Rock Mechanics and Tunneling
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Lecture No. 33
Griffith Crack Theory and Empirical Failure Criteria for rock

Hello everyone, I welcome you all to the second lecture of module seven. In module seven we will discuss about the rock and rock mass failure criteria. Today we will discuss about the Griffith crack theory and Empirical failure criteria for rock.

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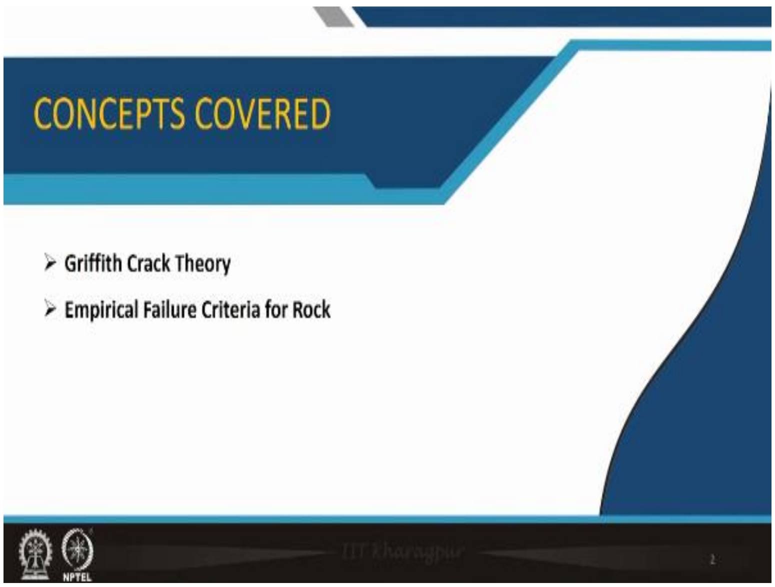

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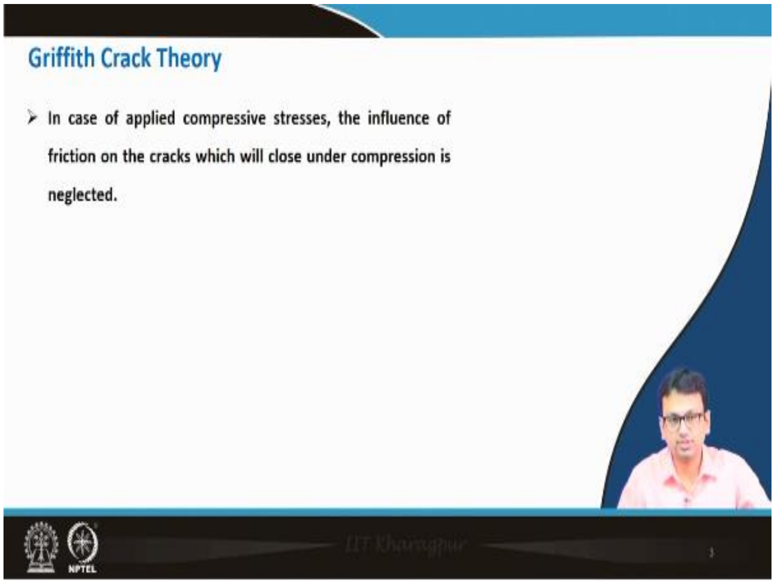

So, Griffith crack theory says that in case of applied compression stresses, the influence of friction on the cracks which will close under compression is neglected.

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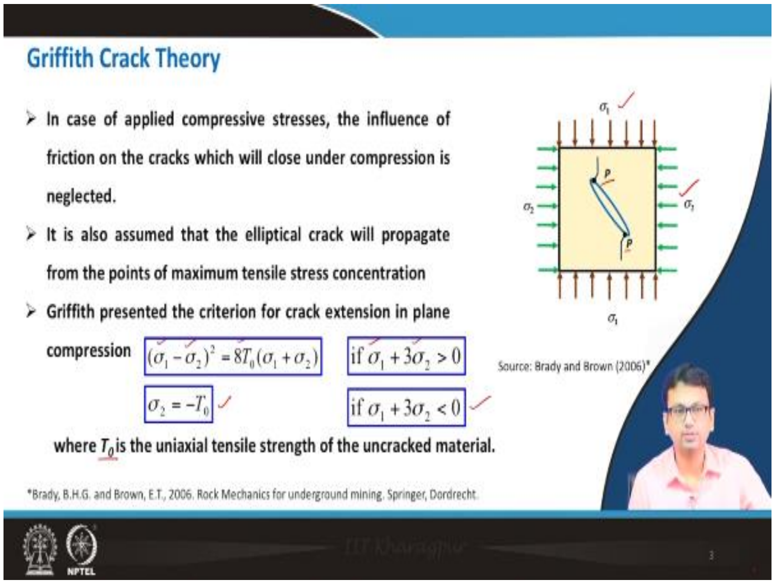

