Rock Mechanics and Tunneling
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Lecture 39
Slopes (continued)

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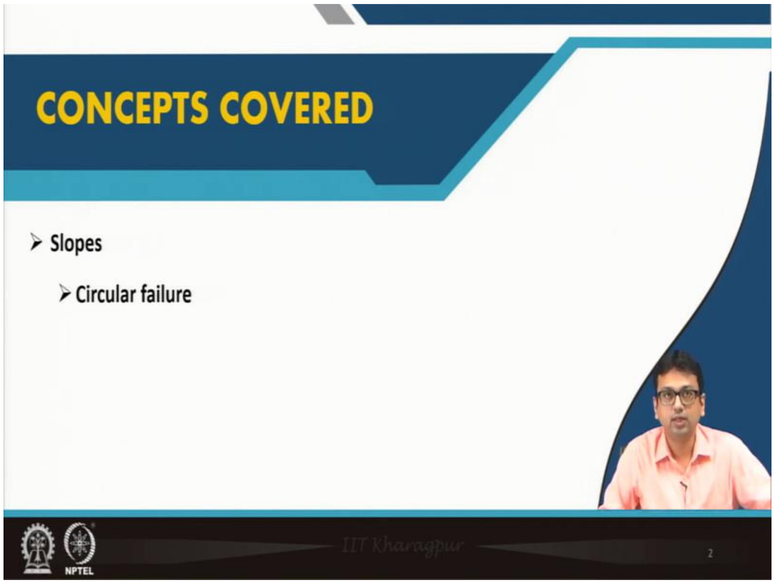

Hello everyone, I welcome all of you to the third lecture of module 8. So, in module 8 we are discussing about slopes, and we will also discuss about the underground excavations briefly. So, today if you remember in our previous classes we have discussed about plane failure, wedge failure and toppling failure. Today we will discuss about the circular failure.

So, regarding circular failure I expect that probably you know what are the different types of circular failure in soil mechanics. You already have learned these things, but for the sake of completeness we should also discuss about this circular failure briefly in this course also. So, I will discuss about that.

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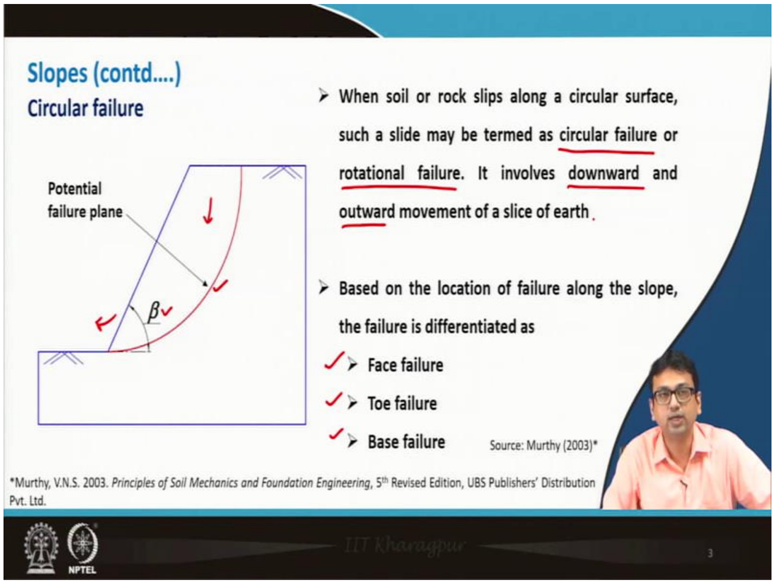

So, as you know probably, this is nothing but the potential failure plane and β is the slope angle, now when soil or rock slips along a circular surface such a slide maybe termed as circular failure or rotational failure. It involves downward and outward movement of a slice of earth. So, it will go downward as well as it will go the outward. So, that is why it is written like that involves downward and outward movement of a slice of earth.

So, based on location of failure along the slope, the failure is differentiated as face failure, toe failure and base failure. So, from these names we can imagine what type of failure occurs in these three cases, but however we will discuss about that in little more detail.

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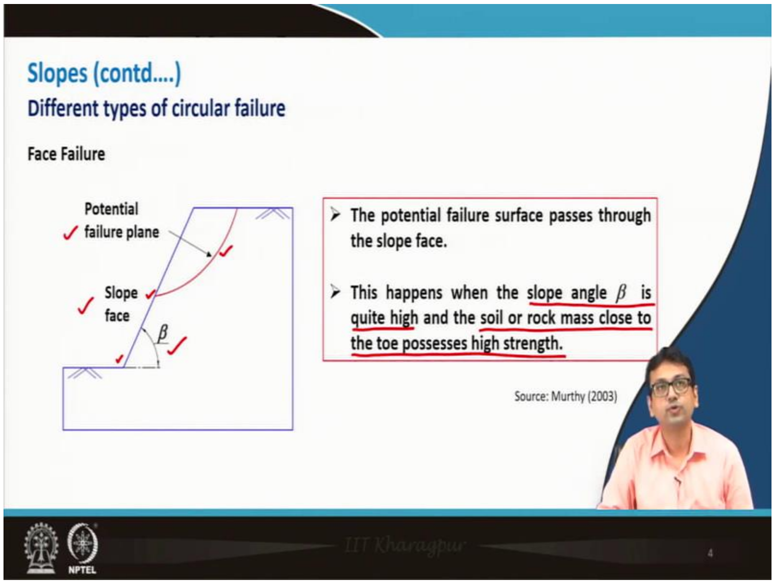

Now, suppose the face failure. So, in case, this is the β angle but you see the potential failure plane is here, whereas this is the slope face and what we can see that potential failure plane is ending here, in the slope face only. Now, the potential failure surface that what we can see over here or failure plane passes through the slope phase in this case, and this happens when slope angle β is quite high and the soil or rock mass close to the toe possesses high strength. This is nothing but the toe. So, it is saying that the slope angle β is quite high as well as soil or rock mass close to the toe possesses high strength. So, in that case this type of face failure may be observed.

