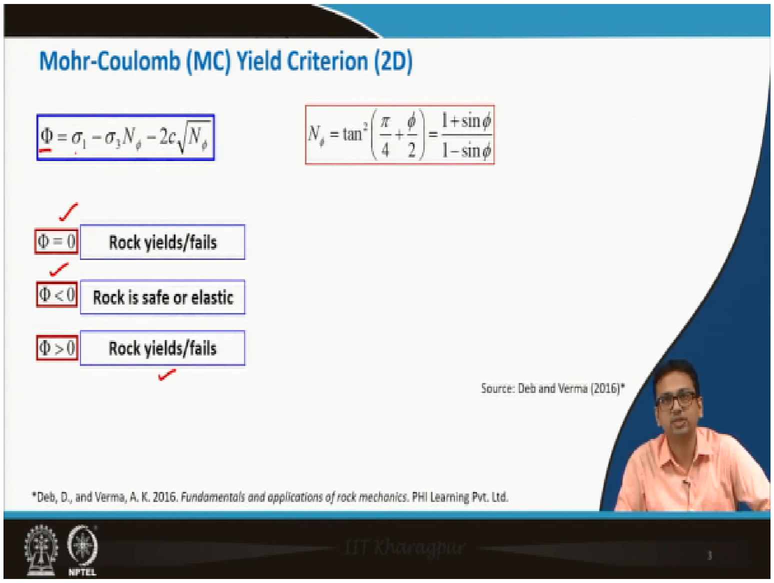
Rock Mechanics and Tunneling
Professor. Debarghya Chakraborty
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture No. 36
Failure Criterion in Deviatoric plane

Hello everyone, I welcome all of you to the fifth lecture of our module 7. So, in module 7 we are discussing about the rock and rock mass failure criteria and this is the last lecture of this module.

(Refer Slide Time: 0:48)

So, in this lecture we will mainly discuss about the failure criterion in Deviatoric plane and under that we will cover like Mohr-Coulomb Yield Criteria which we have discussed earlier in details but we will try to see it in deviatoric plane, then we will discuss about the Drucker-Prager Yield Criteria, then Hoek-Brown Yield Criteria as well as an Alternative Yield Criteria for Hoek-Brown Yield Criteria.

(Refer Slide Time: 1:25)



So, first you see the Mohr-Coulomb Yield criteria in 2D

$$\Phi = \sigma_1 - \sigma_3 N_\phi - 2c\sqrt{N_\phi}$$

$$N_\phi = \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) = \frac{1+\sin\phi}{1-\sin\phi}$$

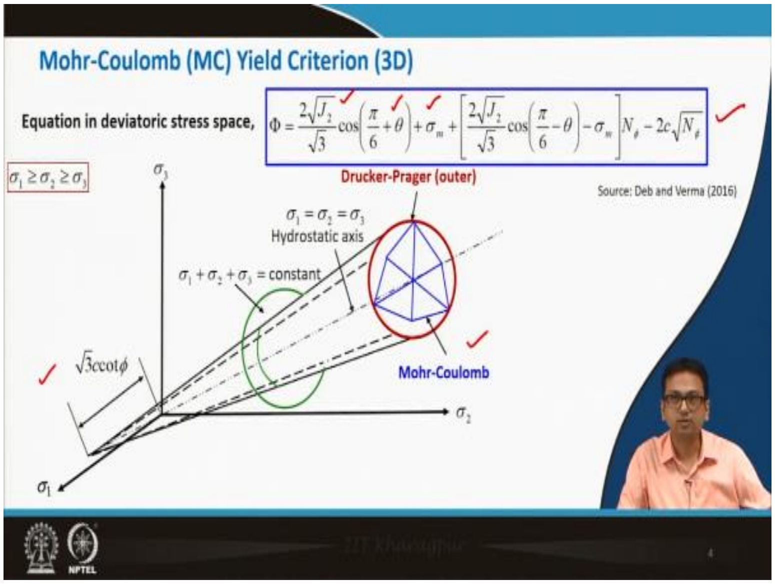
$\Phi = 0$ rock yields or fail

$\Phi > 0$ rock is safe or elastic

$\Phi < 0$ rock yields or fail

So, this is the Mohr-Coulomb yield criteria in 2D which is in the σ_1 and σ_3 plane i.e. principle stress plane and not in deviatoric stress plane.