In the pictorial representation we can see that element is subjected to sigma 1 and sigma 2. Here it is assumed that the elliptical crack will propagate from the points of maximum tensile stress concentration ‘P’, as can be seen in the picture (on slide). Griffith presented the criteria for crack extension in plane compression as

$$(\sigma_1 - \sigma_2)^2 = 8T_o(\sigma_1 + \sigma_2) \qquad \text{if } \sigma_1 + 3\sigma_2 > 0$$

$$\sigma_2 = -T_o \qquad \text{if } \sigma_1 + 3\sigma_2 > 0$$

where T_o is the uniaxial tensile strength of uncracked material.

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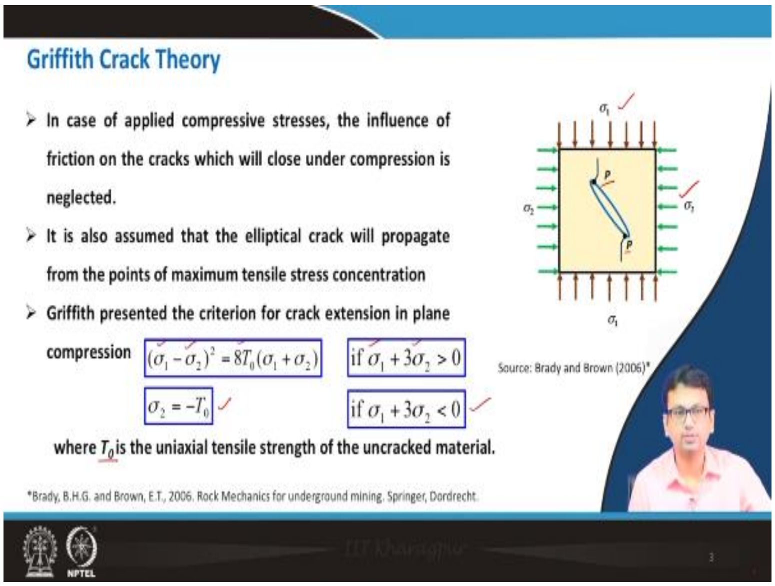

And again in terms of the shear stress, τ and the normal stress, σ_n acting on the plane containing the major axis of the crack Griffith (1924) presented the criteria as

$$\tau^2 = 4T_o(\sigma_n + T_o)$$

For uniaxial compressive stress,

$$\sigma_2 = 0 \text{ and } \sigma_1 = \sigma_{cs}$$

$$\sigma^2{}_{cs} = 8T_o\sigma_{cs}$$

$$\sigma_{cs} = 8T_o$$

So, these some of the important equations which we get from the Griffith crack theory, thus the uniaxial compressive stress at crack extension will always be 8 times the uniaxial tensile strength.

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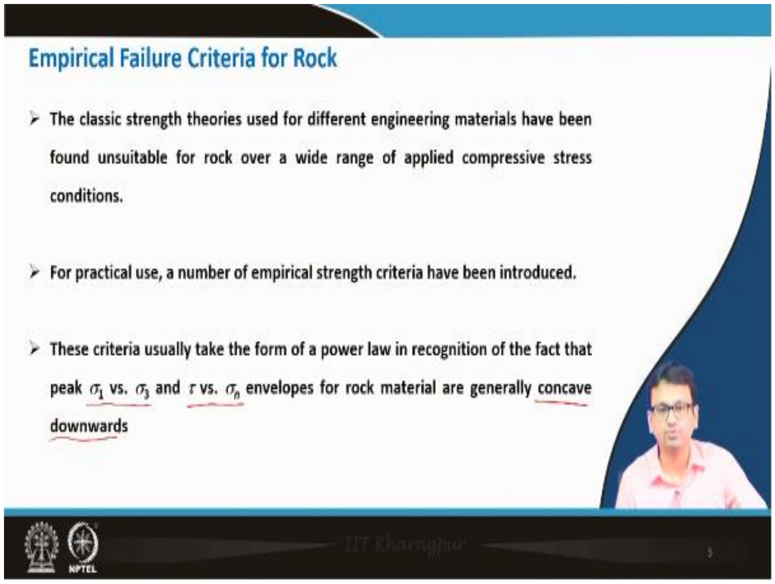