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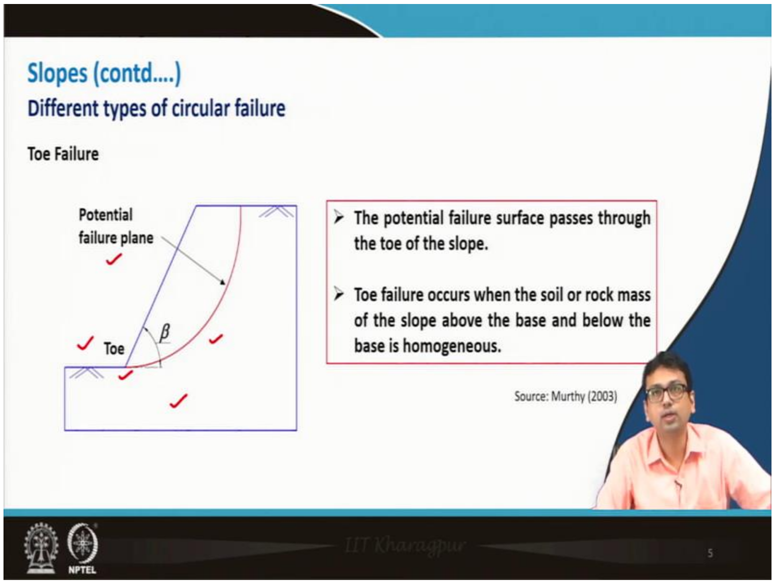

Now, toe failure, so this is our toe as it is marked over here and now you see, what is happening? The potential failure plane is passing through this toe. So, that potential failure surface passes through that toe of the slope in this case and toe failure occurs when the soil or rock mass of the slope above the base and below the base is homogeneous. So, above this is the base so above the base and below the base is homogeneous, then generally toe failure is observed. Now another type of failure is base failure.

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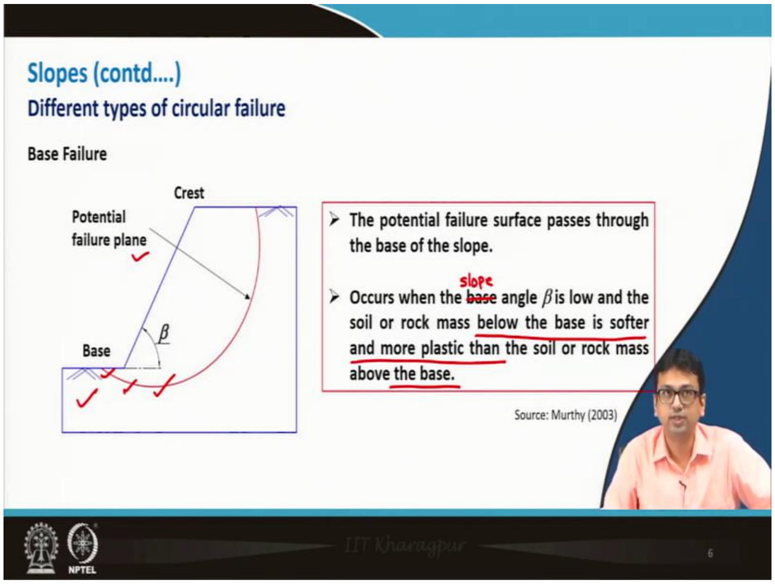

As the name suggests, this is the base. You see the potential failure plane is passing through the base. So, now the potential failure surface passes through the base of the slope and it occurs when the base angle β is low. Other case maybe the slope angle we can write since we are writing again a slope and let us write it a slope angle, so that you should not get confused.

So, it occurs when the slope angle β is low and soil or rock mass below the base is softer and more plastic than the soil or rock mass above the base. So, I hope the idea is now clear at least you can seeing the how the failure pattern or how it is failed you can comment that whether that is a face failure, toe failure or base failure.

