Rock Mechanics and Tunneling
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Lecture No. 32
Mohr-Coulomb (MC) Failure Criterions

Hello everyone, I welcome you all to the first lecture of module 7. In module 7 we will continue our discussion related to the rock and rock mass failure criteria. So, in our first lecture of module seven we will discuss about the Mohr-Coulomb Failure Criterion.

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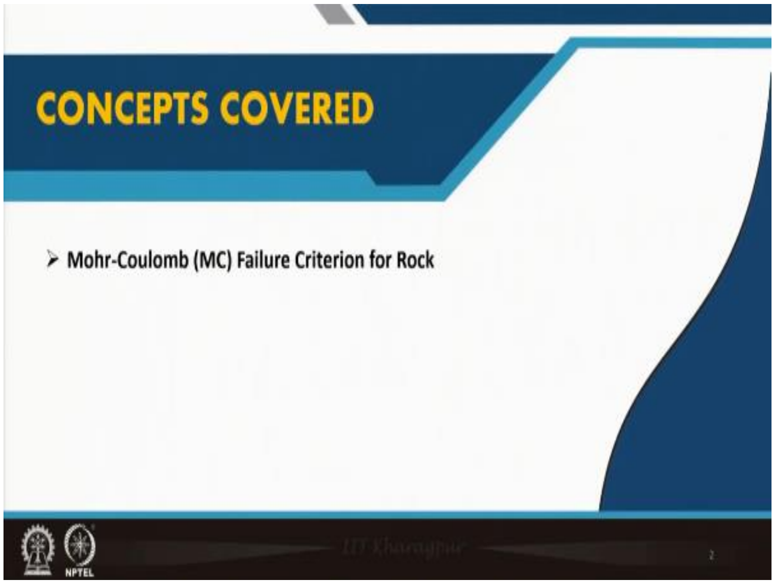

So, our focus will be on the Mohr-Coulomb Failure Criterion for Rock.

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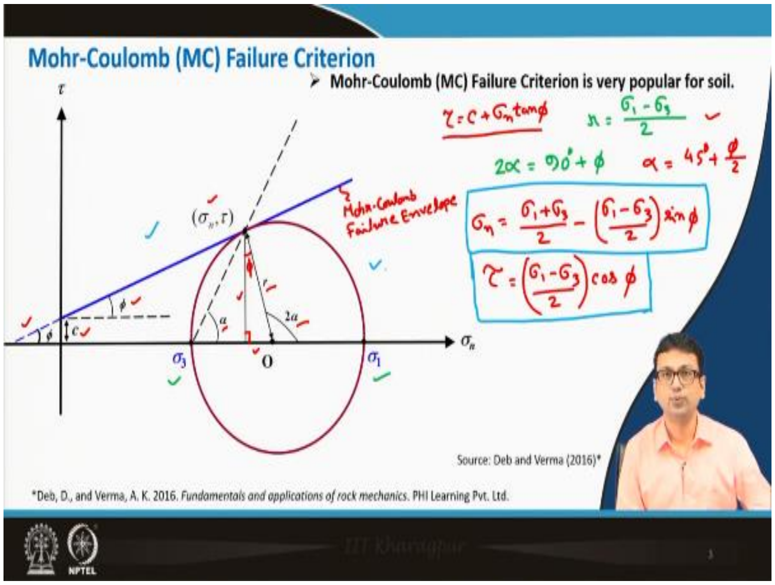

We consider a $\sigma - \tau$ plot and draw a Mohr circle by identifying σ_1, σ_3 and O (centre of the circle). Then draw the tangent to the circle based on the experimental data received from the Triaxial test or Direct shear test. This tangent is well known as the Mohr-Coulomb failure envelope in the case of soil. Tangent cuts the τ axis and the intercept made is cohesion, c whereas the angle made by the tangent with the σ axis is the angle of internal friction, ϕ. The point of contact of the Mohr circle and the failure envelope gives the σ_n and τ values

Now, for the limiting case when magnitude of shear stress is equal to shear strength, we can write,

$$\tau = c + \sigma_n \tan\phi$$

The above equation is the equation of the failure envelope.

Using simple geometry, it is seen that the angle between the radius touching the tangent and the normal from the point of contact to the horizontal is equal to ϕ.

From the figure in the slide it can be seen that,

$$r = \frac{\sigma_1 - \sigma_3}{2}$$

$$2\alpha = 90^\circ + \phi \rightarrow \alpha = 45^\circ + \frac{\phi}{2}$$

σ_n and τ in terms of radius, r

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} - \left(\frac{\sigma_1 - \sigma_3}{3}\right)\sin\phi$$

$$\tau = \left(\frac{\sigma_1 - \sigma_3}{3}\right)\cos\phi$$

Also, region below Mohr-Coulomb failure envelope indicates safe zone and above it indicates failure zone.

