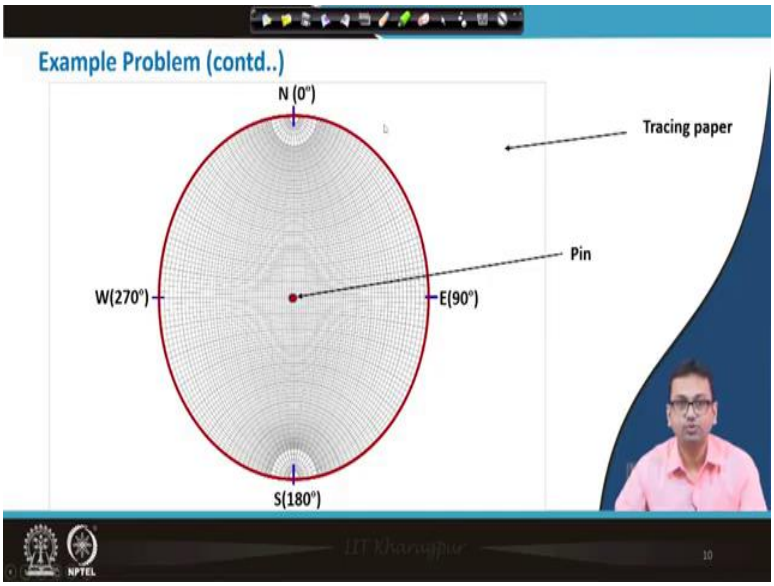
(Refer Slide Time: 28:49)

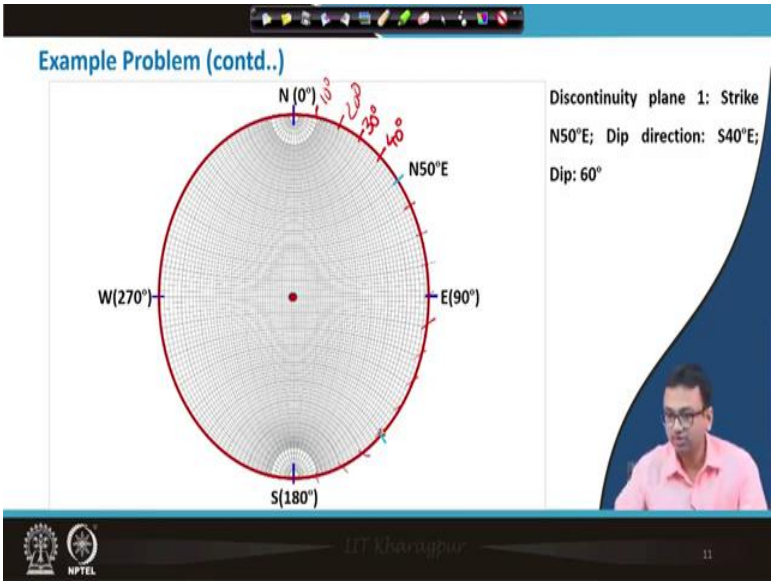


In order to understand that let us take an example problem, and why we are doing it, this will be cleared also through this example problem. You see, the problem is to plot the stereonet projection of the following two discontinuity planes. This is one discontinuity plane, this is one discontinuity plane, and it is shown in 3D, and the problem is to plot the stereonet projection of the following two discontinuity planes having the strike, dip direction and dip as given below. So, here, the strike, dip direction, dip are provided for discontinuity plane 1, likewise this should be the discontinuity plane 2.

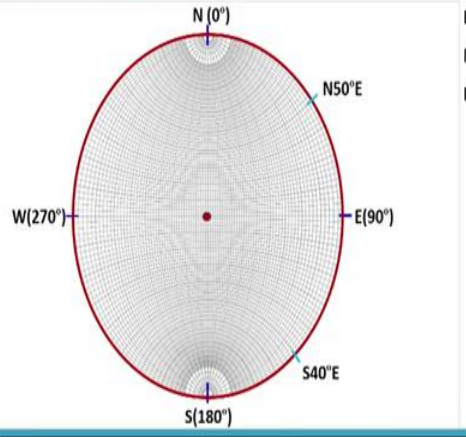
So, it is the discontinuity plane 2 and let us marks it as 2. So, the strike, dip direction, dip are provided for discontinuity plane 2 over there. So now, first, we have to plot and then determine the plunge and trend of the line of intersection. Now what is this? This is a plane and this is a plane, they are intersecting at this line. So now, we can present the plane with the help of strike, dip and dip direction, but to present orientation of the line we need to give the plunge and trend. So, how to do that? We will see that.