(Refer Slide Time: 2:14)



Now equation in deviatoric stress space 3D

$$\Phi = \frac{2\sqrt{J_2}}{\sqrt{3}}\cos\left(\frac{\pi}{6}+\theta\right)+\sigma_m+\left[\frac{2\sqrt{J_2}}{\sqrt{3}}\cos\left(\frac{\pi}{6}-\theta\right)-\sigma_m\right]N_\phi-2c\sqrt{N_\phi}$$

So all the terms are basically known to us

σ_m is the mean stress

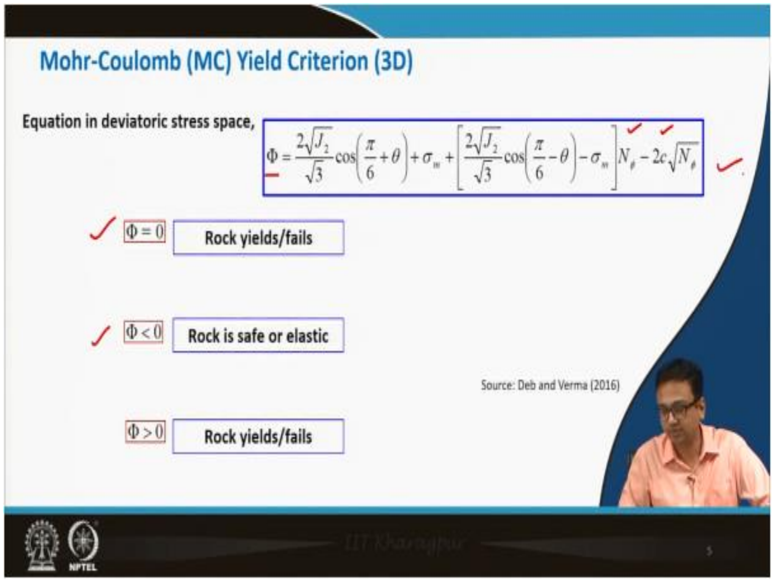
J_2 is the second invariant of the deviatoric stress

θ is the lode angle (you need to have knowledge of J_2 and J_3).

Now let us see in this 3D how it looks. So, there is this hydrostatic axis $\sigma_1 = \sigma_2 = \sigma_3$.

In σ_1, σ_2 and σ_3 space the Mohr-Coulomb yield criteria looks like an irregular hexagonal pyramid.

(Refer Slide Time: 4:47)