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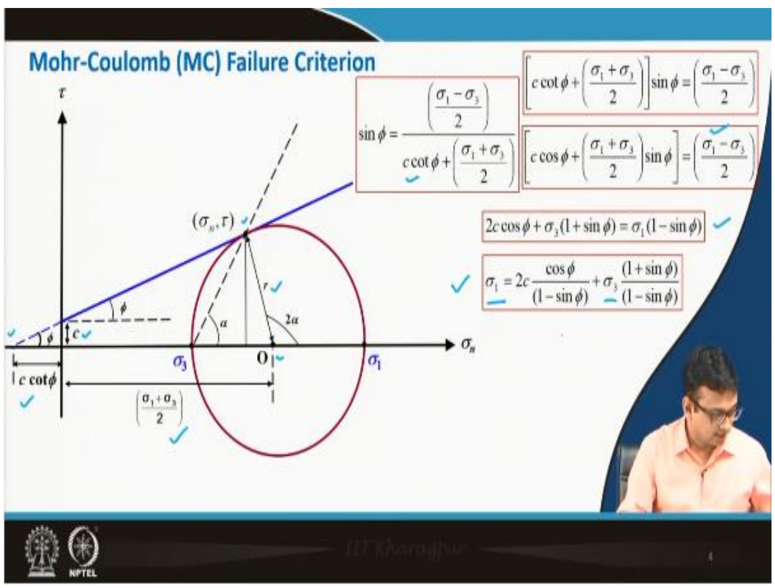

Now if we again look into the figure, three points- the point where the failure envelope cuts the σ_n - axis at an angle ϕ, point where the envelope meets the Mohr circle and point O. These three points make a right angled triangle. Using this triangle, we can observe that-

$$\sin\phi = \frac{\frac{\sigma_1 - \sigma_3}{2}}{c\cot\phi + \left(\frac{\sigma_1 + \sigma_3}{2}\right)}$$

$$\left[c\cot\phi + \left(\frac{\sigma_1+\sigma_3}{2}\right)\right]\sin\phi = \frac{\sigma_1-\sigma_3}{2}$$

$$c\cos\phi + \left(\frac{\sigma_1+\sigma_3}{2}\right)\sin\phi = \frac{\sigma_1-\sigma_3}{2}$$

$$2c\cos\phi + \sigma_3(1+\sin\phi) = \sigma_1(1-\sin\phi)$$

$$\sigma_1 = 2c\frac{\cos\phi}{(1-\sin\phi)} + \sigma_3\frac{(1+\sin\phi)}{(1-\sin\phi)}$$

$$\sigma_1 = 2c\frac{\sqrt{1-\sin^2\phi}}{(1-\sin\phi)} + \sigma_3\frac{(1+\sin\phi)}{(1-\sin\phi)}$$

$$\sigma_1 = 2c\sqrt{\frac{(1+\sin\phi)}{(1-\sin\phi)}} + \sigma_3\frac{(1+\sin\phi)}{(1-\sin\phi)} \qquad (1)$$

Now,

$$\frac{1+\sin\phi}{1-\sin\phi} = \frac{\sin^2\frac{\phi}{2}+\cos^2\frac{\phi}{2}+2\cos\frac{\phi}{2}\sin\frac{\phi}{2}}{\sin^2\frac{\phi}{2}+\cos^2\frac{\phi}{2}-2\cos\frac{\phi}{2}\sin\frac{\phi}{2}}$$

$$\frac{1+\sin\phi}{1-\sin\phi} = \frac{\left(\cos\frac{\phi}{2}+\sin\frac{\phi}{2}\right)^2}{\left(\cos\frac{\phi}{2}-\sin\frac{\phi}{2}\right)^2}$$

$$\frac{1+\sin\phi}{1-\sin\phi} = \left(\frac{\tan 45^\circ+\tan\frac{\phi}{2}}{1-\tan 45^\circ\tan\frac{\phi}{2}}\right)^2$$

$$\frac{1+\sin\phi}{1-\sin\phi} = \tan^2(45^\circ+\frac{\phi}{2}) \qquad (2)$$

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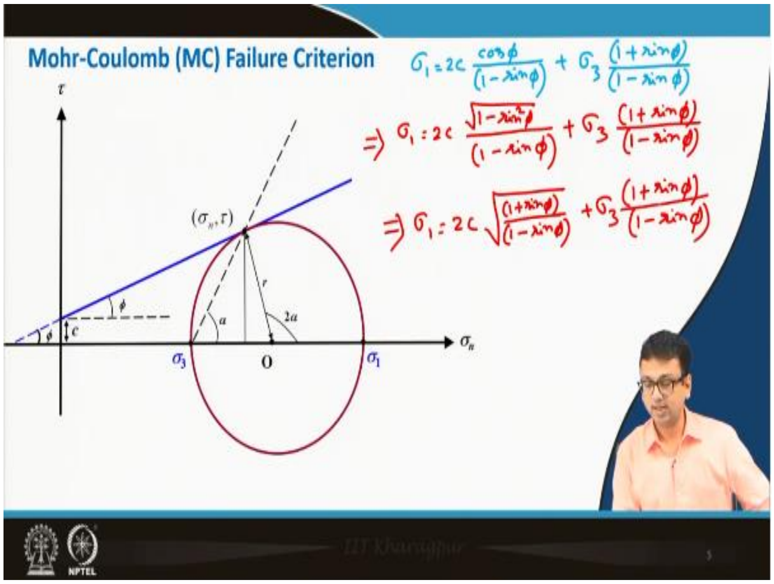