(Refer Slide Time: 30:34)



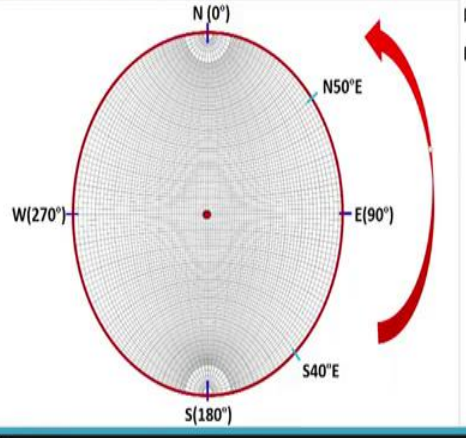


Example Problem (contd..)



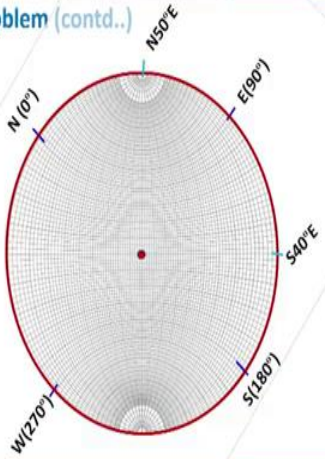
Discontinuity plane 1: Strike N50°E; Dip direction: S40°E; Dip: 60°

Example Problem (contd..)



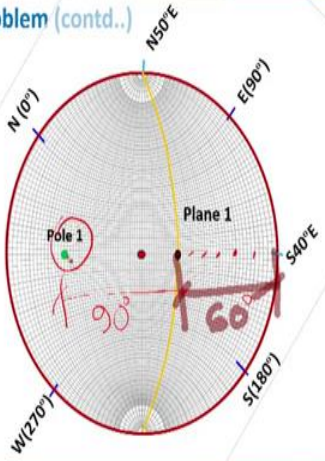
Discontinuity plane 1: Strike N50°E; Dip direction: N40°E; Dip: 60°

Example Problem (contd..)

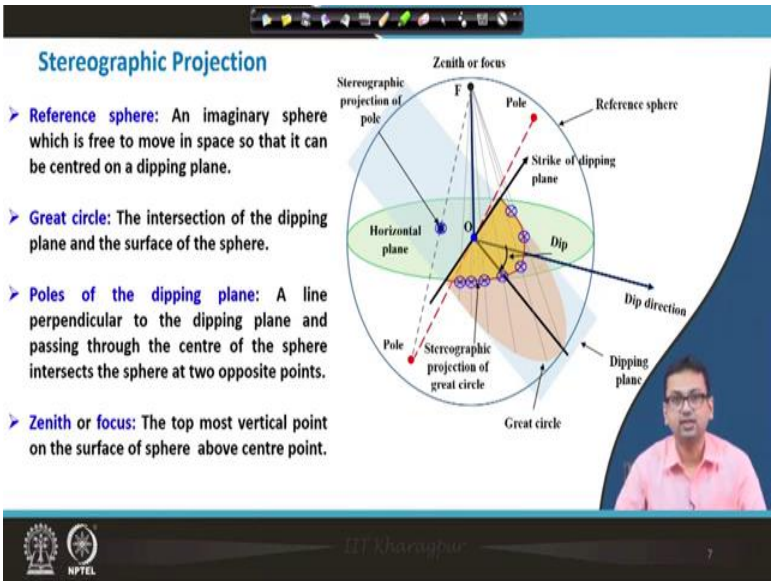


Discontinuity plane 1: Strike N50°E; Dip direction: S40°E; Dip: 60°

Example Problem (contd..)



Discontinuity plane 1: Strike N50°E; Dip direction: S40°E; Dip: 60°



So, first what will we do? This is the stereonet what we will get from the market, then what will we do? You will place a tracing paper over there. You can see that we have to place a tracing paper on this stereonet. After placing the tracing paper, then we have to mark the center and also use a pin to fix it.

The tracing paper is needed to be fixed at the center of the stereonet. Then, you have to identify the North, write there north, then east, then south and west; and also draw this line, around the circle. So, you have to draw this line, clear?

Place the tracing paper, identify center, put a pin over there and mark on that tracing paper only, this North, East, South, West, besides 0^o actually at this top the North, 90^o East, South, West and this is the red circle what I have shown.

Then what will you do? First one is the discontinuity plane 1. So discontinuity plane 1 is having the strike = N50^{o}E, dip direction = S40^{o}E and dip = 60^o. So this is what we have already seen. So now let us identify first the N50^{o}E. So you see if you look at here, in this stereonet, this is 10^o, this is 20^o, this is 30^o, this is 40^o, this is 50^o.

So, I have identified the N50^{o}E. Likewise, dip direction is S40^{o}E or you can go again from here 90o and you can identify this one. So, you see, this is 90 and 50 is here, then from here again 90; 10, 20, 30, 40, 50, 60, 70, 80, 90, or from South also you can come to the 40^o like, 10, 20, 30, 40. So if you do that, you will see that the dip direction is here.