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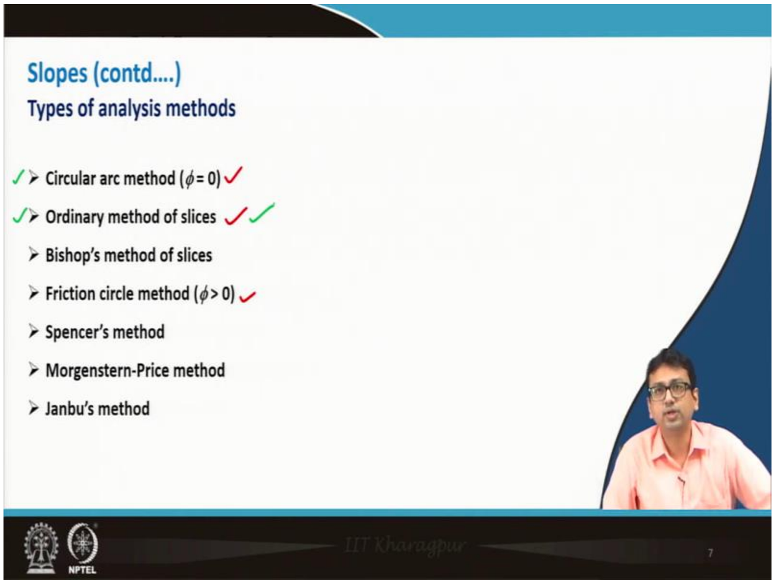

Now, there are the different types of analysis methods. So, I will just show some of the names. So first one is circular arc method. So, it is for frictionless soil, it is our block where ϕ will be considered as 0. Then ordinary method of slice which is very popular. Other than that we have Bishop's method of slices. So, Bishop's method of slices is obviously even better than the ordinary method of slices.

So, it is much more versatile probably you know that already you have learnt it earlier, friction circle method when $\phi > 0$ and other methods are Spencer's method, Morgenstern price method Janbu's method. So, these are different types of slope stability analysis methods. So, out of these methods maybe I will here discuss about two things circular arc method and ordinary method of slices in detail. I will focus more on the ordinary method of slices because that is very commonly used method.

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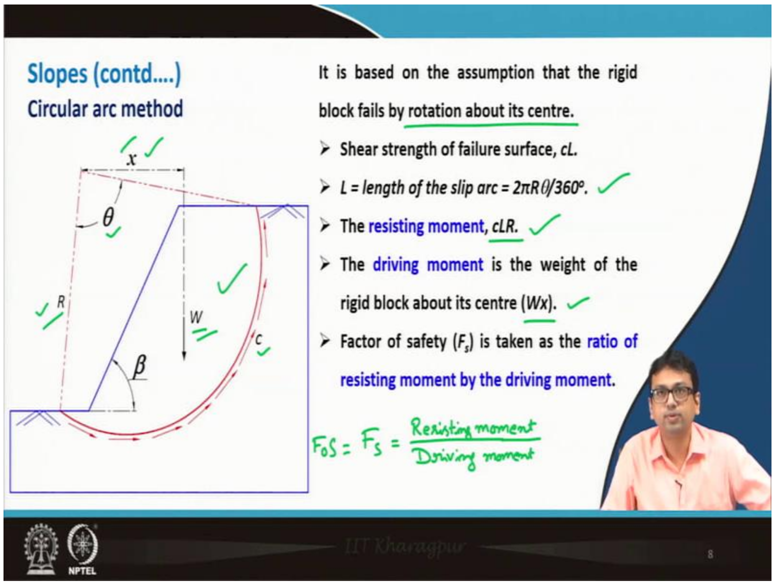

So, circular arc method, it is actually very simple the concept is since ϕ is equal to 0. Only c is present over here you see and this W is the weight of this failure, the rock mass what is failing the weight of that and you see R is the radius of this circular arc and theta is the angle. You have to notice x also, x is nothing but this distance.

So, the centroid of failing mass and the center of the arc distance is x. Now, we can very easily find out factor of safety for this type of configuration. So, what we have to do or what we can say is, it is based on the assumption that rigid block fails by rotation about its center. So, this is assumed as a rigid block and it is means it fails by rotation about its center.

The shear strength of the failure surface will be obviously what c L, where L is nothing but the length of the slip arc which we can very easily find out because if R is the radius then 2πR is the perimeter, so it will be $2\pi R\theta/360$ degree. So, the resisting moment will be what? Resisting moment will simply be cLR. Now, the driving moment is the weight of the rigid block about its center, so it is the weight of the rigid block W and the distance from the center is x, so it is nothing but Wx. So, obviously, now we know factor of safety is taken as the ratio of the resisting moment by the driving moment.

So, basically factor of safety if I write FoS or F_s is nothing but the resisting moment/driving moment

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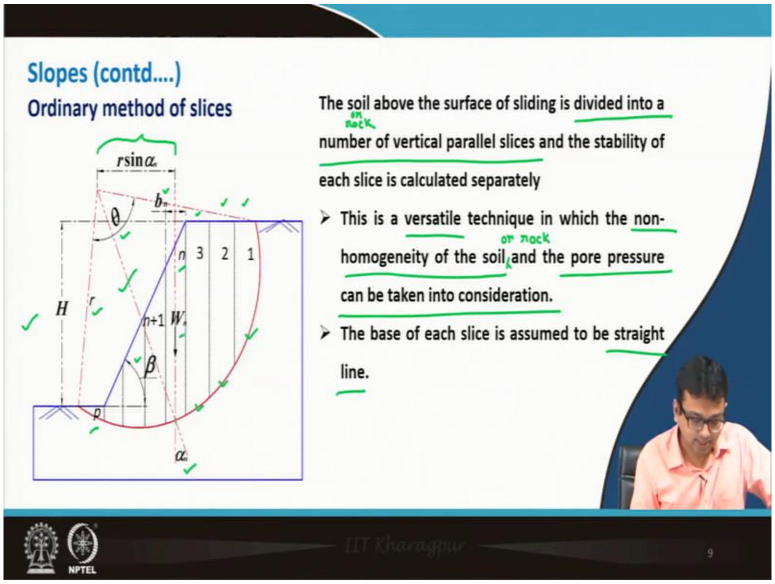


So, it was very simple now, let us discuss about the ordinary method of slices, this is much more versatile obviously, and it can take into account both c and ϕ effect of both cohesion and angle of internal friction and let us see what we can understand from the above diagram. For both soil and rock cases it is applicable. Above the surface of sliding is divided into number of vertical parallel slices.