Now, by using equation 1 and 2 we get

$$\sigma_1 = 2c \tan\left(45° + \frac{\phi}{2}\right) + \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) \tag{3}$$

Equation 3 is the Mohr-Coulomb failure criterion in principal stress space.

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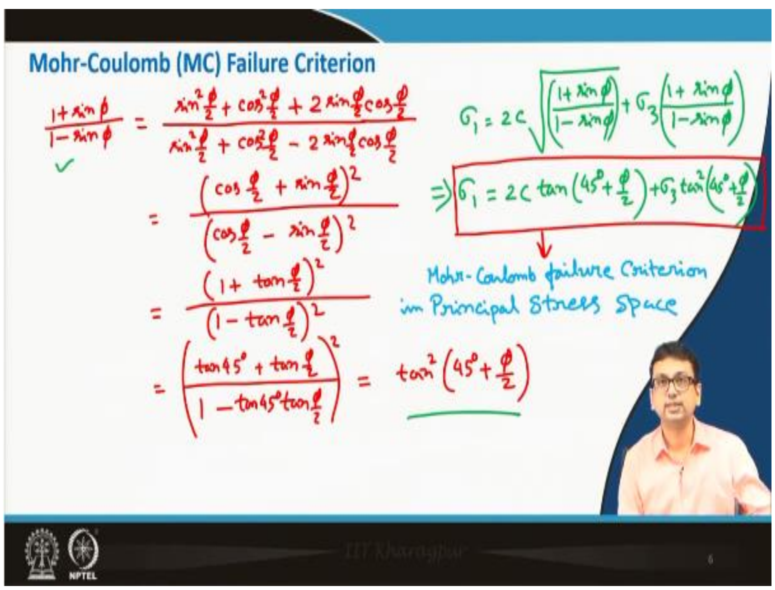

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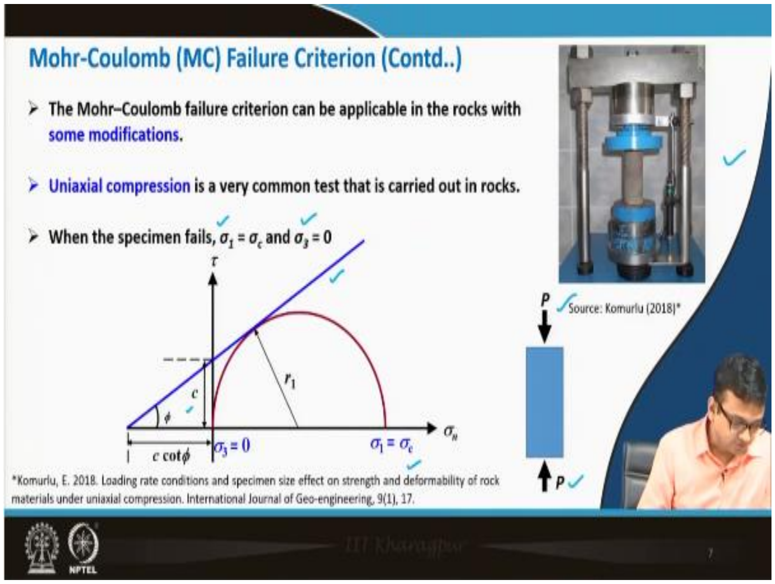

So, to apply Mohr-Coulomb failure criteria to the rock some modifications are needed. We have already seen that in Uniaxial compressive strength confining pressure is not applied and only the axial loading is applied.

$$\sigma_1 = \sigma_c \ ; \ \sigma_3 = 0$$

The Mohr-circle and the failure envelope of UCS is given in the slide.