Now again it can be said that

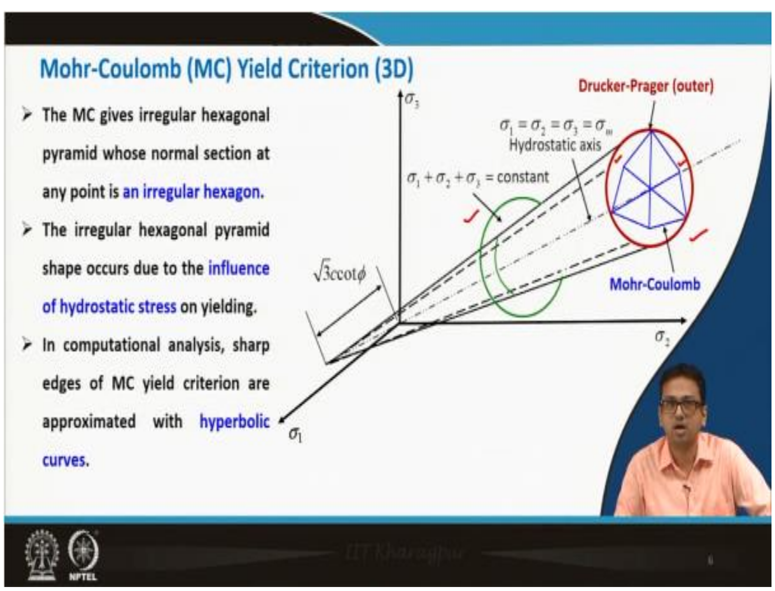
$\Phi = 0$ rock yields or fail

$\Phi > 0$ rock is safe or elastic

$\Phi < 0$ rock yields or fail

By giving the inputs in the above equation you can get the magnitude of Φ and you can know whether the rock is safe or will fail. So, it is whether it is yielding or failing or it is safe or elastic.

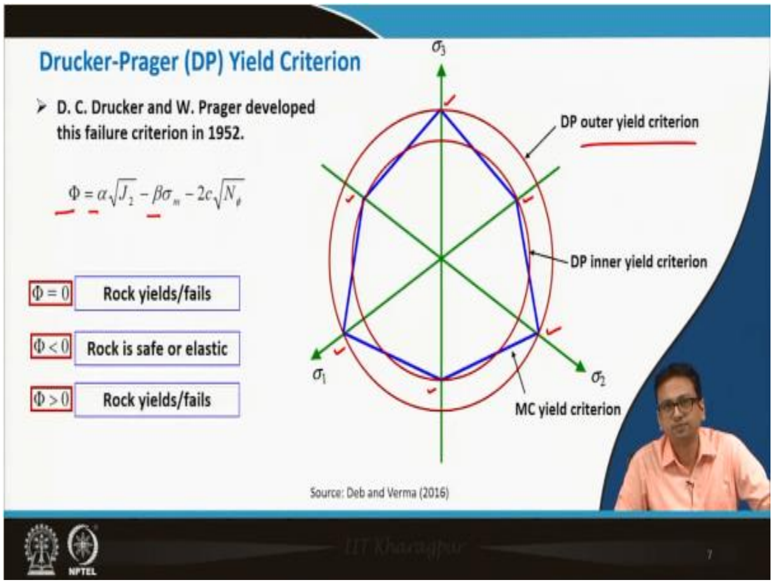
(Refer Slide Time: 6:42)



And again some of the things like as I have mentioned the Mohr-Coulomb gives irregular hexagonal pyramid whose normal section at any point is an irregular hexagon. The irregular hexagonal pyramid shape occurs due to the influence of hydrostatic stress on yielding.

Now the important thing is in computational analysis sharp edges of MC yield criterion creates problem so these edges are smoothed by using hyperbolic curve or the hyperbolic approximations are done for smoothly carrying out the computational analysis, so that is one of the disadvantage of this criteria.

(Refer Slide Time: 8:09)



Now, so in order to avoid that the problem of this sharp edges this you can utilize Drucker-Prager yield criteria. So, D. C. Drucker and W. Prager developed this failure criterion in 1952.

$$\Phi = \alpha\sqrt{J_2} - \beta\sigma_m - 2c\sqrt{N_\phi}$$

$\Phi = 0$ rock yields or fail

$\Phi > 0$ rock is safe or elastic

$\Phi < 0$ rock yields or fail

So, now if you see it in deviatoric stress plane or pi plane, it looks like a circle. There is DP outer yield criteria and DP inner yield criteria with respect to the Mohr-Coulomb criteria.