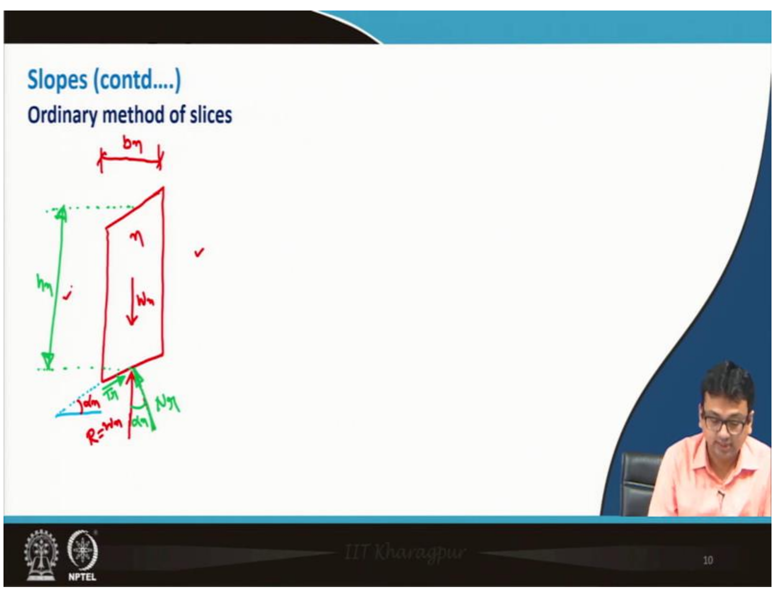
So, you see it is the potential failure surface as we already know. So, the mass above the surface of the sliding is divided into a number of vertical parallel slices as you can see like 1, 2, 3 you can clearly notice from here. And stability of each slice is calculated separately, so we can find out the stability of each slice.

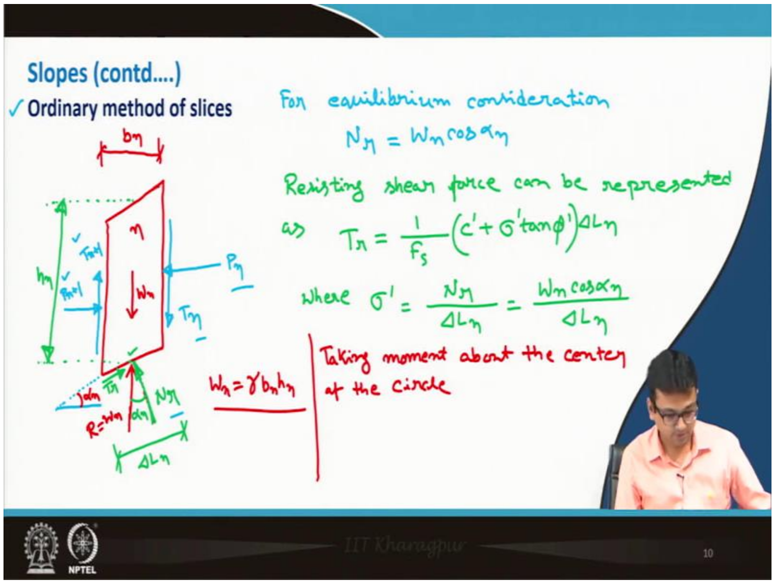
So, for that what we have to do? Obviously, we have to draw the free body diagram of the individual slides. So, I will draw that in next slide only. This is a versatile technique in which the non-homogeneity of the soil or rock both you can write so soil or rock and the pore pressure can be taken into consideration. So, this is as I have mentioned very much versatile method and very popular, it is a versatile technique in which the non-homogeneity of soil or rock and the pore pressure can be taken into consideration.

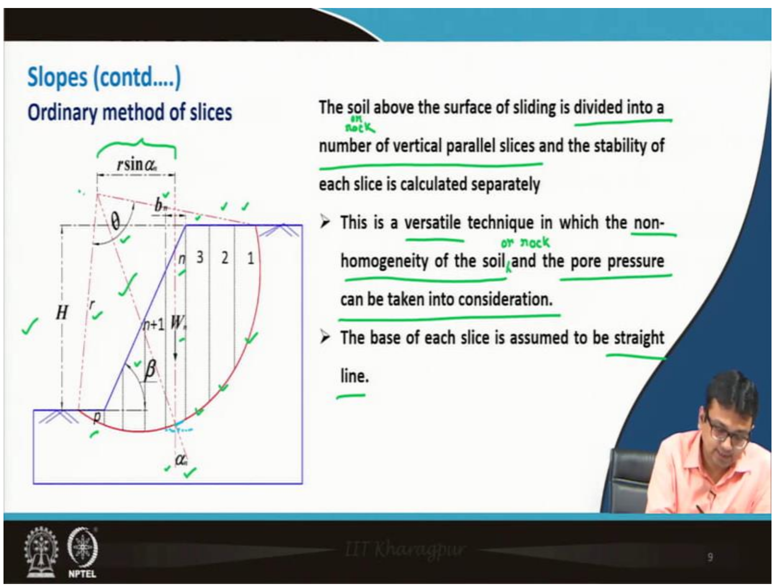
What is considered here is the base of each slice is assumed to be straight line. So, this base though there we can notice some curvature but it is assumed to be straight line. Now, if obviously if we increase the number of the slices obviously my assumptions of this straight line will be much more valid in place of if I consider only less number of slices, so that will not be a true justice that will not do true justice. So, actually if we increase the number of slices my accuracy will increase because this arc will become almost like is this arc if you consider that we will do it is a curved line we can see but it can be assumed as a straight line. So, that is what one assumption you have to do.