Now, I will discuss about different empirical failure criteria for rock. The classic stress theories used for different engineering materials such as steel support etc, have been found unsuitable for rock over a wide range of applied compressive stress conditions.

For practical use a number of empirical strength criteria have been introduced by several researchers. These criteria usually take the form of a power law in recognition of the fact that σ_1 vs. σ_3 and τ vs. σ_n envelops for rock material are generally concave downward, concave downward.

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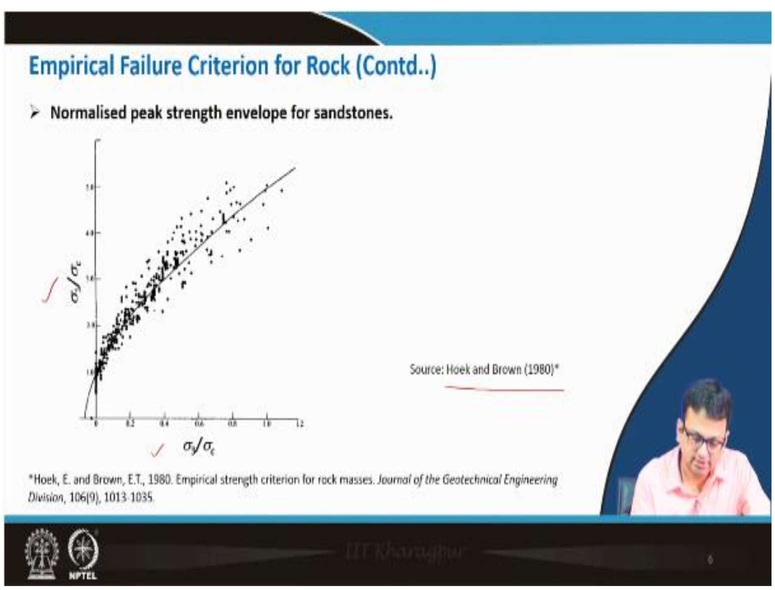

Now in the slide there is a plot of normalized peak strength envelope for sandstone which is taken from Hoek Brown (1980). The plot is between σ_1 / σ_c and σ_3 / σ_c, σ_c is the uniaxial compressive strength. It can be seen that for few data points (black dots) a line passes through it. It is basically done by curve fitting.

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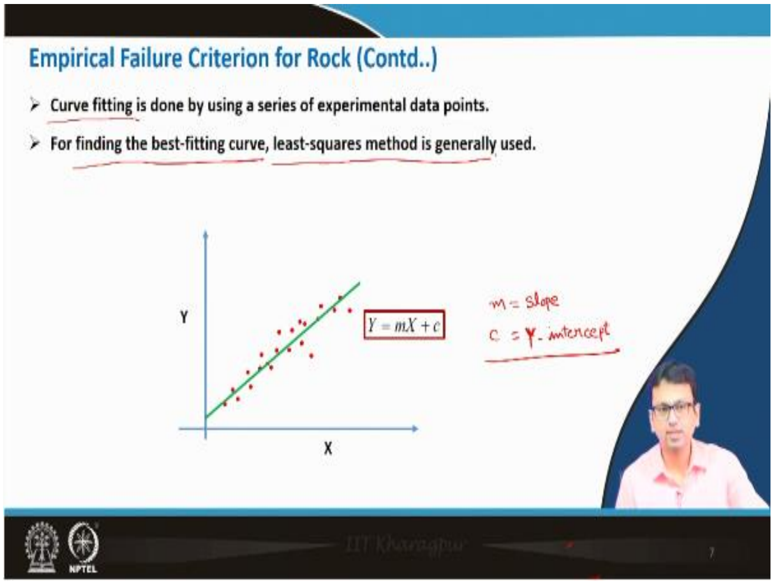

Curve fitting is done by using a series of experimental data points. For finding the best fitting curve generally the least squares method is used. One example is presented in the lecture slide which takes few data points and a line is fitted to the points. The equation of the line is given as

$$Y = mX + c$$

where,

m is the slope and

c is the y-intercept.

