

Rock Mechanics and Tunneling
Professor. Debarghya Chakraborty
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture No. 34
Empirical Failure Criteria for Rock (Continued)

Hello everyone, I welcome all of you to the third lecture of module 7. So, in module 7 we are discussing about the rock and rock mass failure criteria. In our previous lecture we have started discussing about different empirical failure criteria for rock and we will continue that discussion today also.

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek and Brown (1980)*

- Developed $\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^2)^{0.5}$ ✓
- where m and s are constants depend on the properties of the rock.
- σ_c is the uniaxial compressive strength of the intact rock. ✓
- When, $\sigma_3 = 0$ the uniaxial compressive strength of the rock is obtained as $\sigma_{cs} = \sqrt{s\sigma_c^2}$ ✓
- For intact rock, $s = 1$ $\sigma_{cs} = \sigma_c$

*Hoek, E. and Brown, E.T., 1980. Empirical strength criterion for rock masses. *Journal of the Geotechnical Engineering Division*, 106(9), pp.1013-1035.




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Hoek and Brown (1980)

developed $\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^2)^{0.5}$

where σ_1 is major principle stress, σ_3 is minor principle stress, m and s are the constants that depend on the properties of rock. σ_c is the uniaxial compressive strength of intact rock. Now when $\sigma_3 = 0$, the uniaxial compressive strength of the rock mark is obtained as $\sigma_{cs} = \sqrt{s\sigma_c^2}$. Now for intact rock $s = 1$, so $\sigma_{cs} = \sigma_c$.

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek and Brown (1980)

- When $\sigma_1 = 0$
- The uniaxial tensile strength of the rock is obtained as $\sigma_s = \sigma_t$ ✓

✓ $0 = \sigma_t + (m\sigma_c\sigma_t + s\sigma_c^2)^{0.5}$

$\sigma_t^2 = m\sigma_c\sigma_t + s\sigma_c^2$

$\sigma_t^2 - m\sigma_c\sigma_t - s\sigma_c^2 = 0$

$\sigma_t = \frac{m\sigma_c \pm \sqrt{(m\sigma_c)^2 + 4s\sigma_c^2}}{2}$


$\sigma_t = \frac{m\sigma_c \pm \sigma_c \sqrt{m^2 + 4s}}{2}$ ✓


$\sigma_t = \frac{\sigma_c}{2} (m \pm \sqrt{m^2 + 4s})$

$\sigma_t = \frac{\sigma_c}{2} (m - \sqrt{m^2 + 4s})$ ✓

Source: Hoek and Brown (1980)

➤ Since $m > 1$, neglecting the equation having '-' sign




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Now for further discussion when $\sigma_1 = 0$, the uniaxial tensile strength of the rock is obtained as $\sigma_3 = \sigma_t$ or by further simplifying the previous equation

$$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^2)^{0.5}$$

$$0 = \sigma_t + (m\sigma_c\sigma_t + s\sigma_c^2)^{0.5}$$

$$\sigma_t^2 = m\sigma_c\sigma_t + s\sigma_c^2$$

$$\sigma_t^2 - m\sigma_c\sigma_t - s\sigma_c^2 = 0$$

$$\sigma_t = \frac{m\sigma_c \pm \sigma_c \sqrt{m^2 + 4s}}{2}$$

$$\sigma_t = \frac{\sigma_c}{2} (m \pm \sqrt{m^2 + 4s})$$

Now since m is greater than 1, (as suggested by Hoek and Brown)

$$\sigma_t = \frac{\sigma_c}{2} (m - \sqrt{m^2 + 4s})$$

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek and Brown (1980)

Rock type	m
Limestone	5.4 ✓
Dolomite	6.8
Mudstone	7.3
Marble	10.6 ✓
Sandstone	14.3
Dolerite	15.2
Quartzite	16.8 ✓

Rock type	m
Chert	20.3
Norite	23.2
Quartz-diorite	23.4
Gabbro	23.9
Gneiss	24.5
Amphibolite	25.1
Granite	27.9 ✓