Now, based on these let me as I have stated or mentioned over here that the number of vertical slices we need to divide and we have to draw the free body diagram of one of the slices. So, what you can notice from here is the suppose the total height of the slope is H, then R is the radius and what we can see θ is this total angle and β is nothing but the slope angle and likewise nth sliced up to slices pth slice are there.

And we know this nth slices supposed to b_n and this if you look at this line, so this one you see will pass through if we consider this point, this angle will be let us consider α_n, then obviously this distance will be what? This distance will be $R\sin\alpha_n$. So, now, based on this understanding let us draw the free body diagram for this nth slice suppose where the weight is W_n and this is α_n. So, let us do that. (Refer Slide Time: 15:26)







So, let me maybe use red color first maybe, so suppose roughly this is the slice, now so this is supposed to horizontal maybe a little smaller length I will take, now what we can see is so as we can notice from here that this is α_n means this one is making an angle α_n with this line, and this curved line is assumed to be a straight line.

So, now what we can see over here is one way it is acting like W_n of nth slice, this angle is α_n and width of this slice is suppose b_n and suppose maybe use some other color. let me use suppose from here suppose this one is the height of the slice suppose h_n.

So, h_n, b_n, α_n, and weight of the slices W_n. Now, let us see what are the forces act over here. There will be this force, then normal to these suppose another one, so let me maybe use some other color for this. So this is another one, this is normal. So, okay and another one is here the tangential.

So, what we can write over here? This angle is α_n and this is nothing but the suppose this one is my N_r normal reaction, this is my T_r tangential reaction and this red one little bit extreme, so this one is nothing but my reaction which is equal to the W_n and this is the normal component and this is the tangential component of this R.

Now, what we can see you see between two slices, so here it is nth slice suppose in this side there will be the n-1th slice this side and this side will be $n+1^{th}$ slice, what will happen in between these two slices. What will happen the normal force or reaction P_n and the tangential one T_n will act.

Similarly, on this side also what will happen obviously one normal will act that is suppose P_{n+1} and similarly this one will act tangential $T_{n+1.}$ These are nothing but the presence of adjacent slices, so this type in this side T_n,P_n and here P_{n+1} T_{n+1} forces will also act. So this actually completes the free body diagram of this one individual slice which we have given a name like n^{th} slice.

Now, what is done over here you see obtaining these T_n,P_n, P_{n+1} and T_{n+1} are obviously difficult. So, in this ordinary method of slices what is done actually the resultant of P_n and T_n are assumed to be equal in magnitude to the resultant of this P_{n+1} and T_{n+1} and it is also assumed that their lineup actions also coincide with each other. But now considering other forces in this free body diagram we can basically derive the required expression for the factor of safety.

So, what we know right from here is for equilibrium suppose consideration we can write N_r is equal $W_n\cos\alpha_n$. Now, another thing is resisting shear force can be represented as follows.

$$T_r = \frac{1}{F_s}\left(c' + \sigma' \tan\phi'\right)\Delta L_n ;$$

where $\sigma' = \dfrac{N_r}{\Delta L_n} = \dfrac{W_n \cos\alpha_n}{\Delta L_n}$

$$W_n = \gamma b_n h_n$$

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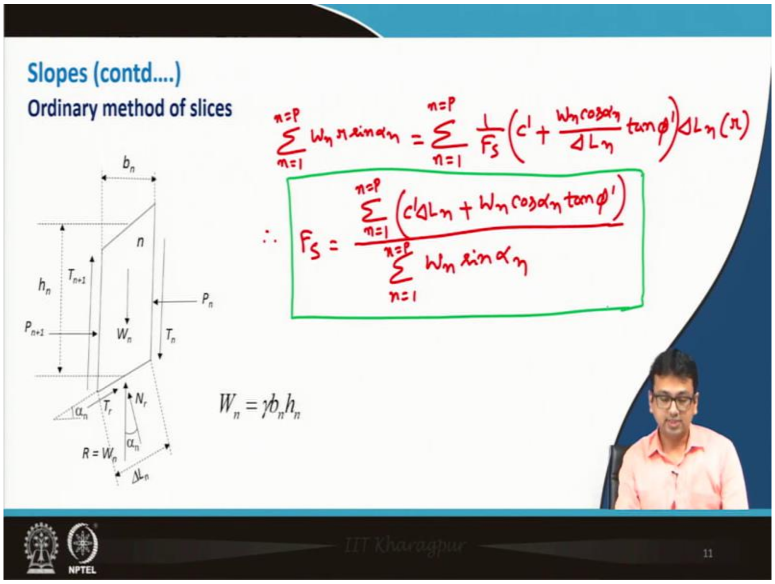