This is nothing but the best fit line for the given data point.

As already mentioned for finding the best fit curve generally the least squares method is used. So, following this procedure of best fitting curve some researchers have given several empirical failure criteria.

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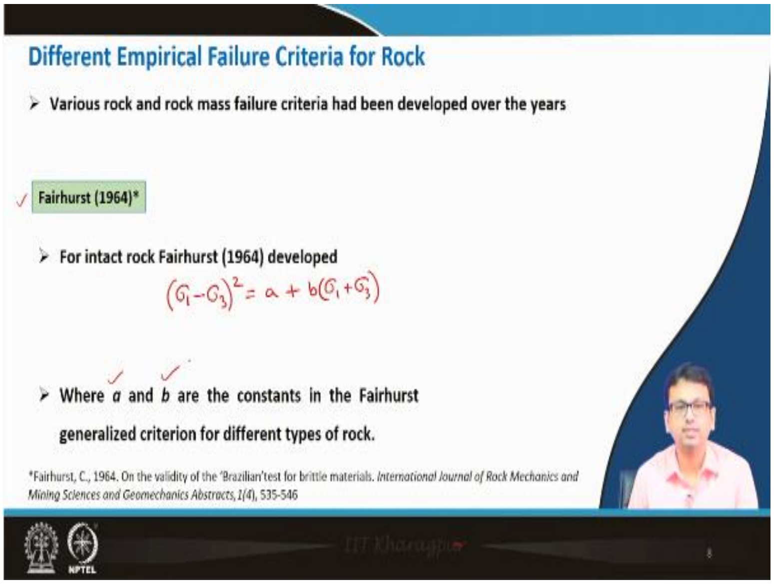

So, let us discuss about different empirical failure criteria.

Fairhurst (1964)

So, for intact rock Fairhurst 1964 developed an empirical equation i.e.

$$(\sigma_1 - \sigma_3)^2 = a + b(\sigma_1 + \sigma_3)$$

where a and b are the constant in the Fairhurst generalized criteria for different types of rock.

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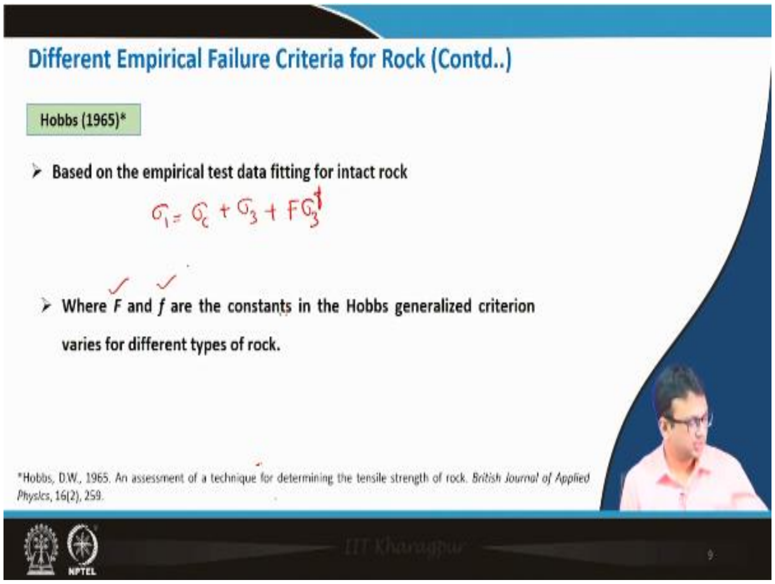

Similarly,

Hobbs (1965)

Based on the empirical test data fitting for intact rock, one equation was given i.e.

$$\sigma_1 = \sigma_C + \sigma_3 + F\sigma_3{}^f$$

where F and f are the constant in the Hobbs generalized criteria and varies for different types of rock.