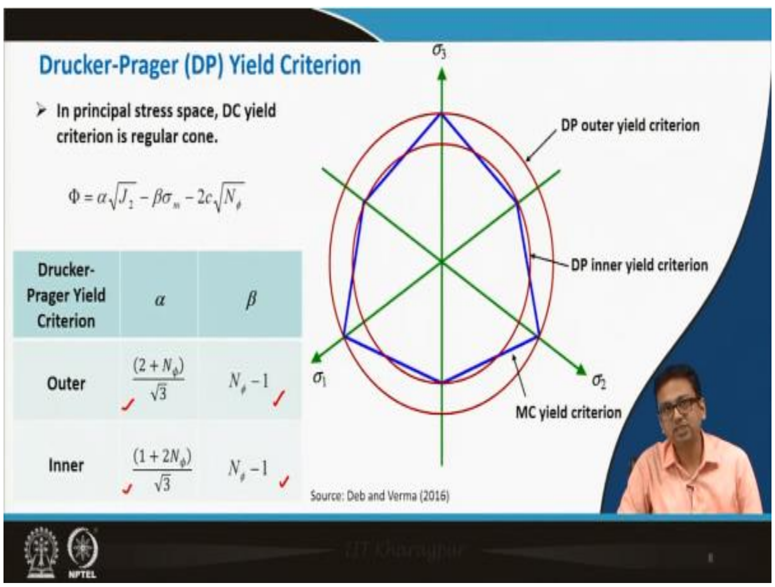
Basically, by connecting outer three apex you obtain your outer yield criteria by considering other three apexes you get inner yield criteria.

Drucker-Prager yield criterion eliminates the problem of sharp edges; in the pi plane it is a circle. So for computational requirement it is very good and we can very easily implement this.

Only problem is, if you consider outer it may overestimate the strength of your material and if you use the inner one it may underestimate the strength but still it is very popular. Like it is outer cone, inner cone, a compromise cone is also a concept which is in between the outer and the inner circle, this maintains a balance between this overestimation and underestimation.

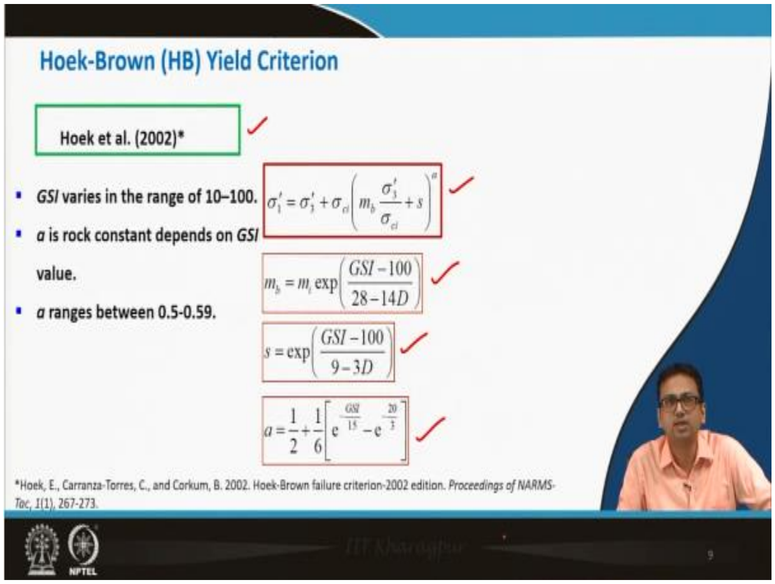
For outer and inner this α and β will actually change for which refer to the table in slide 8 of this lecture.

(Refer Slide Time: 11:52)



Basically β is same for both the cases and only α is changing.

(Refer Slide Time: 13:51)



Now again the very important yield criteria as I have mentioned for rock mass modeling is Hoek et al 2002.