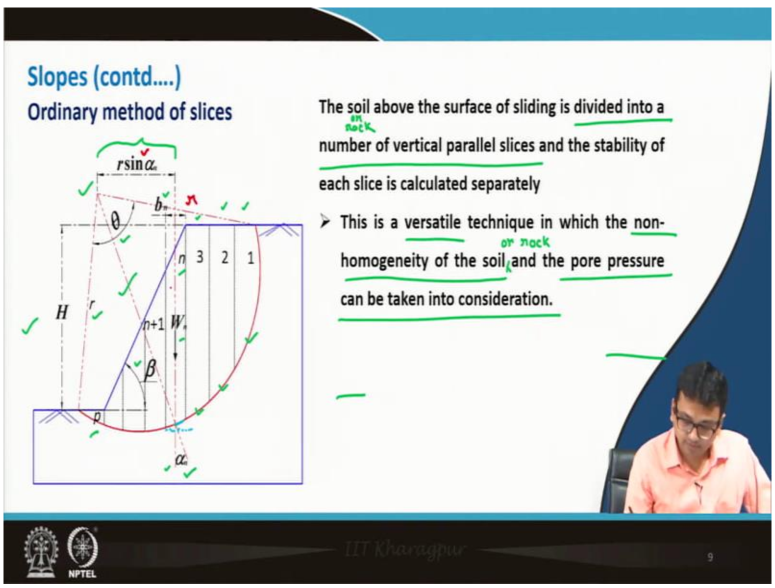

So, this is already I have kept it ready. So, if we do that it will be summation W_n into you see this distance we can see from here that this distance is $W_n R\sin\alpha_n$. Now, summation will be included because it means there are like p number of slices as you can see from here.

So, you see from P_1 to P_n^{th} slice. So, same thing will be applicable for other slices also. So, I will write over here $\sum_{n=1}^{n=p} W_n r \sin\alpha_n = \sum_{n=1}^{n=p} \frac{1}{F_s}\left(c' + \frac{W_n \cos\alpha_n}{\Delta L_n}\tan\phi'\right)\Delta L_n r$

So, this is my desired equation, which we need to utilize for obtaining the factor of safety where we will apply this ordinary method of slices. So, that is what we have derived.

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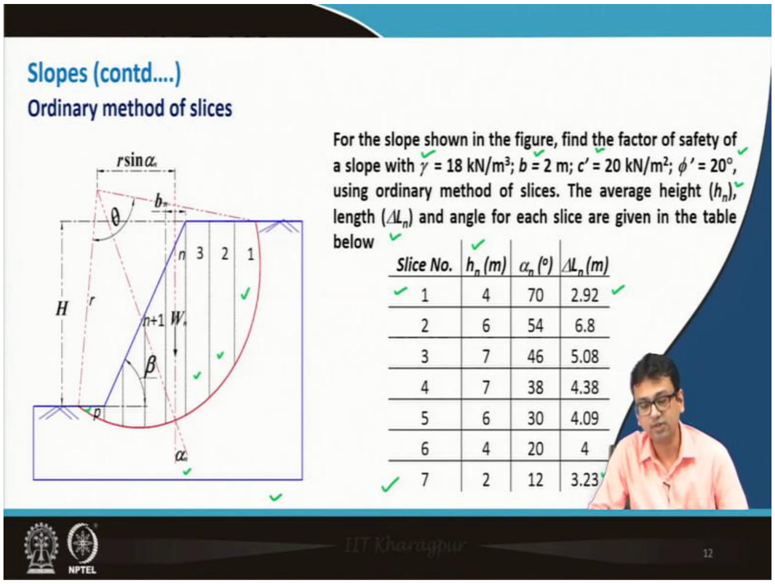

Now, let us do a problem and let us clear our doubt completely. So, the problem says for the slope shown in this figure, this figure as you can see this is actually what I have shown you earlier also the same figure only just now we have done the α_n then β all these things you can refer from this figure. So, find the factor of safety of the slope with suppose γ is equal to 18 kN/m^3, b is nothing but the mean for all the width of this slicer. b_n is 2 as well as b_1 b_2 b_3 all are 2 meters suppose. c' is 20 kN/m^2 and ϕ' is 20 degree, using ordinary method of slices we have to find out the factor of safety. The average height h_n and this length ΔL_n and angle of each slice are given in the table. So, that means h_n is the average height of each slice, α_n is this one. So, there are basically 7 slices, h_n are provided for 7 slices, α_n are provided in degree for 7 slices and ΔL_n is also provided for all these 7 slices also stated b_n is equal to 1 2 3 4 5 6 7 are 2 meters each.