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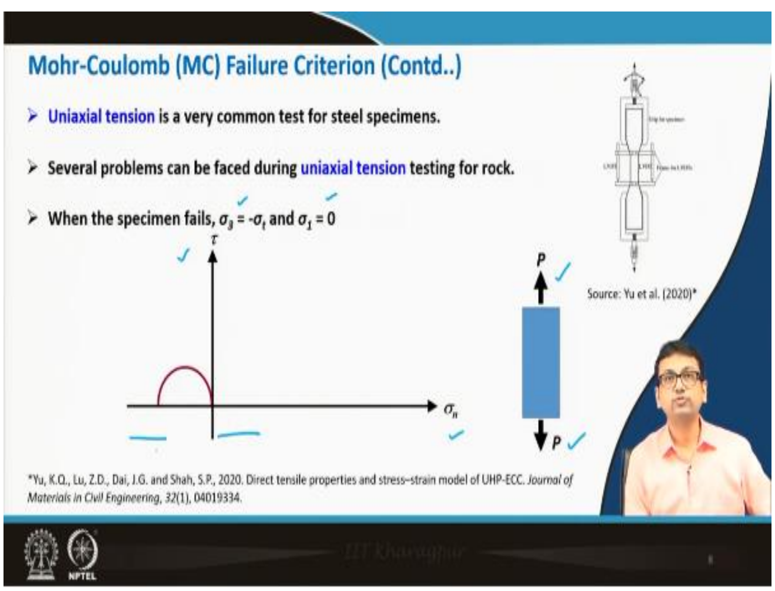

Uniaxial tension is a very common test for steel specimen, but in case of rock there may be some difficulties regarding gripping. Now fundamentally in this test, only the uniaxial tension is applied, no confining pressure; in this situation $\sigma_3 = -\sigma_t$ (minor principal stress) and $\sigma_1 = 0$ (major principal stress), in $\tau - \sigma$ plane if Mohr circle is drawn for uniaxial tensile strength test. Remember in all our discussions compression is considered as positive.

It is assumed that Mohr-Coulomb Yield criterion is valid in uniaxial tension also i.e. in the tensile region also.

Now let us play with both the failure envelopes (tension and compression)

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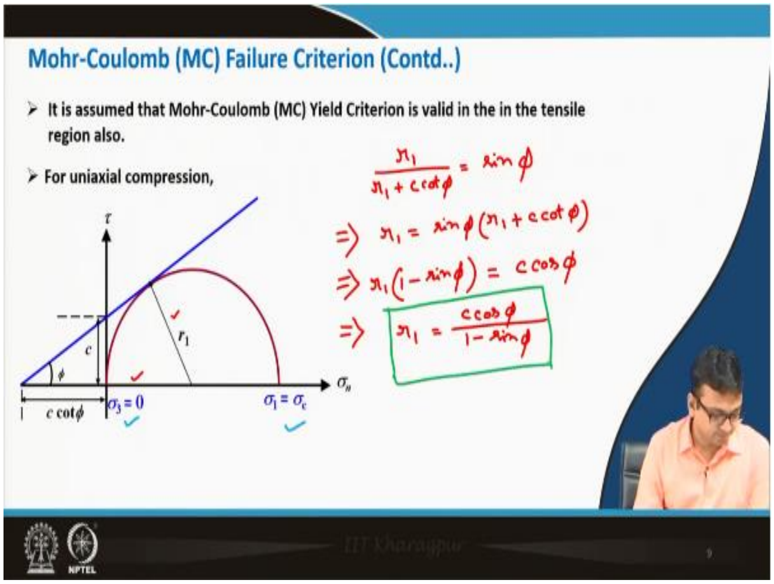

So, here what can be written that it is assumed that Mohr-Coulomb yield criteria is valid in the tension region, means tensile region also.

So, now for uniaxial compression

$\sigma_1 = \sigma_c, \quad \sigma_3 = 0$

Let there be a distance r_1 in the Mohr's circle (radius touches the failure envelope) (refer slides).

$$\frac{r_1}{r_1 + c\cot\phi} = \sin\phi$$

$$r_1 = \sin\phi(r_1 + c\cot\phi)$$

$$r_1(1 - \sin\phi) = c\cos\phi$$

$$r_1 = \frac{c\cos\phi}{1-\sin\phi} \tag{4}$$

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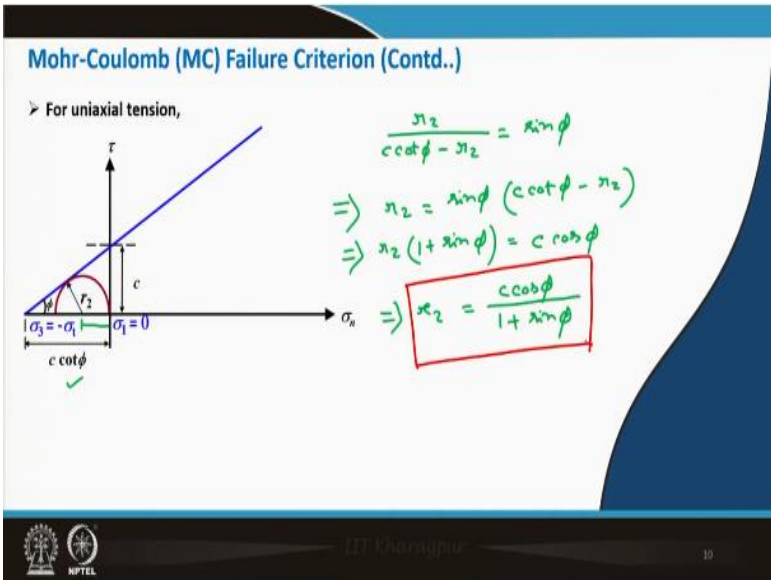

