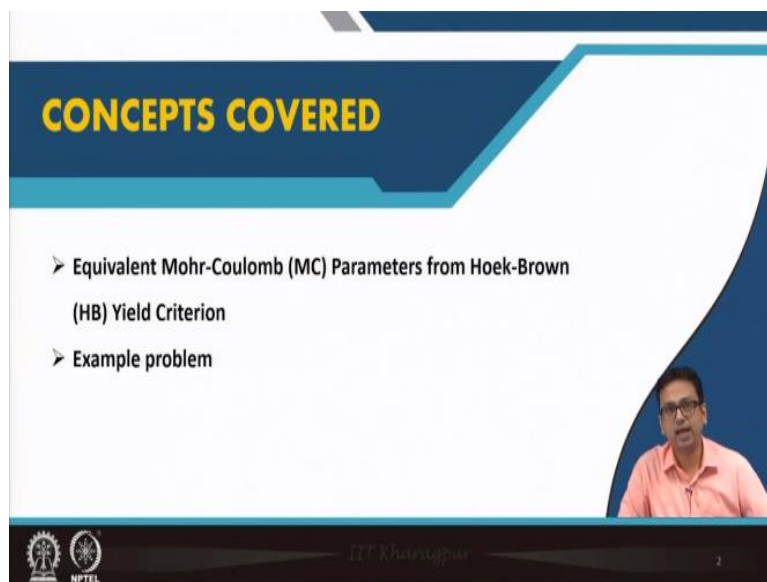


Rock Mechanics and Tunneling
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Lecture No. 35
Equivalent Mohr-Coulomb (MC) Parameters

Hello everyone, I welcome all of you to the fourth lecture of module 7. So, in module 7 we are discussing about the rock and rock mass failure criterion and today we will discuss about the equivalent Mohr-coulomb parameters.

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So, we will discuss about equivalent Mohr-coulomb parameters from Hoek-Brown yield criterion and see an example problem also.

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Equivalent Mohr-Coulomb (MC) Parameters from Hoek-Brown (HB) Yield Criterion

- In geotechnical engineering, the Mohr-Coulomb failure criterion is very much popular and there is a tendency to apply this criterion in rocks too.
- Therefore, it is necessary to determine equivalent angle of internal friction and cohesive strength of the rock mass.

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Mohr-coulomb failure criterion is quite popular and there is a tendency to apply this criterion in rock mass too. Though Hoek et al 2002 has been discussed thoroughly and that criterion definitely is appropriate for rock mass modeling but as geotechnical engineers like to use Mohr-coulomb criterion for modelling so there is a need of finding out the equivalent Mohr-coulomb parameters from the Hoek-Brown yield criterion.

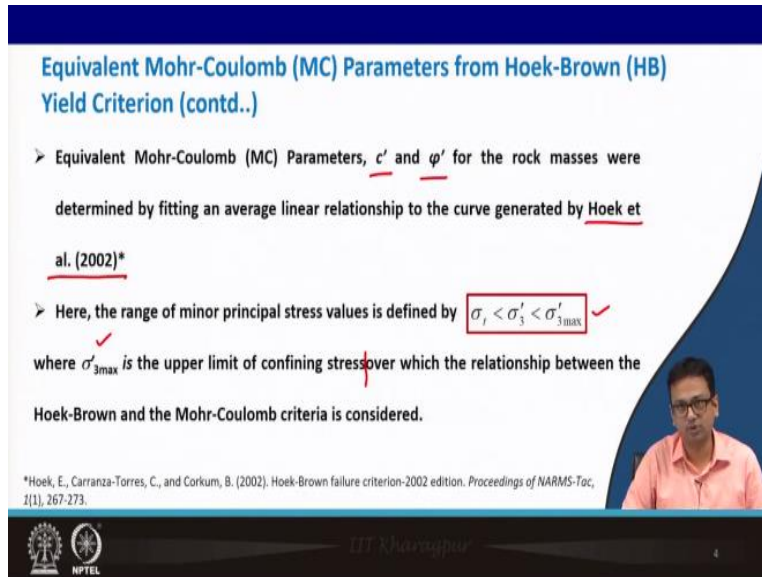
So, that is what it is written therefore it is necessary to determine the equivalent angle of internal friction and cohesive strength of the rock mass, so that is the reason.