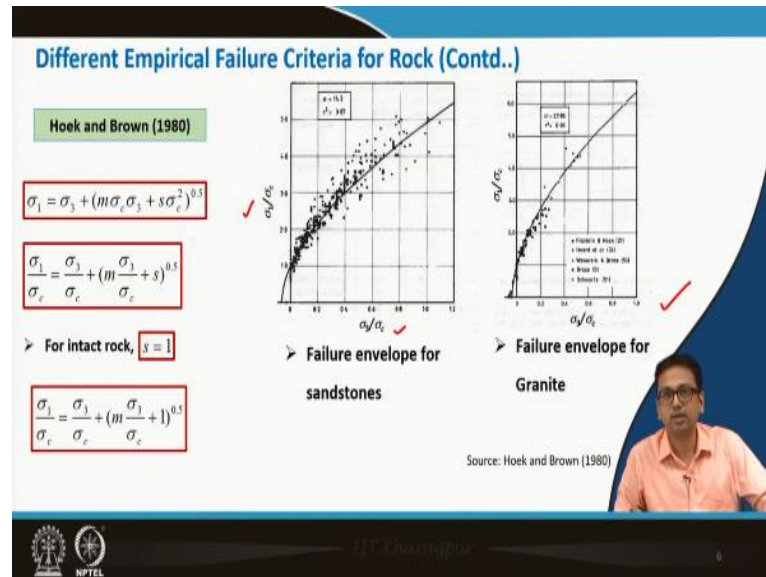
Source: Hoek and Brown (1980)

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Dr. Khosroghani

Refer to the tables given in slide 6 of this lecture for m values of different types of rock so by giving these as the input you can utilize the equation of Hoek and Brown for any numerical calculation.

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Now by further playing with the following equation

$$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^2)^{0.5}$$

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \left(m\frac{\sigma_3}{\sigma_c} + s\right)^{0.5}$$

For intact rock $s = 1$

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \left(m\frac{\sigma_3}{\sigma_c} + 1\right)^{0.5}$$

The above equation can be used to fit the failure envelope plot for different rocks. Refer to the plots in slide 6 and you can see that the fit is quite good.

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Different Empirical Failure Criterion for Rock (Contd..)


Yudhbir et al. (1983)*

- Following the Bieniawski (1974), this criterion was developed, which is applicable for the intact as well as highly jointed soft rock.

$$\frac{\sigma_1}{\sigma_c} = \beta + A \left(\frac{\sigma_3}{\sigma_c} \right)^k$$

- The magnitude of constant k is suggested to be 0.65.
- $\beta = 1$, for intact rock
- $\beta = 0$, for totally disintegrated rock
- A value can be considered for Bieniawski (1974) for soft rocks.

*Yudhbir, Y., Lemanza, W. and Prindl, F., 1983, January. An empirical failure criterion for rock masses. 5th ISRM Congress. International Society for Rock Mechanics and Rock Engineering.



Yudhbir et al. (1983)

Following Bieniawski (1974), this criterion was developed, which is applicable for the intact as well as highly jointed soft rock.

$$\frac{\sigma_1}{\sigma_c} = \beta + A \left(\frac{\sigma_3}{\sigma_c} \right)^k$$

the magnitude of the constant k is suggested to be 0.65.

So, $\beta = 1$ for intact rock and $\beta = 0$ for totally disintegrated rock. A value can be considered, from Bieniawski 1974 for soft rock.

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Different Empirical Failure Criteria for Rock (Contd..)