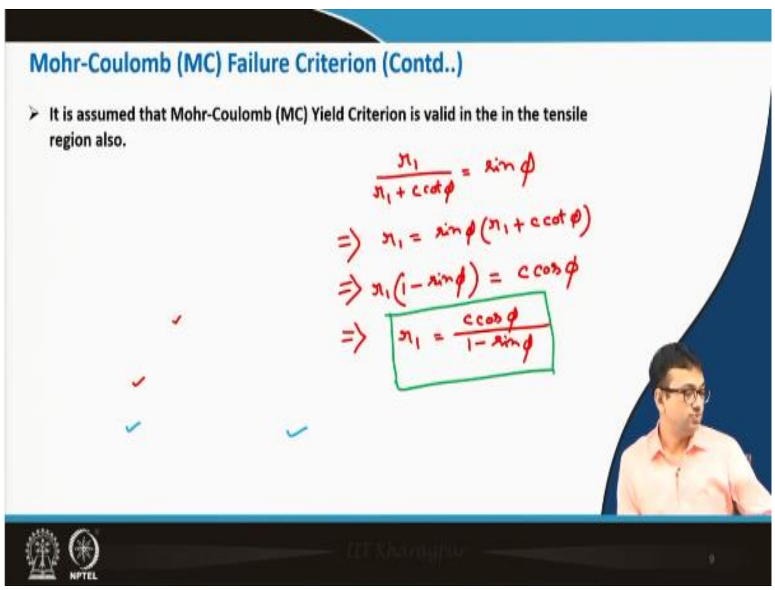

So, now for uniaxial tension

$\sigma_1 = 0, \quad \sigma_3 = -\sigma_t$

Let there be a distance r_2 in the Mohr's circle (radius touches the failure envelope) (refer slides).

$$\frac{r_2}{c\cot\phi - r_2} = \sin\phi$$

$$r_2 = \sin\phi(c\cot\phi - r_2)$$

$$r_2(1+\sin\phi) = c\cos\phi$$

$$r_2 = \frac{c\cos\phi}{1+\sin\phi} \tag{5}$$

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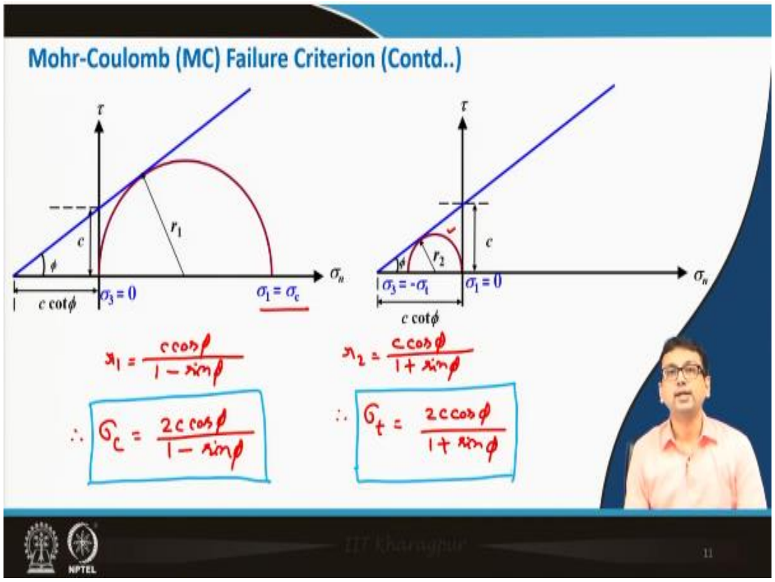

Further using equations 4 and 5 we can write

$$\sigma_c = \frac{2c\cos\phi}{1-\sin\phi} \tag{6}$$

$$\sigma_t = \frac{2c\cos\phi}{1+\sin\phi} \tag{7}$$

We know that σ_c and σ_t are the diameters of the Mohr circles in uniaxial compression and tension respectively.

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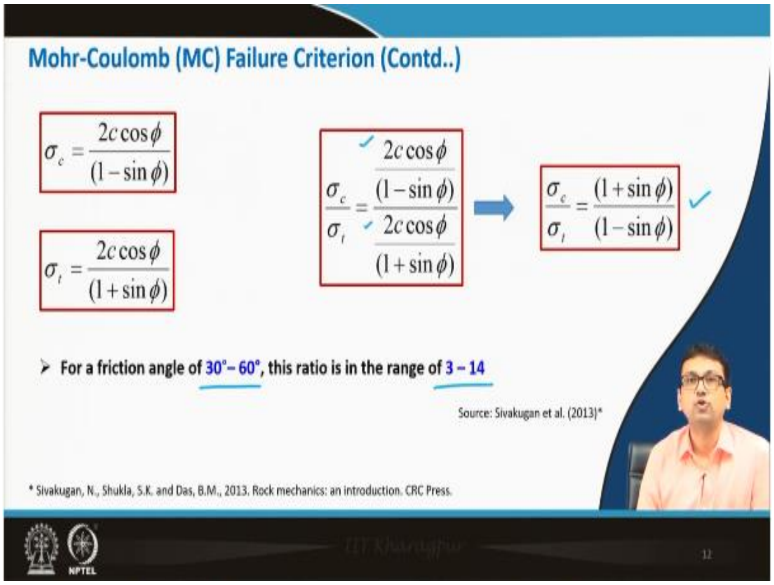