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^a$$

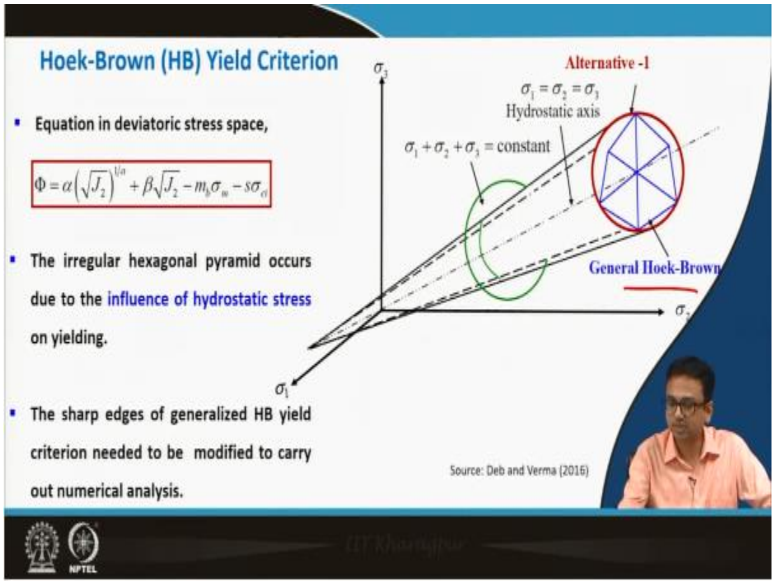
$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right)$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}}\right)$$

GSI varies in the range of generally 10 to 100, a is rock constant depends on GSI value, a ranges between 0.5 to 0.59.

(Refer Slide Time: 15:18)

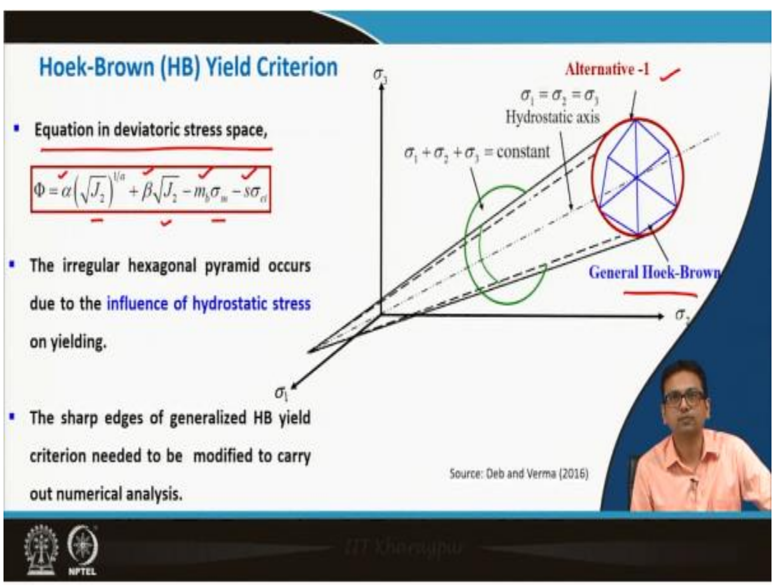


Now equation in deviatoric stress space,

$$\Phi=\alpha\left(\sqrt{J_2}\right)^{1/a}+\beta\sqrt{J_2}-m_b\sigma_m-s\sigma_{ci}$$

So, it looks similar to the Mohr-Coulomb yield criteria only you see, irregular hexagon in pi plane and in 3D it is irregular hexagonal pyramid.

(Refer Slide Time: 16:34)



As we saw in case of Mohr-Coulomb yield criterion problem was the sharp edges, same problem was noticed in case of Hoek-Brown yield criterion which imposes difficulty at the time of numerical analysis.

