

Rock Mechanics and Tunneling
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Lecture No. 33
Griffith Crack Theory and Empirical Failure Criteria for rock

Hello everyone, I welcome you all to the second lecture of module seven. In module seven we will discuss about the rock and rock mass failure criteria. Today we will discuss about the Griffith crack theory and Empirical failure criteria for rock.


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Griffith Crack Theory

- In case of applied compressive stresses, the influence of friction on the cracks which will close under compression is neglected.



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So, Griffith crack theory says that in case of applied compression stresses, the influence of friction on the cracks which will close under compression is neglected.

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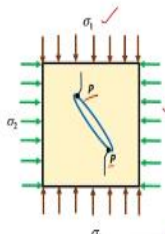
Griffith Crack Theory

- In case of applied compressive stresses, the influence of friction on the cracks which will close under compression is neglected.
- It is also assumed that the elliptical crack will propagate from the points of maximum tensile stress concentration
- Griffith presented the criterion for crack extension in plane compression

compression $(\sigma_1 - \sigma_2)^2 = 8T_0(\sigma_1 + \sigma_2)$ if $\sigma_1 + 3\sigma_2 > 0$


$\sigma_2 = -T_0$ if $\sigma_1 + 3\sigma_2 < 0$

where T_0 is the uniaxial tensile strength of the uncracked material.



Source: Brady and Brown (2006)*

*Brady, B.H.G. and Brown, E.T., 2006. Rock Mechanics for underground mining. Springer, Dordrecht.



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In the pictorial representation we can see that element is subjected to sigma 1 and sigma 2. Here it is assumed that the elliptical crack will propagate from the points of maximum tensile stress concentration 'P', as can be seen in the picture (on slide). Griffith presented the criteria for crack extension in plane compression as

$$(\sigma_1 - \sigma_2)^2 = 8T_o(\sigma_1 + \sigma_2) \quad \text{if } \sigma_1 + 3\sigma_2 > 0$$

$$\sigma_2 = -T_o \quad \text{if } \sigma_1 + 3\sigma_2 > 0$$

where T_o is the uniaxial tensile strength of uncracked material.

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Griffith Crack Theory

- In case of applied compressive stresses, the influence of friction on the cracks which will close under compression is neglected.
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- Griffith presented the criterion for crack extension in plane compression

$$(\sigma_1 - \sigma_2)^2 = 8T_o(\sigma_1 + \sigma_2)$$

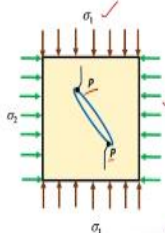
$$\sigma_2 = -T_o$$

$$\text{if } \sigma_1 + 3\sigma_2 > 0$$

$$\text{if } \sigma_1 + 3\sigma_2 < 0$$

where T_o is the uniaxial tensile strength of the uncracked material.

Source: Brady and Brown (2006)*



*Brady, B.H.G. and Brown, E.T., 2006. Rock Mechanics for underground mining. Springer, Dordrecht.

And again in terms of the shear stress, τ and the normal stress, σ_n acting on the plane containing the major axis of the crack Griffith (1924) presented the criteria as

$$\tau^2 = 4T_o(\sigma_n + T_o)$$

For uniaxial compressive stress,

$$\sigma_2 = 0 \text{ and } \sigma_1 = \sigma_{cs}$$

$$\sigma_{cs}^2 = 8T_o \sigma_{cs}$$

$$\sigma_{cs} = 8T_o$$

So, these some of the important equations which we get from the Griffith crack theory, thus the uniaxial compressive stress at crack extension will always be 8 times the uniaxial tensile strength.

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Empirical Failure Criteria for Rock

- The classic strength theories used for different engineering materials have been found unsuitable for rock over a wide range of applied compressive stress conditions.
- For practical use, a number of empirical strength criteria have been introduced.
- These criteria usually take the form of a power law in recognition of the fact that peak σ_1 vs. σ_3 and τ vs. σ_n envelopes for rock material are generally concave downwards



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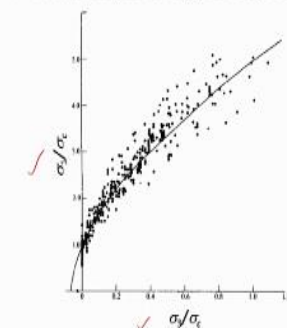
Now, I will discuss about different empirical failure criteria for rock. The classic stress theories used for different engineering materials such as steel support etc, have been found unsuitable for rock over a wide range of applied compressive stress conditions.