Now on taking the ratios of both the values we get (divide equation 6 by equation 7)

$$\frac{\sigma_c}{\sigma_t} = \frac{\frac{2c\cos\phi}{1-\sin\phi}}{\frac{2c\cos\phi}{1+\sin\phi}}$$

$$\frac{\sigma_c}{\sigma_t} = \frac{1+\sin\phi}{1-\sin\phi} \qquad (8)$$

For a friction angle of 30 to 60 degrees above ratio (eq. 8) is in the range of 3 to 14, equation 8 interprets that σ_c is always greater than σ_t as expected from Mohr circle.

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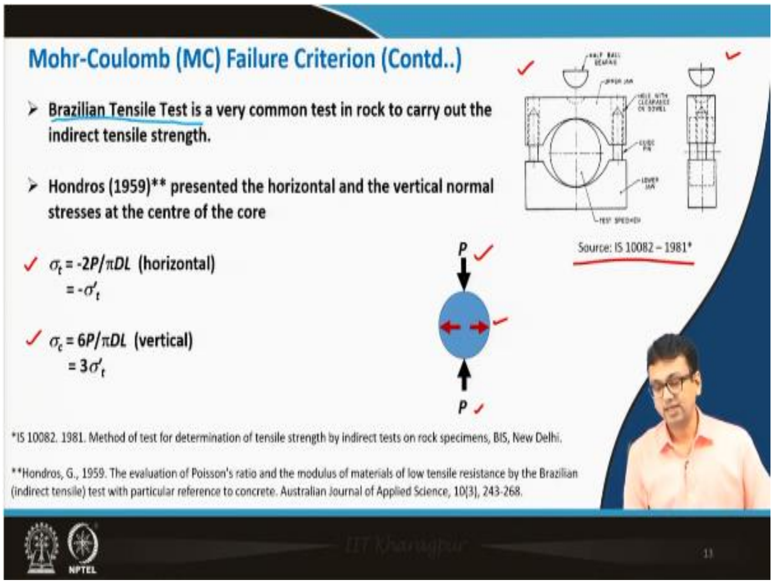

Now as stated that conducting direct tensile strength test is difficult due to some issues and also we learnt about Brazilian Tensile Test. So, now with respect to that, we will see how we can utilize this Mohr circle and can get a useful expression.

So, we know Brazilian tensile test is a very common test in rock to carry out the indirect tensile strength. Now we can see that load P is applied on the sample using Brazilian tensile strength test apparatus, as a result of which cracks will develop in between i.e. along the line of load cracks will develop; in horizontal direction tensile force will develop as a result of which the sample will fail. So, it is an indirect way of determining the tensile strength of the rock sample.

So, now Hondros in the year 1959 presented the horizontal and vertical normal stresses at the center of the core, as

$$\sigma_t = \frac{-2P}{\pi DL} = -\sigma_t' \quad \text{(horizontal)}$$

$$\sigma_c = \frac{6P}{\pi DL} = 3\sigma_t' \quad \text{(vertical)}$$

where D is the diameter of the sample and the L is the thickness of the sample.

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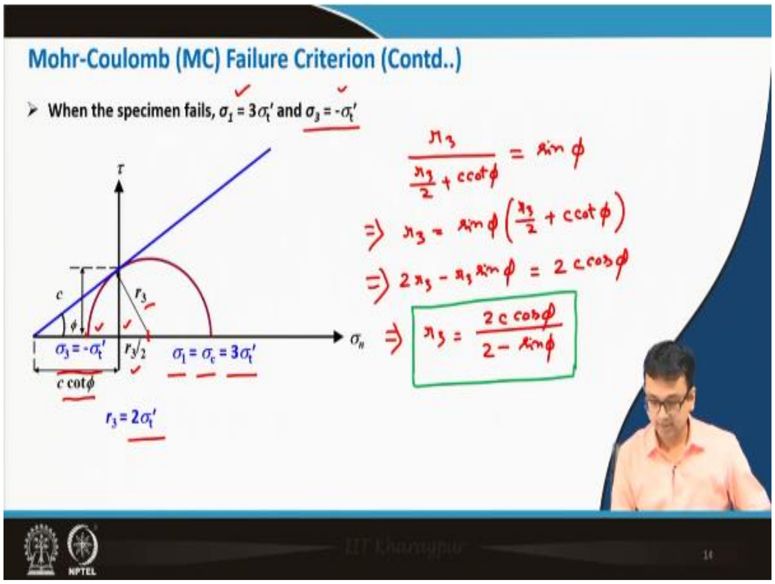

Now as per this whatever we have understood, when the specimen fails then

$$\sigma_1 = 3\sigma_t' \text{ and } \sigma_3 = -\sigma_t'$$

Where σ_1 and σ_3 are major and minor principal stresses respectively.