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Equivalent Mohr-Coulomb (MC) Parameters from Hoek-Brown (HB) Yield Criterion (contd..)

- Equivalent Mohr-Coulomb (MC) Parameters, c' and ϕ' for the rock masses were determined by fitting an average linear relationship to the curve generated by Hoek et al. (2002)*
- Here, the range of minor principal stress values is defined by $\sigma_1 < \sigma_3' < \sigma_{3\max}'$ ✓
where $\sigma_{3\max}'$ is the upper limit of confining stress over which the relationship between the Hoek-Brown and the Mohr-Coulomb criteria is considered.

*Hoek, E., Carranza-Torres, C., and Corkum, B. (2002). Hoek-Brown failure criterion-2002 edition. *Proceedings of NARMS-Tac*, 2(1), 267-273.



Now we know the Mohr-coulomb parameters are internal friction and cohesive strength. So, the equivalent Mohr-coulomb parameters like c' and ϕ' for the rock masses were determined by fitting an average linear relationship to the curve generated by Hoek et al. 2002. So, in Hoek et al. 2002 paper authors have discussed about the failure criterion.

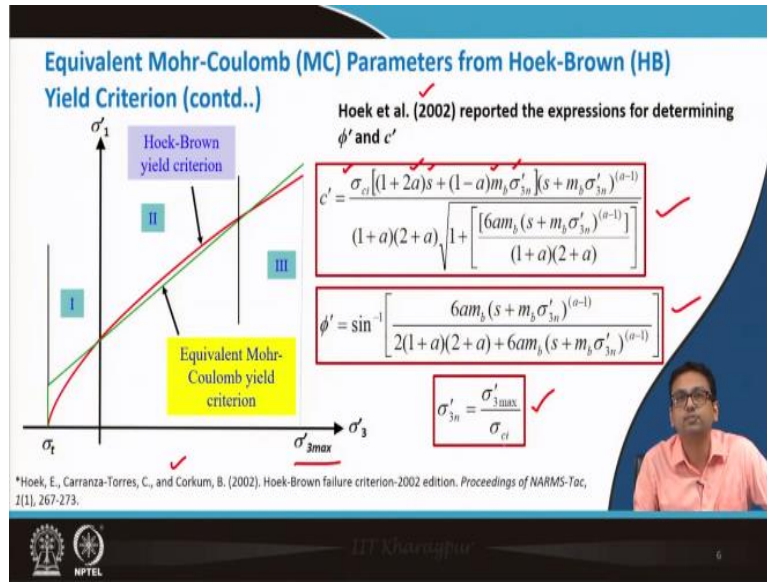
So, here the range of minor principal stress values is defined by

$$\sigma_1 < \sigma_3' < \sigma_{3\max}'$$

where $\sigma_{3\max}'$ is the upper limit of confining stress over which the relationship between the Hoek and Brown and the Mohr-coulomb criterion is considered.

Roughly we can say that the fitting process contains the balancing of the area above and below the Mohr-coulomb plot.

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So, in σ_1' and σ_3' plane average linear relationship curve was fitted to the curve generated by Hoek et al 2002, and equivalent Mohr Coulomb parameters c' and ϕ' for rock masses were determined.