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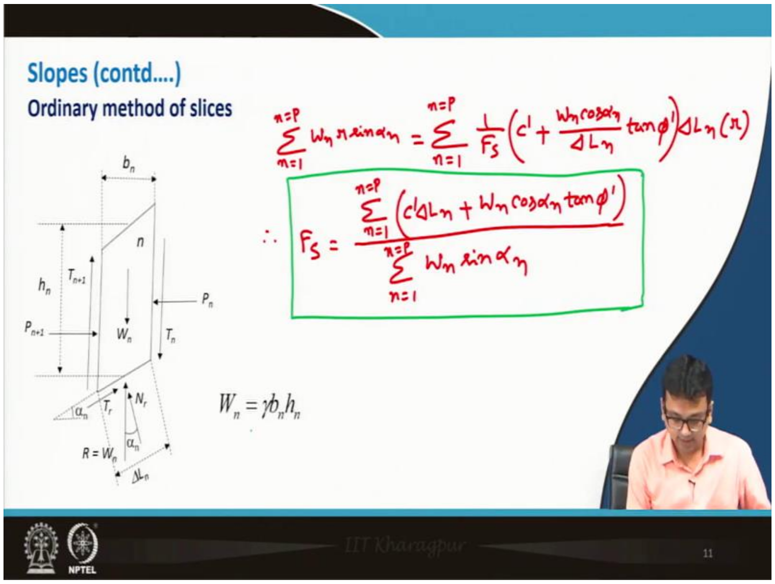

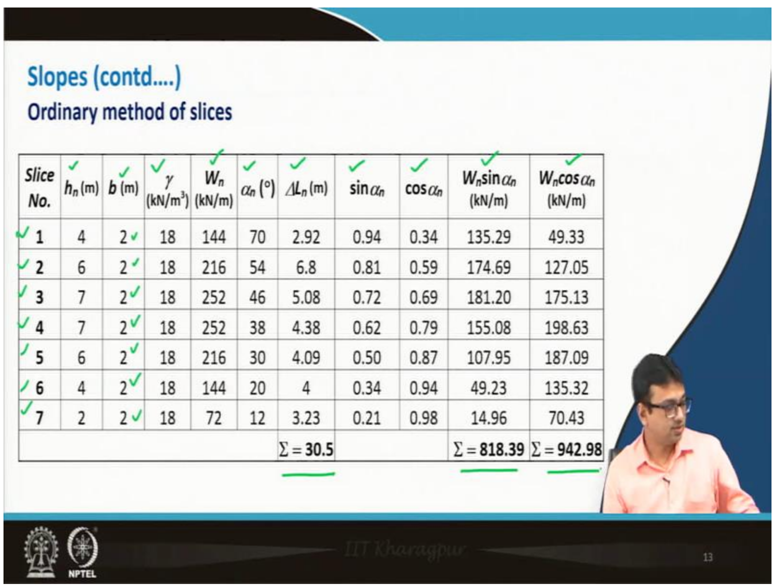

Slice No.	h_n (m)	b (m)	γ (kN/m³)	W_n (kN/m)	α_n (°)	ΔL_n (m)	$\sin\alpha_n$	$\cos\alpha_n$	$W_n\sin\alpha_n$ (kN/m)	$W_n\cos\alpha_n$ (kN/m)
1	4	2	18	144	70	2.92	0.94	0.34	135.29	49.33
2	6	2	18	216	54	6.8	0.81	0.59	174.69	127.05
3	7	2	18	252	46	5.08	0.72	0.69	181.20	175.13
4	7	2	18	252	38	4.38	0.62	0.79	155.08	198.63
5	6	2	18	216	30	4.09	0.50	0.87	107.95	187.09
6	4	2	18	144	20	4	0.34	0.94	49.23	135.32
7	2	2	18	72	12	3.23	0.21	0.98	14.96	70.43
						$\Sigma = 30.5$			$\Sigma = 818.39$	$\Sigma = 942.98$

Now, what we can do very easily we can make this type of table if you create your work will be very simple. So, you see 7 slices are there 1 2 3 4 5 6 7, now as given input h_n is written, now b you see all are 2 as it is written, given data γ is same, now using this we can very easily obtain in our W_n.

We know from equation

$$F_s = \frac{\sum_{n=1}^{n=p}\left(c' + \frac{W_n \cos\alpha_n}{\Delta L_n}\tan\phi'\right)\Delta L_n}{\sum_{n=1}^{n=p} W_n \sin\alpha_n}$$

So, we can obtain it very easily W_n is known, α_n known, cos α_n is known so W_n sin α_n and W_n cos α_n. So, now you see I need in that equation mainly del summation of ΔL_n is required and summation of W_n sin α_n is required and summation of W_n cos α_n is required.

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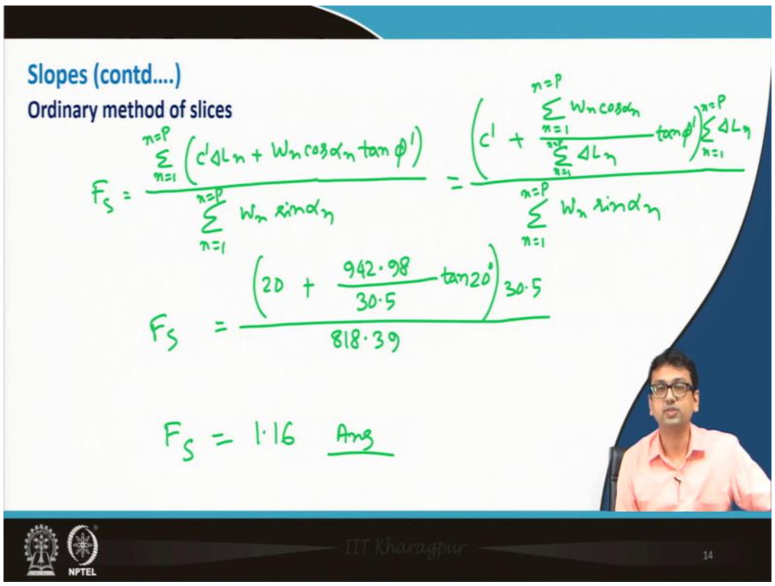