So, anyway, if you identify any one of the strike and the dip direction, which will be good enough, but here, we have shown the both. Now, you have to rotate your tracing paper, not the stereonet, stereonet is lying as it is, below the tracing paper, you have to rotate the tracing paper in counter clockwise direction and bring the S40^{o}E at this location or you can tell that N50^{o}E at the North location.

So, let us do that. Yes. So now the tracing paper is in this position. Now what will we do? Initially it was the East-West axis and this is the North, or East-West line and this is the North-South line, right? Or you can say an axis. So, now, it will look like this after rotation and now here, the dip direction is S40^{o}E and the strike is over here, 90^{o} apart and the North has shifted here.

No problem. But what can we see here? You see the great circles are now looking like these, as we can identify over here. So, they are now looking like these. These are the great circles present over here. So, no problem, we can see that. Now what will we do? It is given dip is 60^{o}. So what we have to do, you see there, they are 10, 20, 30, 40, 50, 60; so each of these distances between these deep great circles are 10^{o} and difference between each two small, these two consecutive great circles are 2^{o} but this deep line, after actually every 10^{o} the lines are made a little bit deep for the easy identification.

So you see dip is here, 60^{o}. So from this end, East point, we have to go up to 60, so if we go and do that. So you can see that this point we can identify. So, we are identifying this one great circle which is actually passing through this point. So, you see, this great circle is passing through this point. Now basically, this line is giving us the information or representing basically the discontinuity plane 1. And here you see this distance, this distance is basically 60.

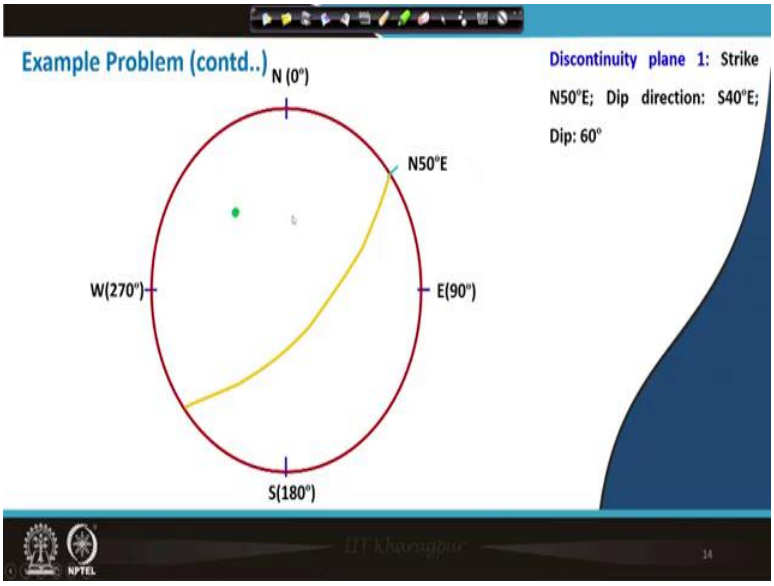
Now if you further proceed, let us try to identify the pole. So how will you get the pole? You see what we have seen that pole is; when we have seen the pole, what we have seen, just quickly see. Pole of a dipping plane what we wrote ‘A line perpendicular to the dipping plane and passing through the center of the sphere, intersects the sphere at two opposite points’. So basically they are perpendicular to each other.

The plane and the line, they are perpendicular to each other. So, it is stated basically here in this diagram. Now, you see, from this point if you go another, these are again 10, 10, 10, so this is up to the 90, means three points are further, then further if you go, means another 60^{o} in this side, so what will we get? You will be able to identify another point which is nothing but the pole. So this is the pole and this is again 90^{o} actually.

So, in this way, we could able to represent the plane 1 with the help of this great circle and also we have identified another thing that is pole. Now this pole itself, if we only present that will give us all the information related to this discontinuity plane. So only this point is good enough to represent the orientation of a plane.

Now similarly, next what will we do? So, next we will again rotate it back and we will bring this North to its original home position. So, we are doing this. So, in this way we can identify the discontinuity plane 1 on this stereonet. You see here this one, as we have discussed that this is the pole 1 and plane 1, it is showing over here, and then here, what we are showing, the discontinuity plane 1 is shown with the help of this diagram.

(Refer Slide Time: 40:53)



If I leave the tracing paper from the stereonet it will look like this. After identifying the discontinuity plane 1, which is having the strike as N50^{o}E, dip direction as S40^{o}E and the dip = 60^{o} and the stereonet will look like this. So we will continue this problem in our next class. So,

first we will identify the second plane and also subsequently the intersection line and the orientation of the intersection line. Thank you.