For practical use a number of empirical strength criteria have been introduced by several researchers. These criteria usually take the form of a power law in recognition of the fact that σ_1 vs. σ_3 and τ vs. σ_n envelopes for rock material are generally concave downward, concave downward.

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Empirical Failure Criterion for Rock (Contd..)

- Normalised peak strength envelope for sandstones.



Source: Hoek and Brown (1980)*

*Hoek, E. and Brown, E.T., 1980. Empirical strength criterion for rock masses. *Journal of the Geotechnical Engineering Division*, 106(9), 1013-1035.

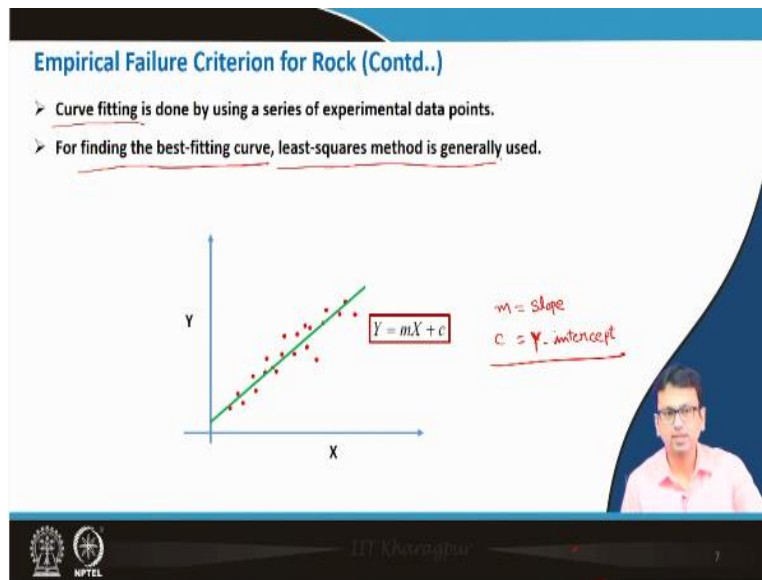


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Now in the slide there is a plot of normalized peak strength envelope for sandstone which is taken from Hoek Brown (1980). The plot is between σ_1/σ_c and σ_3/σ_c , σ_c is the uniaxial compressive strength. It can be seen that for few data points (black dots) a line passes through it. It is basically done by curve fitting.

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Curve fitting is done by using a series of experimental data points. For finding the best fitting curve generally the least squares method is used. One example is presented in the lecture slide which takes few data points and a line is fitted to the points. The equation of the line is given as

$$Y = mX + c$$

where,

m is the slope and

c is the y-intercept.

This is nothing but the best fit line for the given data point.

As already mentioned for finding the best fit curve generally the least squares method is used. So, following this procedure of best fitting curve some researchers have given several empirical failure criteria.

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Different Empirical Failure Criteria for Rock


- Various rock and rock mass failure criteria had been developed over the years

✓ Fairhurst (1964)*

- For intact rock Fairhurst (1964) developed

$$(\sigma_1 - \sigma_3)^2 = a + b(\sigma_1 + \sigma_3)$$
- Where a and b are the constants in the Fairhurst generalized criterion for different types of rock.

*Fairhurst, C., 1964. On the validity of the 'Brazilian' test for brittle materials. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts*, 1(4), 535-546



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So, let us discuss about different empirical failure criteria.

Fairhurst (1964)

So, for intact rock Fairhurst 1964 developed an empirical equation i.e.

$$(\sigma_1 - \sigma_3)^2 = a + b(\sigma_1 + \sigma_3)$$

where a and b are the constant in the Fairhurst generalized criteria for different types of rock.

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Different Empirical Failure Criteria for Rock (Contd..)

Hobbs (1965)*

- Based on the empirical test data fitting for intact rock

$$\sigma_1 = \sigma_c + \sigma_3 + F\sigma_3^f$$
- Where F and f are the constants in the Hobbs generalized criterion varies for different types of rock.

*Hobbs, D.W., 1965. An assessment of a technique for determining the tensile strength of rock. *British Journal of Applied Physics*, 16(2), 259.



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Similarly,

Hobbs (1965)

Based on the empirical test data fitting for intact rock, one equation was given i.e.

$$\sigma_1 = \sigma_c + \sigma_3 + F\sigma_3^f$$

where F and f are the constant in the Hobbs generalized criteria and varies for different types of rock.