So, in order to avoid that difficulty in MC yield criterion we saw that Drucker-Prager outer cone and inner cone concept was introduced therefore, likewise in case of Hoek-Brown yield criterion alternative-1 and alternative-2 criteria were introduced. The alternative-1 is very much similar to the Drucker-Prager outer and alternative-2 is similar to the Drucker-Prager inner.

$$\Phi = \alpha\left(\sqrt{J_2}\right)^{1/a} + \beta\sqrt{J_2} - m_b\sigma_m - s\sigma_{ci}$$

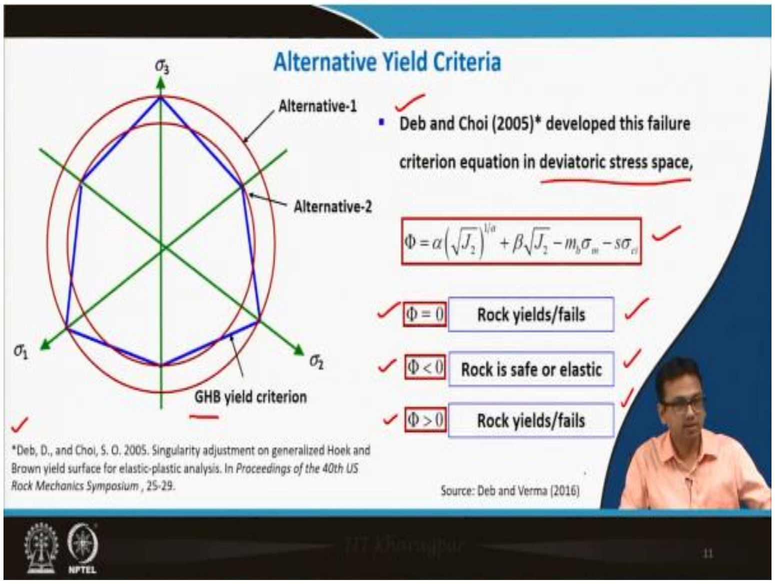
$\Phi = 0$ rock yields or fail

$\Phi > 0$ rock is safe or elastic

$\Phi < 0$ rock yields or fail

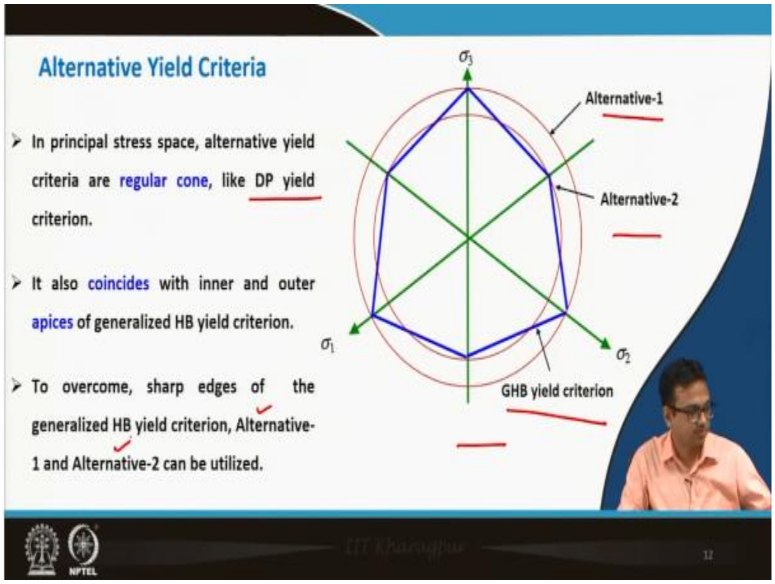
Now α and β for this Hoek-Brown yield criterion and as well as for the alternative-1 or alternative-2 is given in the table in slide number 12 of this lecture.

(Refer Slide Time: 19:40)



So, now this alternative 1 and 2 was developed by Deb and Choi in 2005 in the deviatoric stress space.