From the previous understanding of Mohr's circle and geometry, from the figure in the slide we can get

Radius, $r_3 = 2\sigma_t'$

Therefore,

$$\frac{r_3}{\frac{r_3}{2} + c\cot\phi} = \sin\phi$$

$$r_3 = \frac{2c\cos\phi}{2 - \sin\phi}$$

As $r_3 = 2\sigma_t'$, hence

$$\sigma_t' = \frac{c\cos\phi}{2 - \sin\phi}$$

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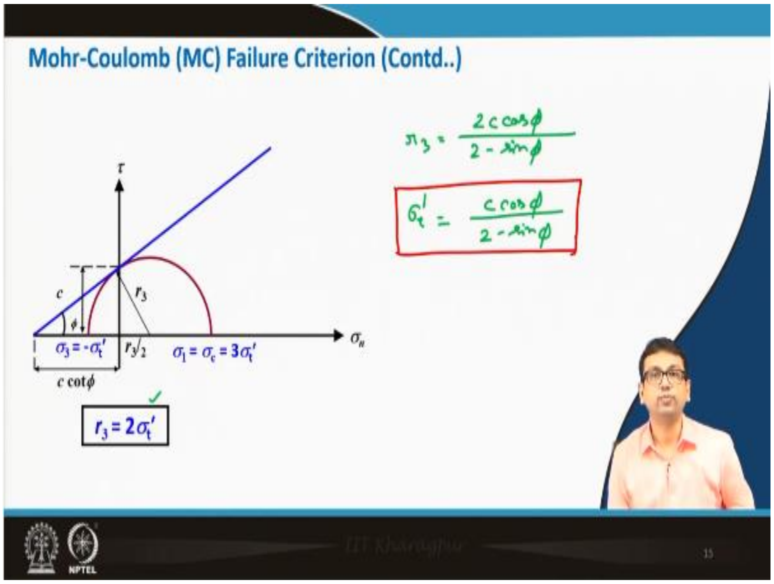

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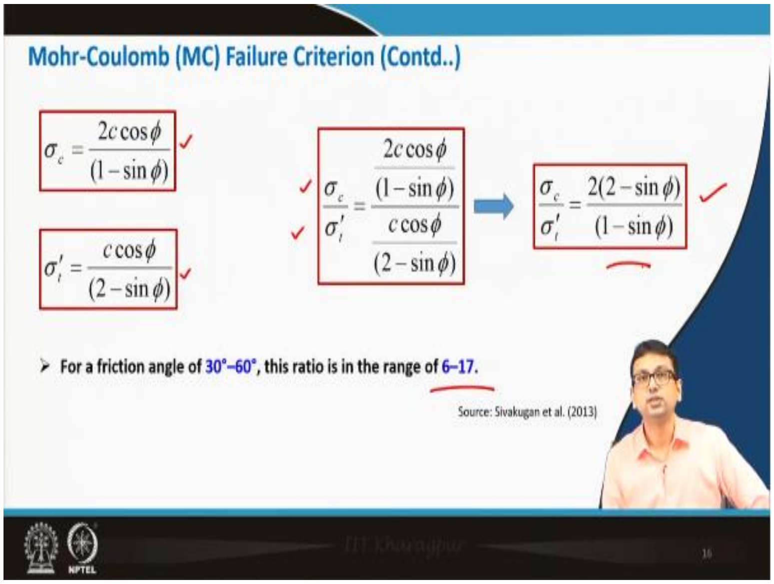

Now, as we know

$$\sigma_c = \frac{2c\cos\phi}{1-\sin\phi} \text{ and } \sigma_t' = \frac{c\cos\phi}{2-\sin\phi}$$

$$\frac{\sigma_c}{\sigma_t'} = \frac{2(2-\sin\phi)}{(1-\sin\phi)}$$

For ϕ=30° to 60° the above ratio will lie in the range 6-17.