Hoek Brown yield criterion

$$\sigma_1' = \sigma_3' + \sigma_{ci} \left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^a$$

Equivalent Mohr-Coulomb yield criterion

$$\sigma_1' = \frac{2c' \cos \phi'}{1 - \sin \phi'} + \frac{1 + \sin \phi'}{1 - \sin \phi'} \sigma_3'$$

In the plot we notice three zones. In zone 1, the green line which is indicating the equivalent Mohr-coulomb yield criterion is above the red line which is red is indicating the Hoek-Brown yield criterion, whereas in zone 2 the green line is below the red line and in the zone 3, again green line is above the red line. So, you can see it is balanced. I hope the idea of fitting the linear average curve is clear. Based on this Hoek et al 2002 reported the expressions for determining c' and ϕ' , in their paper.

$$c' = \frac{\sigma_{ci}[(1+2a)s + (1-a)m_b\sigma'_{3n}](s + m_b\sigma'_{3n})^{(a-1)}}{(1+a)(2+a)\sqrt{1 + \left[\frac{[6am_b(s + m_b\sigma'_{3n})^{(a-1)}]}{(1+a)(2+a)} \right]}}$$

$$\phi' = \sin^{-1} \left[\frac{6am_b(s + m_b\sigma'_{3n})^{(a-1)}}{2(1+a)(2+a) + 6am_b(s + m_b\sigma'_{3n})^{(a-1)}} \right]$$

$$\sigma'_{3n} = \frac{\sigma'_{3\max}}{\sigma_{ci}}$$

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Equivalent Mohr-Coulomb (MC) Parameters from Hoek-Brown (HB) Yield Criterion (contd..)

As per Hoek et al. (2002),

For deep tunnels or equivalent subsidence profiles for shallow tunnels,

$$\sigma'_{3\max} = 0.47\sigma_{cm}^{0.06}\sigma_{insitu}^{0.94}$$

For slopes,

$$\sigma'_{3\max} = 0.72\sigma_{cm}^{0.09}\sigma_{insitu}^{0.91}$$

$$\sigma'_{cm} = \sigma_{ci} \frac{\{m_b + 4s - a(m_b - 8s)\} \left(\frac{m_b}{4} + s \right)^{(a-1)}}{2(1+a)(2+a)}$$

$$\sigma_{insitu} = \gamma H$$

Now as per Hoek et al 2002 for deep tunnels or equivalent subsidence profiles for shallow tunnels,

$$\sigma'_{3\max} = 0.47\sigma_{cm}^{0.06}\sigma_{insitu}^{0.94}$$

Now similarly, for slopes also Hoek et al 2002 is providing another useful equation

$$\sigma'_{3\max} = 0.72\sigma_{cm}^{0.09}\sigma_{insitu}^{0.91}$$

$$\sigma'_{cm} = \sigma_{ci} \frac{\{m_b + 4s - a(m_b - 8s)\} \left(\frac{m_b}{4} + s \right)^{(a-1)}}{2(1+a)(2+a)}$$

$$\sigma_{insitu} = \gamma H$$

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Problem

The rock mass at 70 m depth for a hydroelectric powerhouse deep tunnel project in Sikkim, India, consists of average GSI of about 30. Triaxial tests on intact rock cores showed $\sigma_{ci} = 14$ MPa and $m_i = 20$. The unit weight of rock mass is found as 24 kN/m^3 and the disturbance factor, D is considered as 0. Estimate the equivalent cohesion and friction angle value using Hoek et al. (2002).

Solution: $\sigma_{ci} = 14$ MPa, $GSI = 30$, $m_i = 20$, $D = 0$, $\gamma = 24 \text{ kN/m}^3$ and $H = 70$ m

Now based on this let us try to solve one problem.

Problem 1: The rock mass of at 70 m depth for a hydroelectric powerhouse deep tunnel project in Sikkim, India consists of average GSI of about 30. Triaxial test on intact rock cores showed $\sigma_{ci} = 14$ MPa and $m_i = 20$. The unit weight of rock mass is found as 24 kN/m^3 and the disturbance factor D is considered as 0. Estimate the equivalent cohesion and friction angle value using Hoek et al 2002.