Sheorey et al. (1989)* ✓

➤ This criterion is applicable to **intact** as well as **highly jointed** coal.

$$\sigma_1 = \sigma_c \left(1 + \frac{\sigma_3}{\sigma_t} \right)^b$$

$$\sigma_{cn} = \frac{\sigma_{cj}}{\sigma_c} \quad \sigma_{tn} = \frac{\sigma_{tj}}{\sigma_c}$$

$$\sigma_1 = \sigma_{cn} \sigma_c \left(1 + \frac{\sigma_3}{\sigma_{tn} \sigma_c} \right)^b$$

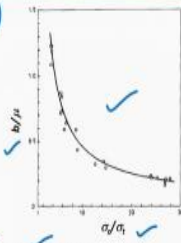

➤ b value can be estimated

$$\frac{b}{\mu} = 2.6 \left(\frac{\sigma_c}{\sigma_t} - 1 \right)^{-0.8}$$

➤ Where μ is coefficient of friction $\mu = \tan \phi$ ✓

➤ σ_{cj} and σ_{tj} are the jointed uniaxial compressive and tensile strengths.

*Sheorey, P.R., Biswas, A. and Choubey, V.D., 1989. An empirical failure criterion for rocks and jointed rock masses. *Engineering Geology*, 26(2), 141-159.

Sheorey et al (1989)

This criterion is applicable to intact as well as highly jointed coal.

For intact coal

$$\sigma_1 = \sigma_c \left(1 + \frac{\sigma_3}{\sigma_t} \right)^b$$

For highly jointed coal

$$\sigma_1 = \sigma_{cn} \sigma_c \left(1 + \frac{\sigma_3}{\sigma_{tn} \sigma_c} \right)^b$$

$$\sigma_{cn} = \frac{\sigma_{cj}}{\sigma_c} \quad \text{and} \quad \sigma_{tn} = \frac{\sigma_{tj}}{\sigma_c}$$

where σ_{cj} and σ_{tj} are the jointed uniaxial compressive and tensile strength of intact rock.

b value can be estimated from the equation provided by Sheorey et al. 1989,

$$\frac{b}{\mu} = 2.6 \left(\frac{\sigma_c}{\sigma_t} - 1 \right)^{-0.8}$$

Refer to the plot in slide 8.

where μ is the coefficient of friction, $\mu = \tan \phi$.

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek and Brown (1997)*

➤ The Generalised Hoek-Brown failure criterion for jointed rock masses is defined by:

$$\sigma_1' = \sigma_3' + \sigma_{ci} \left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^n$$

➤ where σ_1' and σ_3' are the maximum and minimum effective stresses at failure, respectively.

➤ σ_{ci} is the uniaxial compressive strength of the intact rock

➤ m_b is the modified value of the Hoek-Brown constant m_i for the rock mass.

*Hoek, E. and Brown, E.T., 1997. Practical estimates of rock mass strength. *International Journal of Rock Mechanics and Mining Sciences*, 34(8), 1165-1186.

Hoek and Brown (1997)

The generalized Hoek Brown failure criterion for jointed rock masses is defined by the following equation

$$\sigma_1' = \sigma_3' + \sigma_{ci} \left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^n$$

where σ_1' and σ_3' are the maximum and minimum effective stresses at failure, σ_{ci} is the uniaxial compressive strength of the intact rock, m_b is the modified value of Hoek Brown constant m_i for the rock mass.

$$m_b = m_i \exp \left(\frac{GSI - 100}{28} \right)$$

value of the constant m_i can be obtained from the table (refer to slide number 10).

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek and Brown (1997)

$$m_b = m_i \exp\left(\frac{GSI - 100}{28}\right)$$

Values of the constant m_i for intact rock can be obtained from the Table.

Rock type	Class	Group	Crystalline	Metamorphic	Sedimentary	Volcanic	Key site
SEDIMENTARY	Clastic	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
	New Clastic	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
METAMORPHIC	Slightly foliated	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
	Foliated?	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
VOLCANIC	Light	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
	Dark	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
EXTRUSIVE (igneous) type	Aphanitic	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
	Phanitic	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓

Source: Hoek and Brown (1997)

GSI value can be found from the table given by Hoek Brown 1997 based on structure and surface condition.

Later Hoek and Brown suggested that if $GSI > 25$, $GSI = RMR_{89} - 5$.

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Different Empirical Failure Criterion for Rock (Contd..)