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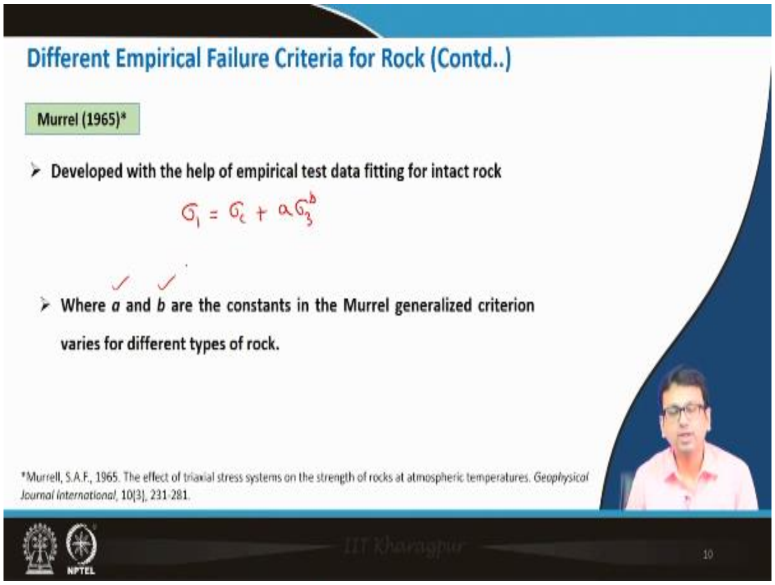

Murrel (1965)

Developed with the help of empirical test data fitting for intact rock.

$$\sigma_1 = \sigma_C + a\sigma_3{}^b$$

where a and b are the constants in the Murrel generalized criteria varies is for different types of rock.

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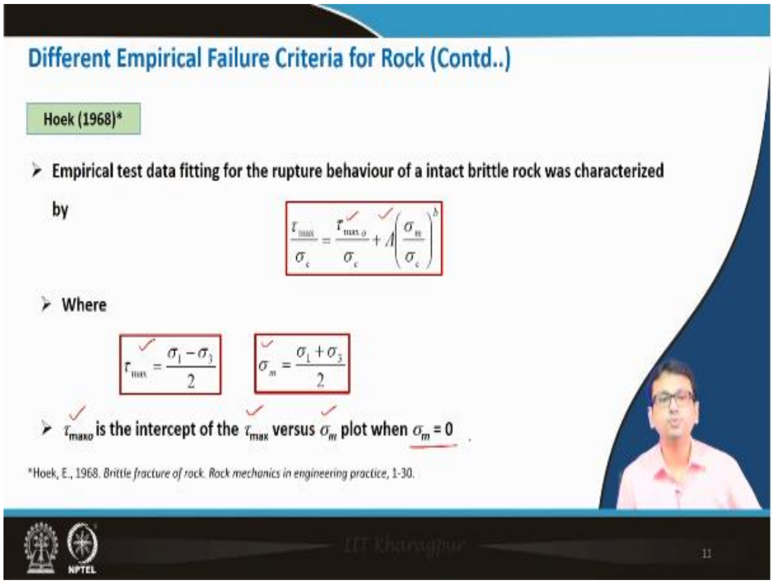

Hoek (1968)

Empirical test data fitting for the rupture behavior of intact brittle rock was characterized by following equation,

$$\frac{\tau_{\max}}{\sigma_c} = \frac{\tau_{\max o}}{\sigma_c} + A\left(\frac{\sigma_m}{\sigma_c}\right)^b$$

where, $\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$; $\sigma_m = \frac{\sigma_1 + \sigma_3}{2}$

$\tau_{\max o}$ is the intercept of the $\tau_{\max}$ versus σ_m plot when $\sigma_m = 0$