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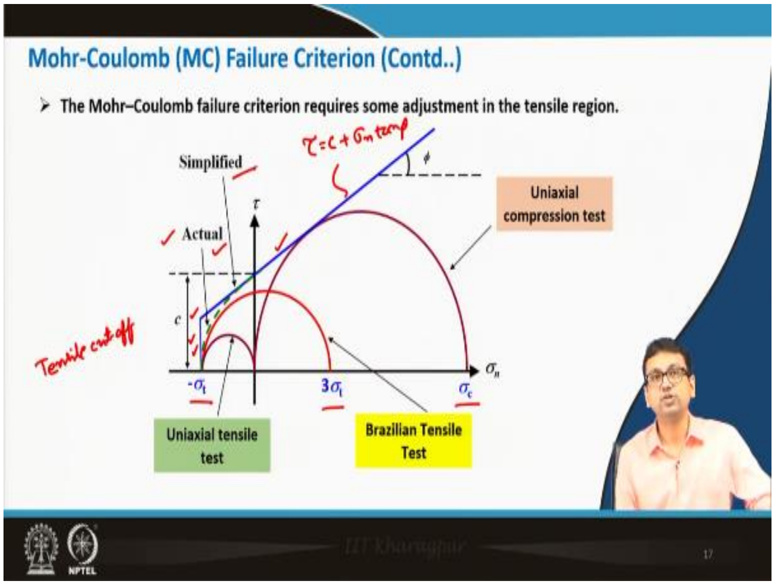

Now the Mohr-Coulomb yield criteria requires some adjustment in the tensile region, to avoid the overestimation of the performance of the rock in the tensile region.

In the figure it can be seen that the failure envelope is not extended and is cut at $-\sigma_t$, it is called tensile cut off.

If the actual data of failure envelope is plotted then the non-linearity can be observed but even though tensile cut off is considered simplified MC envelope overestimates the strength in the tensile region due to linearity assumption.

So to summarize,

$$\sigma_c = 2c\tan(45+\phi/2) = 2c\frac{\cos\phi}{1-\sin\phi} = 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\sigma_t = 2c\tan(45^\circ+\phi/2)+\sigma_3\tan^2(45+\phi/2) = \sigma_c+\sigma_3\tan^2(45+\phi/2)$$

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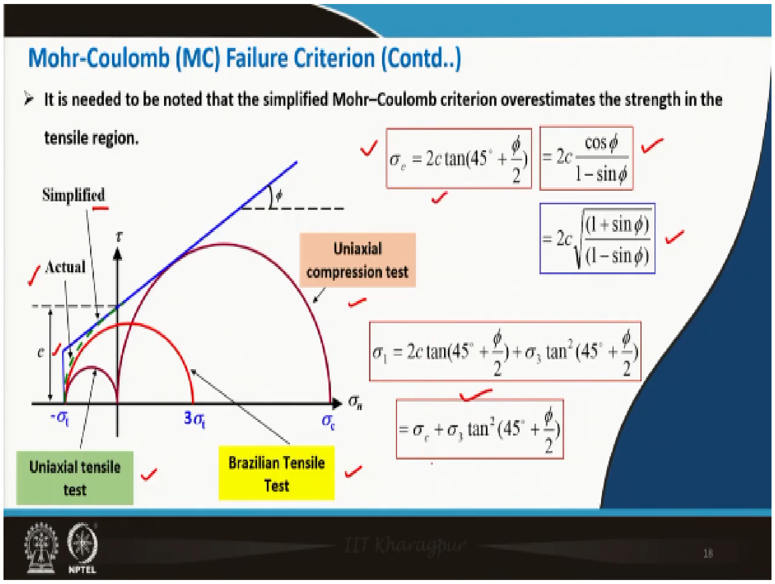

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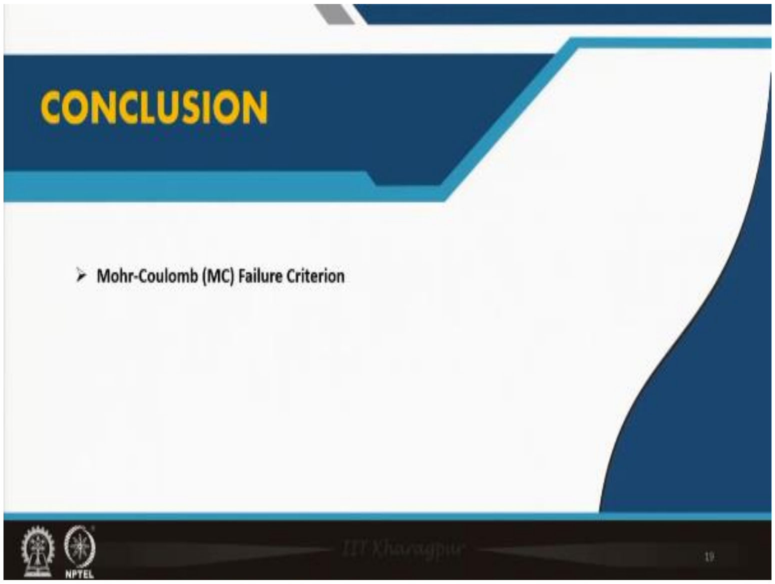

So, today basically we have discussed in detail about the Mohr-Coulomb failure criteria, we have derived a number of things, so I think in the next lecture we will start discussing about the other failure criteria, so let us conclude here, thank you.