Hoek and Brown (1997)

$$m_b = m_i \exp\left(\frac{GSI - 100}{28}\right)$$

$$GSI = RMR_{89} - 5 \quad \text{if } GSI > 25$$

Rock type	Class	Group	Crystalline	Metamorphic	Sedimentary	Volcanic	Key site
SEDIMENTARY	Clastic	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
	New Clastic	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
METAMORPHIC	Slightly foliated	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
	Foliated?	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
VOLCANIC	Light	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
	Dark	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
EXTRUSIVE (igneous) type	Aphanitic	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓
	Phanitic	✓	✓	✓	✓	✓	✓
		✓	✓	✓	✓	✓	✓

Source: Hoek and Brown (1997)

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Different Empirical Failure Criteria for Rock (Contd..) Source: Hoek and Brown (1997)

Hoek and Brown (1997)



➤ s and a are constants which depend upon the characteristics of the rock mass.

✓ $s = \exp\left(\frac{GSI - 100}{9}\right)$ if $GSI > 25$

$s = 0$ if $GSI < 25$

$a = 0.5$ if $GSI > 25$ ✓

$a = 0.65 - \frac{GSI}{200}$ if $GSI < 25$ ✓

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Now as per this Hoek and Brown there are other two parameters s and a .

$$s = \exp\left(\frac{GSI - 100}{9}\right) \quad \text{if } GSI > 25$$

$$s = 0 \quad \text{if } GSI < 25$$

$$s = \exp\left(\frac{GSI - 100}{9}\right) \quad \text{if } GSI > 25$$

$$a = 0.65 - \frac{GSI}{200} \quad \text{if } GSI < 25$$

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek et al. (2002)*

➤ The Generalised Hoek-Brown failure criterion for jointed rock masses is defined by:

$$\sigma_1' = \sigma_3' + \sigma_{ci} \left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^a$$

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right)$$

$$a = \frac{1}{2} + \frac{1}{6} \left(e^{\frac{GSI}{15}} - e^{\frac{20}{3}} \right)$$

➤ D is the disturbance factor

*Hoek, E., Carranza-Torres, C. and Corkum, B., 2002. Hoek-Brown failure criterion-2002 edition. Proceedings of NARMS-Tac, 1(1), 267-273.

Hoek et al 2002, is the most widely used criterion as it takes into account almost all the flaws that were there in the previous version. So, the generalized Hoek Brown failure criteria for jointed rock masses is defined by

$$\sigma_1' = \sigma_3' + \sigma_{ci} \left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^a$$

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad a = \frac{1}{2} + \frac{1}{6} \left(e^{\frac{GSI}{15}} - e^{\frac{20}{3}} \right)$$

The above equations are quite similar to Hoek and Brown 1997 but with some modifications. Authors have introduced a new factor, D . D is the disturbance factor.

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek et al. (2002)

➤ m_i value can be obtained from Marinos and Hoek (2000)**.

**Marinos, P. and Hoek, E., 2000. GSI: a geologically friendly tool for rock mass strength estimation. In ISRM International symposium. International Society for Rock Mechanics and Rock Engineering.

Rock type	Class	Group	Grain size			
			Coarse	Medium	Fine	Very fine
Sedimentary	Clastic	Conglomerates	7 < 4	4 < 3	3 < 2	2 < 1
		Breccias	7 < 4	4 < 3	3 < 2	2 < 1
Sedimentary	Non-clastic	Carbonates	7 < 4	4 < 3	3 < 2	2 < 1
		Evaporites	7 < 4	4 < 3	3 < 2	2 < 1
Metamorphic	Foliated	Metapelite	7 < 4	4 < 3	3 < 2	2 < 1
		Metagranite	7 < 4	4 < 3	3 < 2	2 < 1
Igneous	Foliated**	Granite	7 < 4	4 < 3	3 < 2	2 < 1
		Diorite	7 < 4	4 < 3	3 < 2	2 < 1
Igneous	Non-foliated	Granite	7 < 4	4 < 3	3 < 2	2 < 1
		Diorite	7 < 4	4 < 3	3 < 2	2 < 1

*Conglomerates and breccias may present a wide range of m_i values depending on the nature of the cementing material and the degree of cementation

Now for finding out m_i , in 1997, a table was given different types of rock like sedimentary, metamorphic, igneous but again Marinos and Hoek in 2000 modified it and they have given a new table which is a little different from the previous one. In this table for different rocks m_i can fall in a range.