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The slide is titled "Different Empirical Failure Criteria for Rock (Contd..)" and features a green box labeled "Murrell (1965)*". Below this, a bullet point states: "Developed with the help of empirical test data fitting for intact rock". The equation $\sigma_1 = \sigma_c + a\sigma_3^b$ is written in red. Another bullet point, preceded by two red checkmarks, states: "Where a and b are the constants in the Murrell generalized criterion varies for different types of rock." At the bottom left, a small text block provides a reference: "*Murrell, S.A.F., 1965, The effect of triaxial stress systems on the strength of rocks at atmospheric temperatures. *Geophysical Journal International*, 10(3), 231-281." The bottom of the slide includes the IIT Kharagpur logo and the NPTEL logo, with the text "IIT Kharagpur" and the number "19" on the right.

Murrell (1965)

Developed with the help of empirical test data fitting for intact rock.

$$\sigma_1 = \sigma_c + a\sigma_3^b$$

where a and b are the constants in the Murrell generalized criteria varies is for different types of rock.

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek (1968)*

- Empirical test data fitting for the rupture behaviour of a intact brittle rock was characterized by

$$\frac{\tau_{\max}}{\sigma_c} = \frac{\tau_{\max o}}{\sigma_c} + A \left(\frac{\sigma_m}{\sigma_c} \right)^b$$

- Where

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sigma_m = \frac{\sigma_1 + \sigma_3}{2}$$

- $\tau_{\max o}$ is the intercept of the τ_{\max} versus σ_m plot when $\sigma_m = 0$

*Hoek, E., 1968. Brittle fracture of rock. Rock mechanics in engineering practice, 1-30.



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Hoek (1968)

Empirical test data fitting for the rupture behavior of intact brittle rock was characterized by following equation,

$$\frac{\tau_{\max}}{\sigma_c} = \frac{\tau_{\max o}}{\sigma_c} + A \left(\frac{\sigma_m}{\sigma_c} \right)^b$$

$$\text{where, } \tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} ; \sigma_m = \frac{\sigma_1 + \sigma_3}{2}$$

$\tau_{\max o}$ is the intercept of the τ_{\max} versus σ_m plot when $\sigma_m = 0$

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek (1968)

- Plotting the experimental results on logarithmic scales permits a direct evaluation of the constants A and b .
- The value of b is given by the slope of the straight line through the experimental points

$$\frac{\tau_{\max}}{\sigma_c} = \frac{\tau_{\max o}}{\sigma_c} + A \left(\frac{\sigma_m}{\sigma_c} \right)^b$$

$$\log_{10} \frac{\tau_{\max} - \tau_{\max o}}{\sigma_c} = \log_{10} A + b \log_{10} \left(\frac{\sigma_m}{\sigma_c} \right)$$

$$\frac{\sigma_m}{\sigma_c} = 1 \quad \log_{10} \left(\frac{\sigma_m}{\sigma_c} \right) = 0$$

$$A = \frac{\tau_{\max} - \tau_{\max o}}{\sigma_c}$$

Source: Hoek (1968)



Now by plotting this experimental results on logarithmic scales permits a direct evaluation of the constants A and b . Now the equation was,

$$\frac{\tau_{\max}}{\sigma_c} = \frac{\tau_{\max o}}{\sigma_c} + A \left(\frac{\sigma_m}{\sigma_c} \right)^b$$

if we take log in both the sides

$$\log_{10} \frac{\tau_{\max} - \tau_{\max o}}{\sigma_c} = \log_{10} A + b \log_{10} \left(\frac{\sigma_m}{\sigma_c} \right)$$

$$\text{Now if } \frac{\sigma_m}{\sigma_c} = 0 \text{ then } \log_{10} \left(\frac{\sigma_m}{\sigma_c} \right) = 0$$

therefore,

$$A = \frac{\tau_{\max} - \tau_{\max o}}{\sigma_c}$$

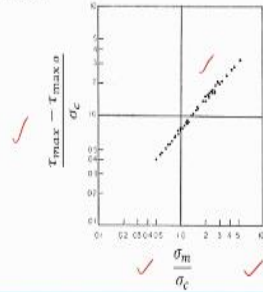
So the value of B is given by the slope of the straight line through the experimental point. So, in this way A and b can be obtained for this Hoek 1968 empirical expression.