Solution:

Given $\sigma_{ci} = 14$ MPa, $GSI = 30$, $m_i = 20$, $D = 0$, $\gamma = 24 \text{ kN/m}^3$ and $H = 70$ m

$$m_b = m_i \exp\left(\frac{GSI-100}{28-14D}\right) = 20 \exp\left(\frac{30-100}{28-14 \times 0}\right) = 1.64$$

$$s = \exp\left(\frac{GSI-100}{9-3D}\right) = \exp\left(\frac{30-100}{9-3 \times 0}\right) = 0.000419$$

$$a = \frac{1}{2} + \frac{1}{6} \left(e^{\frac{GSI}{15}} - e^{\frac{20}{3}} \right) = \frac{1}{2} + \frac{1}{6} \left(e^{\frac{30}{15}} - e^{\frac{20}{3}} \right) = 0.522$$

$$\begin{aligned}
\sigma'_{cm} &= \sigma_{ci} \frac{\{m_b + 4s - a(m_b - 8s)\} \left(\frac{m_b}{4} + s \right)^{(a-1)}}{2(1+a)(2+a)} \\
&= 14000 \times \frac{\{1.64 + 4 \times 0.000419 - 0.522(1.64 - 8 \times 0.000419)\} \left(\frac{1.64}{4} + 0.000419 \right)^{(0.522-1)}}{2(1+0.522)(2+0.522)} \\
&= 2195.90 \text{ kPa}
\end{aligned}$$

$$\begin{aligned}
\sigma_{insitu} &= \gamma H \\
&= 24 \times 70 = 1680 \text{ kPa}
\end{aligned}$$

For deep tunnels

$$\begin{aligned}
\sigma'_{3\max} &= 0.47 \sigma_{cm}'^{0.06} \sigma_{insitu}'^{0.94} \\
&= 0.47 \times (2195.90)^{0.06} \times (1680)^{0.94} \\
&= 802.39 \text{ kPa}
\end{aligned}$$

$$\begin{aligned}
\sigma'_{3n} &= \frac{\sigma'_{3\max}}{\sigma_{ci}} \\
&= \frac{802.39}{14000} \\
&= 0.0573 \text{ kPa}
\end{aligned}$$

The equivalent cohesion

$$c' = \frac{\sigma_{ci} [(1+2a)s + (1-a)m_b \sigma'_{3n}] (s + m_b \sigma'_{3n})^{(a-1)}}{(1+a)(2+a) \sqrt{1 + \left[\frac{[6am_b (s + m_b \sigma'_{3n})^{(a-1)}]}{(1+a)(2+a)} \right]}}$$

$$= \frac{14000 \times [(1 + 2 \times 0.522) \times 0.000419 + (1 - 0.522) \times 1.64 \times 0.0573] \times (0.000419 + 1.64 \times 0.0573)^{(0.522-1)}}{(1 + 0.522)(2 + 0.522) \times \sqrt{1 + \left\{ \frac{6 \times 0.522 \times 1.64 \times (0.000419 + 1.64 \times 0.0573)^{(0.522-1)}}{(1 + 0.522) \times (2 + 0.522)} \right\}}}$$

$$= 227.67 \text{ kPa}$$

The equivalent friction angle

$$\phi' = \sin^{-1} \left[\frac{6am_b(s + m_b\sigma'_{3n})^{(a-1)}}{2(1+a)(2+a) + 6am_b(s + m_b\sigma'_{3n})^{(a-1)}} \right]$$

$$= \sin^{-1} \left[\frac{6 \times 0.522 \times 1.64 \times (0.000419 + 1.64 \times 0.0573)^{(0.522-1)}}{2 \times (1 + 0.522)(2 + 0.522) + 6 \times 0.522 \times 1.64 \times (0.000419 + 1.64 \times 0.0573)^{(0.522-1)}} \right]$$

$$= 42.376^\circ$$

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Solution:

$$m_b = m_i \exp \left(\frac{GSI - 100}{28 - 14D} \right) = 20 \exp \left(\frac{30 - 100}{28 - (14 \times 0)} \right) = 1.64$$