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek et al. (2002)

➤ GSI value can be obtained from Hoek and Marinos (2000)*.

*Hoek, E. and Marinos, P. (2000). Predicting tunnel squeezing. *Tunnels and Tunneling International*, Part 1, 32/11, pp. 45-51, November 2000; Part 2, 32/12, pp. 33-36, December 2000.

**Hoek, E., and Brown, E. T. 2019. The Hoek-Brown failure criterion and GSI - 2018 edition, *Journal of Rock Mechanics and Geotechnical Engineering*, 11(3), 445-463.

Rock type	Class	Group	Grain size			
			Coarse	Medium	Fine	Very fine
Sedimentary	Clastic	Conglomerates	7 < 4	4 < 3	3 < 2	2 < 1
		Breccias	7 < 4	4 < 3	3 < 2	2 < 1
Sedimentary	Non-clastic	Carbonates	7 < 4	4 < 3	3 < 2	2 < 1
		Evaporites	7 < 4	4 < 3	3 < 2	2 < 1
Metamorphic	Foliated**	Metapelite	7 < 4	4 < 3	3 < 2	2 < 1
		Metagranite	7 < 4	4 < 3	3 < 2	2 < 1
Igneous	Foliated**	Granite	7 < 4	4 < 3	3 < 2	2 < 1
		Diorite	7 < 4	4 < 3	3 < 2	2 < 1
Igneous	Non-foliated	Granite	7 < 4	4 < 3	3 < 2	2 < 1
		Diorite	7 < 4	4 < 3	3 < 2	2 < 1

*Hoek and Brown (2019)**

Now for the GSI value again Hoek and Marinos 2000 gave another table (refer to the slides)

Hoek et al 2002 says that now you do not have to use RMR table, you can directly find out your GSI using this table and utilize that for obtaining your required parameters like m_b , s and a .


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Different Empirical Failure Criteria for Rock (Contd..)

Source: Hoek et al. (2002)

Hoek et al. (2002)

Disturbance factor due to blasting, D can be determined from the guideline given in Hoek et al. (2002).

Appearance of rock mass	Description of rock mass	Suggested value of D
	Excellent quality controlled blasting or excavation by Tunnel Boring Machine results in minimal disturbance to the confined rock mass surrounding a tunnel.	$D = 0$

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So, now regarding D , the disturbance factor due to blasting can be determined from the guideline given in Hoek et al 2002. In their paper you will find pictures and description of rocks and D value corresponding to that.

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Different Empirical Failure Criteria for Rock (Contd..)

Source: Hoek et al. (2002)

Hoek et al. (2002)

Appearance of rock mass	Description of rock mass	Suggested value of D
	Mechanical or hand excavation in poor quality rock masses (no blasting) results in minimal disturbance to the surrounding rock mass. Where squeezing problems result in significant floor heave, disturbance can be severe unless a temporary invert, as shown in the photograph, is placed.	$D = 0$ ✓ $D = 0.5$ ✓ No invert


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Different Empirical Failure Criteria for Rock (Contd..)

Source: Hoek et al. (2002)

Hoek et al. (2002)

Appearance of rock mass	Description of rock mass	Suggested value of D
	Very poor quality blasting in a hard rock tunnel results in severe local damage, extending 2 or 3 m, in the surrounding rock mass.	$D = 0.8$ ✓

✓


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Different Empirical Failure Criteria for Rock (Contd..)

Source: Hoek et al. (2002)

Hoek et al. (2002)

Appearance of rock mass	Description of rock mass	Suggested value of D
	Small scale blasting in civil engineering slopes results in modest rock mass damage, particularly if controlled blasting is used as shown on the left hand side of the photograph. However, stress relief results in some disturbance.	$D = 0.7$ ✓ Good blasting
		$D = 1.0$ ✓ Poor blasting

✓


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Different Empirical Failure Criteria for Rock (Contd..)