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Different Empirical Failure Criteria for Rock (Contd..)

Hoek (1968)

- Relation between the maximum shear and mean normal stresses at rupture for sandstones can be determined from



- In the absence of experimental values, the author suggests

$$\tau_{\max}/\sigma_c = 0.1$$

$$\frac{\tau_{\max}}{\sigma_c} = 0.1 + 0.76 \left(\frac{\sigma_m}{\sigma_c} \right)^{0.85}$$

Source: Hoek (1968)



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Now relation between the maximum shear (τ_{\max}) and mean normal stresses (σ_m) at rupture for sandstones can be determined from the plot given by Hoek 1968 (refer to slide no. 13 of this lecture).

Author suggests that in the absence of experimental values

$$\tau_{\max}/\sigma_c = 0.1$$

So, the final expression can be

$$\frac{\tau_{\max}}{\sigma_c} = 0.1 + 0.76 \left(\frac{\sigma_m}{\sigma_c} \right)^{0.85}$$

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Different Empirical Failure Criteria for Rock (Contd..)

Bodonyi (1970)*

- The relation was presented with the help of triaxial tests on soft rock

$$\sigma_1 = \sigma_c + a\sigma_3$$

- The magnitude of constant a in the Bodonyi generalized criterion varies for different types of rock.

Franklin and Hoek (1970)**

- The empirical curve fitting with the help of 1500 triaxial tests on rock sample

$$\sigma_1 = \sigma_3 + \sigma_c^{1-B} (\sigma_1 - \sigma_3)^B$$

- The constant B in the Franklin and Hoek criterion varies between 0.6 and 0.9 for different types of rock.

*Bodonyi, J., 1970. Laboratory tests of certain rocks under axially symmetrical loading conditions. 2nd ISRM International Congress of Rock Mechanics, 2-17, Belgrade.

**Franklin, J.A. and Hoek, E., 1970. Developments in triaxial testing technique. Rock Mechanics, 2(4), 223-228.



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Bodonyi (1970)

The relation was presented with the help of triaxial tests on soft rock; according to this

$$\sigma_1 = \sigma_c + a\sigma_3$$

magnitude of the constant a in the Bodonyi generalized criteria varies for different types of rock as it was varying for other cases earlier.

Franklin and Hoek (1970)

The empirical curve fitting with the help of 1500 triaxial tests on rock sample

$$\sigma_1 = \sigma_3 + \sigma_c^{1-B} (\sigma_1 - \sigma_3)^B$$

The constant B in the Franklin and Hoek criteria varies from 0.6 to 0.9 for different types of rock.

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Different Empirical Failure Criteria for Rock (Contd..)

Bieniawski (1974)*

➤ The peak triaxial strengths of a range of intact rock types were well represented by

$$\frac{\sigma_1}{\sigma_c} = 1 + A \left(\frac{\sigma_3}{\sigma_c} \right)^k \quad \text{or} \quad \frac{\tau_m}{\sigma_c} = 0.1 + B \left(\frac{\sigma_m}{\sigma_c} \right)^c$$

Bieniawski found that, for the range of rock types tested, the magnitude of the constant $k \sim 0.75$ and $c \sim 0.90$.

Rock type	A	B
Norite	5.0	0.80
Quartzite	4.5	0.78
Sandstone	4.0	0.75
Siltstone	3.0	0.70
Mudstone	3.0	0.70

*Bieniawski, Z.T., 1974. Estimating the strength of rock materials. Journal of the Southern African Institute of Mining and Metallurgy, 74(8), 312-320.

Dr. T. Khuram

Bieniawski (1974)

The peak triaxial strength of a range of intact rock types were well represented by

$$\frac{\sigma_1}{\sigma_c} = 1 + A \left(\frac{\sigma_3}{\sigma_c} \right)^k \quad \text{or} \quad \frac{\tau_m}{\sigma_c} = 0.1 + B \left(\frac{\sigma_m}{\sigma_c} \right)^c$$

Where

$$\text{where, } \tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} ; \quad \sigma_m = \frac{\sigma_1 + \sigma_3}{2}$$

Bienawski found that, for the range of rock types tested, the magnitude of constant k is approximately 0.75 and c is approximately 0.9.

Now, for A and B table is provided by Bienawski (refer to slide 15 of this lecture).

So, okay so thank you let us conclude here our today's lecture and we will continue with the empirical failure criteria only in our next lecture, thank you.