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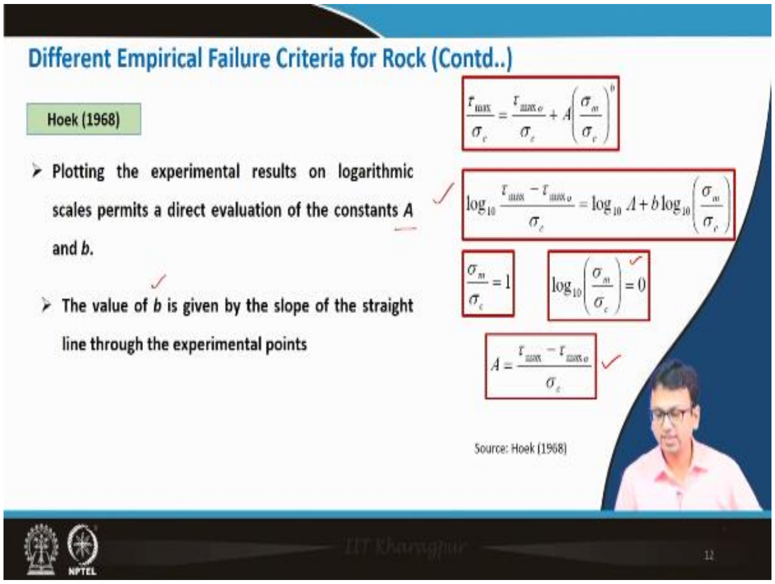

Now by plotting this experimental results on logarithmic scales permits a direct evaluation of the constants *A* and *b*. Now the equation was,

$$\frac{\tau_{\max}}{\sigma_c} = \frac{\tau_{\max o}}{\sigma_c} + A\left(\frac{\sigma_m}{\sigma_c}\right)^b$$

if we take log in both the sides

$$\log_{10}\frac{\tau_{\max} - \tau_{\max o}}{\sigma_c} = \log_{10} A + b\log_{10}\left(\frac{\sigma_m}{\sigma_C}\right)$$

Now if $\frac{\sigma_m}{\sigma_C} = 0$ then $\log_{10}\left(\frac{\sigma_m}{\sigma_C}\right) = 0$

therefore,

$$A = \frac{\tau_{\max} - \tau_{\max o}}{\sigma_c}$$

So the value of B is given by the slope of the straight line through the experimental point. So, in this way A and b can be obtained for this Hoek 1968 empirical expression.

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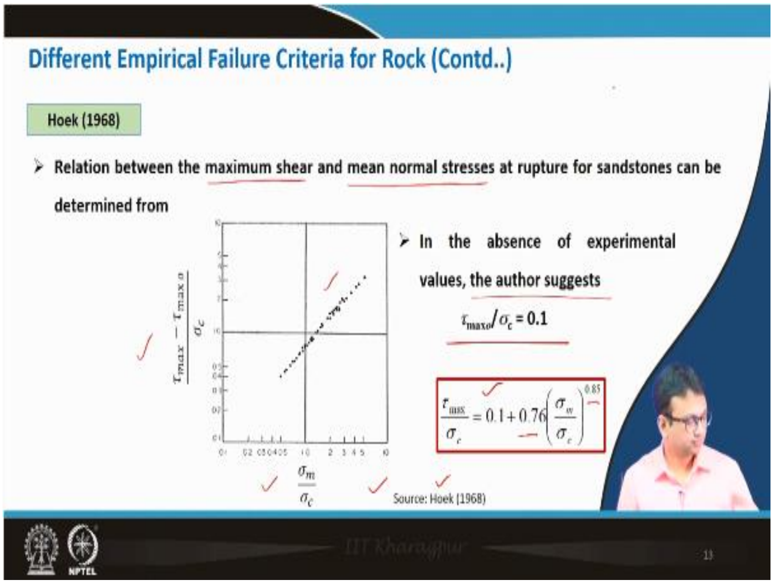

Now relation between the maximum shear (τ_{max}) and mean normal stresses (σ_m) at rupture for sandstones can be determined from the plot given by Hoek 1968 (refer to slide no. 13 of this lecture).