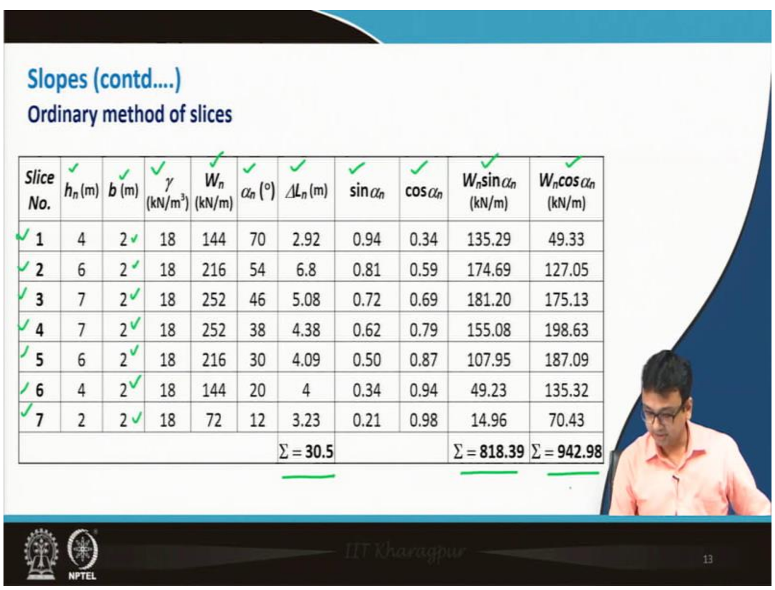

Slice No.	h_n (m)	b (m)	γ (kN/m^3)	W_n (kN/m)	α_n (°)	ΔL_n (m)	sin α_n	cos α_n	$W_n$sin α_n (kN/m)	$W_n$cos α_n (kN/m)
1	4	2	18	144	70	2.92	0.94	0.34	135.29	49.33
2	6	2	18	216	54	6.8	0.81	0.59	174.69	127.05
3	7	2	18	252	46	5.08	0.72	0.69	181.20	175.13
4	7	2	18	252	38	4.38	0.62	0.79	155.08	198.63
5	6	2	18	216	30	4.09	0.50	0.87	107.95	187.09
6	4	2	18	144	20	4	0.34	0.94	49.23	135.32
7	2	2	18	72	12	3.23	0.21	0.98	14.96	70.43
						$\Sigma = 30.5$			$\Sigma = 818.39$	$\Sigma = 942.98$

Using the calculation made in the table one can find the factor of safety or we can directly use the equation to find the F_s without using the table.

$$F_s = \frac{\left(20+\dfrac{942.98}{30.5}\tan 20\right)30.5}{818.39} = 1.16$$

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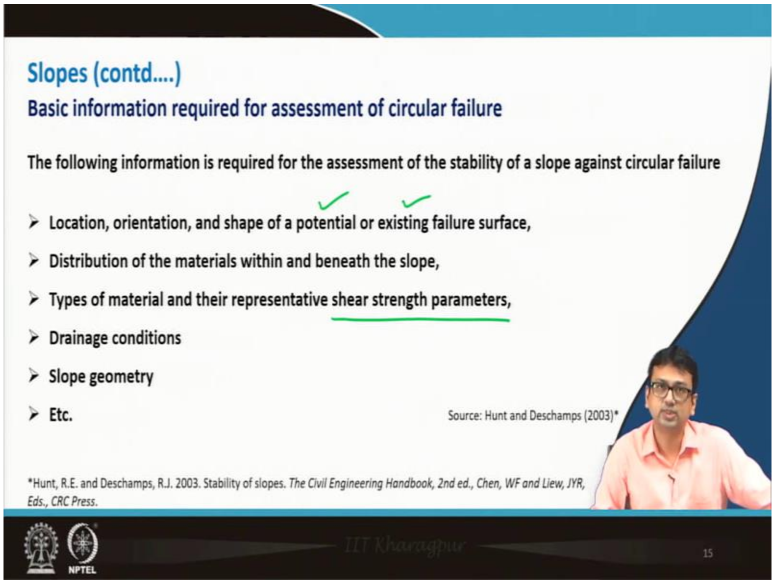

Now, just another basic thing that is basic information required for assessment of circular failure, some of the things are listed over here the following information is required for assessment of the stability of a slope against circular failure, what are they like location, orientation and shape of the potential or existing failure surface, that is very important.

Then distribution of the materials within and beneath the slow means, whether it is homogeneous low or non-homogeneous slope that is very important. Then types of material and their representatives shear strength parameters, so that is very important obviously, it will depend means very much on this c and ϕ parameter. Then drainage conditions obviously whether is developing or not that is also important for analyzing circular failure.

And slope geometry definitely that is also important what is the β angle whether it is very high or low all these things, so et cetera and several like the some of the things only I have listed over here, so these are the basic things I think we can discuss our circular failure but there are I means lot of things are there in this circular failure, but I hope you have learned other things, but these essential things I thought I will discuss in this course, so thank you.