$$S = \exp \left(\frac{GSI - 100}{9 - 3D} \right) = \exp \left(\frac{30 - 100}{9 - (3 \times 0)} \right) = 0.000419$$

$$a = \frac{1}{2} + \frac{1}{6} \left(e^{-GSI/15} - e^{-20/3} \right)$$

$$= \frac{1}{2} + \frac{1}{6} \left(e^{-30/15} - e^{-20/3} \right)$$

$$= 0.522$$


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Solution:


$$\sigma'_{cm} = \sigma_{ci} \frac{\{m_b + 4s - a(m_b - 8s)\} \left(\frac{m_b}{4} + s \right)^{(a-1)}}{2(1+a)(2+a)}$$

$$= 14000 \times \frac{\{1.64 + 4 \times 0.000419 - 0.522(1.64 - 8 \times 0.000419)\} \left(\frac{1.64}{4} + 0.000419 \right)^{(0.522-1)}}{2(1+0.522)(2+0.522)}$$

$$= 2195.90 \text{ kPa}$$

$$\sigma_{min} = \gamma H$$

$$= 24 \times 70$$

$$= 1680 \text{ kPa}$$


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Solution:

For deep tunnels,


$$\sigma'_{3max} = 0.47 \sigma_{cm}^{0.66} \sigma_{min}^{0.94}$$

$$= 0.47 \times (2195.90)^{0.66} \times (1680)^{0.94}$$

$$= 802.39 \text{ kPa}$$

$$\sigma'_{3D} = \frac{\sigma'_{3max}}{\sigma_{ci}}$$

$$= \frac{802.39}{14000}$$

$$= 0.0573 \text{ kPa}$$


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Solution:

The equivalent cohesion ,

$$c' = \frac{\sigma_{ci}[(1+2a)s + (1-a)m_b\sigma'_{3n}]\{s + m_b\sigma'_{3n}\}^{(a-1)}}{(1+a)(2+a)\sqrt{1 + \left[\frac{6am_b(s + m_b\sigma'_{3n})^{(a-1)}}{(1+a)(2+a)} \right]}}$$

$$= \frac{14000 \times [(1+2 \times 0.522) \times 0.000419 + (1-0.522) \times 1.64 \times 0.0573] \times (0.000419 + 1.64 \times 0.0573)^{(0.522-1)}}{(1+0.522)(2+0.522) \times \sqrt{1 + \left[\frac{6 \times 0.522 \times 1.64 \times (0.000419 + 1.64 \times 0.0573)^{(0.522-1)}}{(1+0.522) \times (2+0.522)} \right]}}$$

$$= 227.67 \text{ kPa}$$


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Solution:

The equivalent friction angle,


$$\phi' = \sin^{-1} \left[\frac{6am_b(s + m_b\sigma'_{3n})^{(a-1)}}{2(1+a)(2+a) + 6am_b(s + m_b\sigma'_{3n})^{(a-1)}} \right]$$

$$= \sin^{-1} \left[\frac{6 \times 0.522 \times 1.64 \times (0.000419 + 1.64 \times 0.0573)^{(0.522-1)}}{2 \times (1+0.522)(2+0.522) + 6 \times 0.522 \times 1.64 \times (0.000419 + 1.64 \times 0.0573)^{(0.522-1)}} \right]$$

$$= 42.376^\circ$$



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CONCLUSION

- Equivalent Mohr-Coulomb (MC) Parameters from Hoek-Brown (HB) Yield Criterion
- Example problem



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So, basically today in our today's lecture we have learnt about equivalent Mohr-coulomb parameters from Hoek-Brown yield criterion which is quite important. It is seen that the geotechnical engineers, instead of Hoek-Brown may be comfortable with more Mohr-coulomb criterion because it is very simple yield criterion.

And Mohr-coulomb criterion is being used for many years now, so it makes it comfortable for geotechnical engineers to rely more on it. So, obviously we have learnt about the Hoek-Brown criterion along with Mohr-Coulomb criterion. So thank you, let us conclude here.