Author suggests that in the absence of experimental values

$$\tau_{max}/\sigma_C = 0.1$$

So, the final expression can be

$$\frac{\tau_{max}}{\sigma_c} = 0.1 + .76\left(\frac{\sigma_m}{\sigma_C}\right)^{0.85}$$

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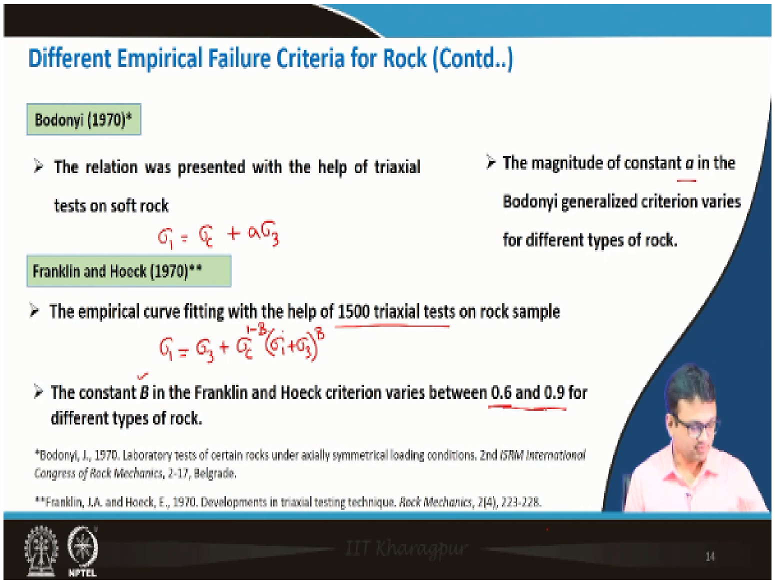

Bodonyi (1970)

The relation was presented with the help of triaxial tests on soft rock; according to this

$$\sigma_1 = \sigma_C + a\sigma_3$$

magnitude of the constant *a* in the Bodonyi generalized criteria varies for different types of rock as it was varying for other cases earlier.

Franklin and Hoeck (1970)

The empirical curve fitting with the help of 1500 triaxial tests on rock sample

$$\sigma_1 = \sigma_3 + \sigma_C{}^{1-B}(\sigma_1 - \sigma_3)^B$$

The constant B in the Franklin and Hoeck criteria varies from 0.6 to 0.9 for different types of rock.

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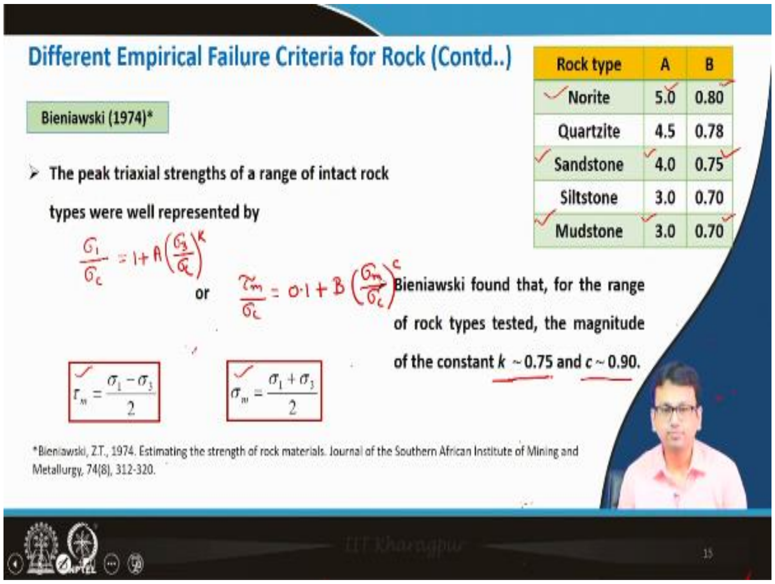

Bieniawski (1974)

The peak triaxial strength of a range of intact rock types were well represented by

$$\frac{\sigma_1}{\sigma_C} = 1 + A\left(\frac{\sigma_3}{\sigma_C}\right)^k \quad \text{or} \quad \frac{\tau_m}{\sigma_C} = 0.1 + B\left(\frac{\sigma_m}{\sigma_C}\right)^c$$

Where

where, $\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2}$; $\sigma_m = \dfrac{\sigma_1 + \sigma_3}{2}$

Bienawski found that, for the range of rock types tested, the magnitude of constant k is approximately 0.75 and c is approximately 0.9.

Now, for A and B table is provided by Bienawski (refer to slide 15 of this lecture).

So, okay so thank you let us conclude here our today's lecture and we will continue with the empirical failure criteria only in our next lecture, thank you.