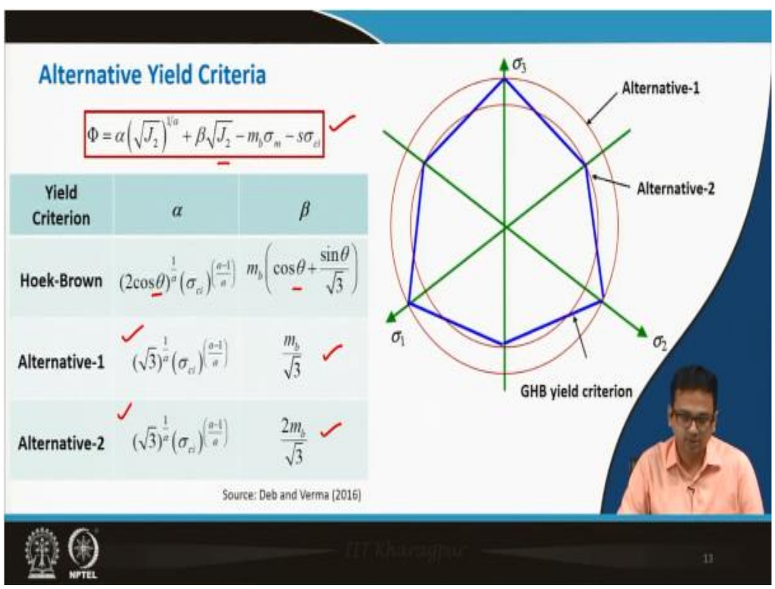
(Refer Slide Time: 21:12)



Now as we can see in principles stress space, alternative yield criteria are regular cone like Drucker-Prager yield criteria and in the pi plane we can see it is like a circle.

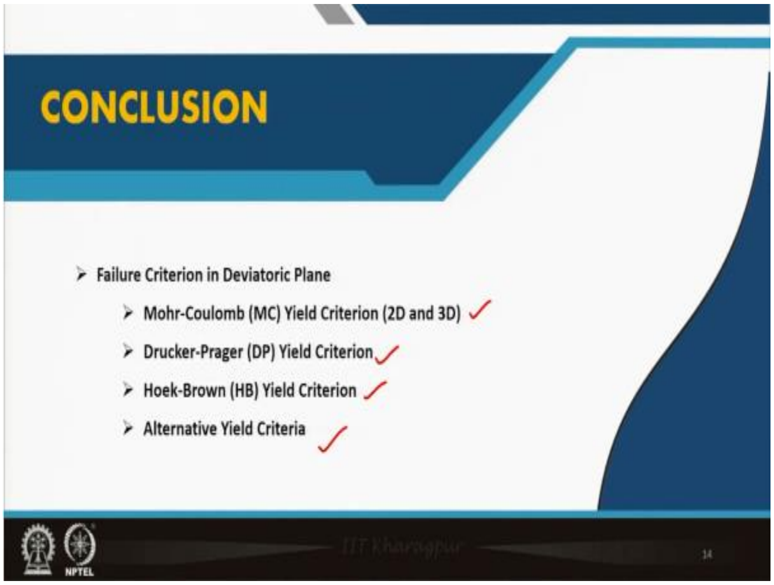
Now it also coincides with inner and outer apices of generalized Hoek-Brown yield criteria.

(Refer Slide Time: 22:11)



So, you are now in a position to find out the magnitude of Φ, based on that we will be in a position to comment that whether it is yielding or not yielding.

(Refer Slide Time: 23:46)



So, you see in our today's lecture we have basically discussed about a very important topic that is the failure criterion in deviatoric plane and its applicability is quite huge because as we have seen that these equations, means from there by simply obtaining the ultimate value we can easily say that at the given state of stress whether the particular rock sample will fail or not.

And we have discussed about different things like Mohr-Coulomb yield criteria, then Drucker-Prager yield criteria, Hoek-Brown yield criteria as well as alternative yield criteria and with this I am concluding my discussion related to this rock and rock mass failure criteria.

So, we have spent two weeks on this topic because it is very much important, fundamental and for future. Do not forget about this Hoek et al 2002 which gives the generalized Hoek-Brown yield criteria which is very much useful for rock mass modeling.

So, with this I am concluding this module, thank you. So, from our next module we will start discussing all the applications of this module’s knowledge in slope, underground excavation, foundations, tunnels etc., thank you.