Source: Hoek et al. (2002)

Hoek et al. (2002)

Appearance of rock mass	Description of rock mass	Suggested value of D
	<p>Very large open pit mine slopes suffer significant disturbance due to heavy production blasting and also due to stress relief from overburden removal.</p> <p>In some softer rocks excavation can be carried out by ripping and dozing and the degree of damage to the slopes is less.</p>	<p>$D = 1.0$ ✓</p> <p>Production blasting ✓</p>
		<p>$D = 0.7$ ✓</p> <p>Mechanical excavation ✓</p>

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Different Empirical Failure Criteria for Rock (Contd..)

Source: Hoek et al. (2002)

Hoek et al. (2002)

➤ When, $\sigma'_1 = 0$ the uniaxial compressive strength of the rock is obtained as

➤ The biaxial tensile strength of the rock is obtained as

✓ $\sigma'_1 = \sigma'_3 = \sigma_t$

✓ $\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a$ ✓

✓ $\sigma_c = \sigma_{ci} s^a$ ✓

✓ $\sigma_t = \sigma_t + \sigma_{ci} \left(m_b \frac{\sigma_t}{\sigma_{ci}} + s \right)^a$ ✓

✓ $\sigma_t = -\frac{s \sigma_{ci}}{m_b}$ ✓

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So, finally from Hoek et al 2002 equations,

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a$$

When, $\sigma'_3 = 0$ the uniaxial compressive strength of the rock is obtained as

$$\sigma_c = \sigma_{ci} s^a$$

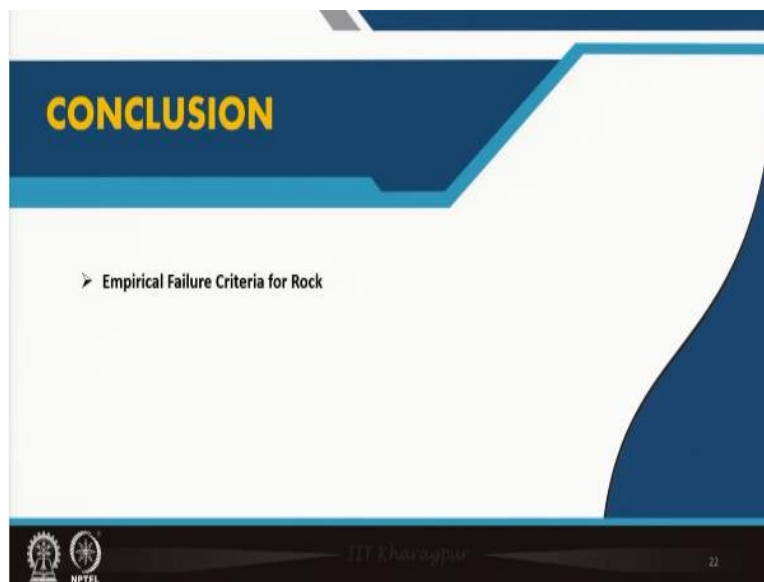
The biaxial tensile strength of the rock is obtained as

$$\sigma'_1 = \sigma'_3 = \sigma_t$$

$$\sigma_t = \sigma_t + \sigma_{ci} \left(m_b \frac{\sigma_t}{\sigma_{ci}} + s \right)^a$$

$$\sigma_t = -\frac{s\sigma_{ci}}{m_b}$$

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So, in conclusion what we can say basically we have today also discussed about the empirical failure criteria for rock and in our previous tasks also we have discussed the few other empirical failure criteria and then we have discussed about the most important failure criteria that is Hoek et al 2002 and we have discussed that in detail which need to be used for rock mass modeling. For a better modeling of the rock mask this criterion is quite acceptable and is used with good confidence. So, thank you, so let us conclude our today's lecture here only.