Rock Mechanics and Tunneling
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Lecture 15
Mechanical Properties

Hello everyone, I welcome all of you to the fourth lecture of module 3. So, in Module 3, we were discussing about physical mechanical properties. In initial 3 lectures, we have covered the physical properties. Now, today we will start discussing about the mechanical properties of intact rock and rock mass.

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So, primarily we will discuss about mechanical properties and under that this week's Hookes law is very important. So, we will a little bit focus on this part Hookes law. So, but the main topic we will discuss is about the mechanical properties of rock.

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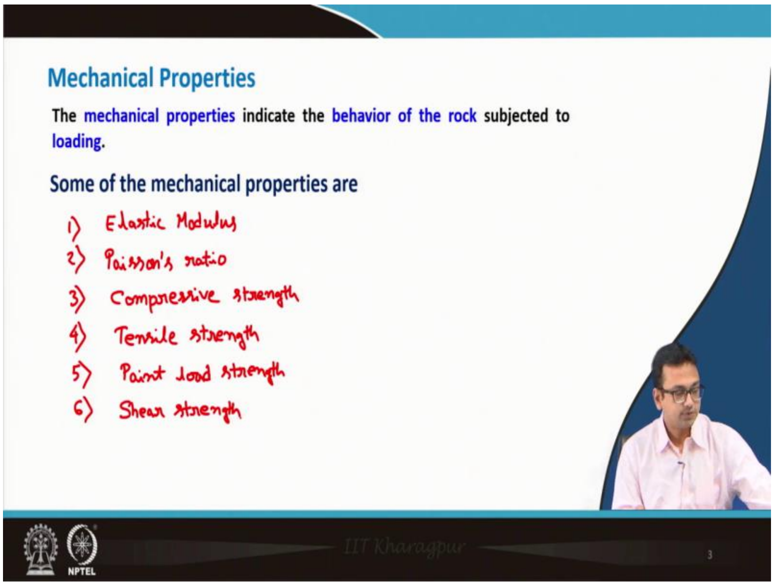

So, now, just as I have stated earlier that mechanical properties indicate the behavior of rock subjected to loading. So, that is already we have discussed in our initial class of this module. So, now, some of the mechanical properties about which we should have some idea are like elastic modulus, then Poison's ratio, then we should also discuss about the compressive strength also the tensile strength then the point load strength, point load strength also definitely we should discuss about the shear strength.

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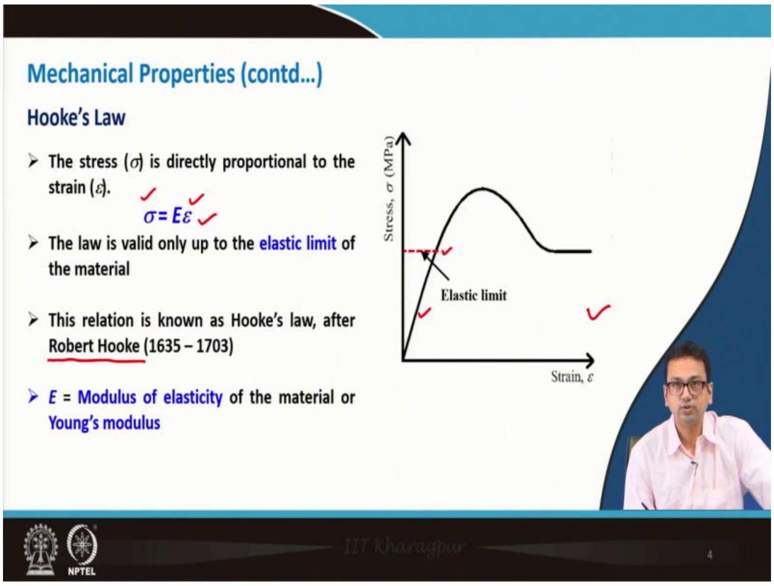

So, let us start discussing about the Hooke's law. About Hooke's law all of us are familiar with this law, in school days also you have learned in your undergraduate also must have learned about it, but it is very much important. So, I think we should little bit discuss about this topic.

So, so, it is a typical stress strain plots if we consider now, suppose this is my elastic limit means stress is proportional to strain here. So, as you can see this portion is almost linear. So, the law is valid only up to the elastic limit of the material, we know that and this relationship that is stress is proportional to strain and if you write in this form stress is equal to E into epsilon, this is my stress and this is my strain is nothing but my Young's modulus.

So, this relationship is known as Hooke’s law and this is after the name of Robert Hooke, as we know E is nothing but the modulus of elasticity of the material or the Young's modulus. So, this is something we know quite well.

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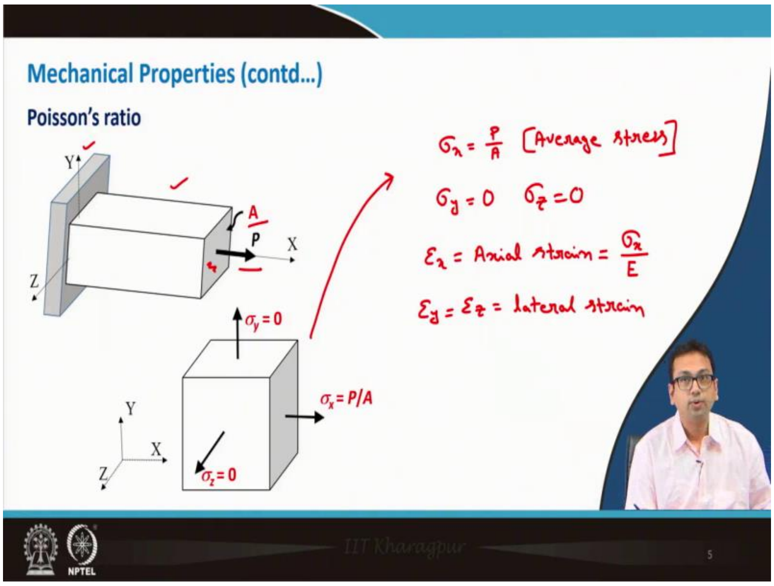

Now, one important thing is Poison’s ratio. It is very important to discuss because that gives us a lot of information. Like if we first think about suppose this is a maybe a beam and fixed at this point here is the beam. Now a load P is acting along x axis only and in y and z direction there is no force acting, let us consider the area of this surface is A.

Now, since, there is no force acting on y or z direction, so, what we can say that sigma x is nothing but p by A now p by is what, P by is sigma x is equal to P by if we write that indicates the average stress that all of us we know, because, if we want to present it the in terms of stress at a point, then we need to present it in terms of the limit as we know, but this is nothing but the average stress.

So, as we can see from this one maybe if touch on this one and now, what are the things we can notice we can write this is nothing but equal to P by A, which is the average stress. Now, what we can notice that since we have not applied any force in Y and Z direction, so, what we can clearly notice sigma y is equal to 0 and sigma z is also equal to 0.

Now, those sigma y and sigma Z are equal to 0 that does not mean my strain in y and z direction will be 0; there will be some strain, some strain will definitely develop. Now, that strain will develop because of the Poison's effect as we know. So, if we consider epsilon x as my actual strain, so, how do we can write from our Hooke's law it is very easily we can write it as this sigma x by E, young modulus. Whereas, mine let us consider epsilon y and epsilon z are my Lateral strains; one is in y direction one is in z direction.

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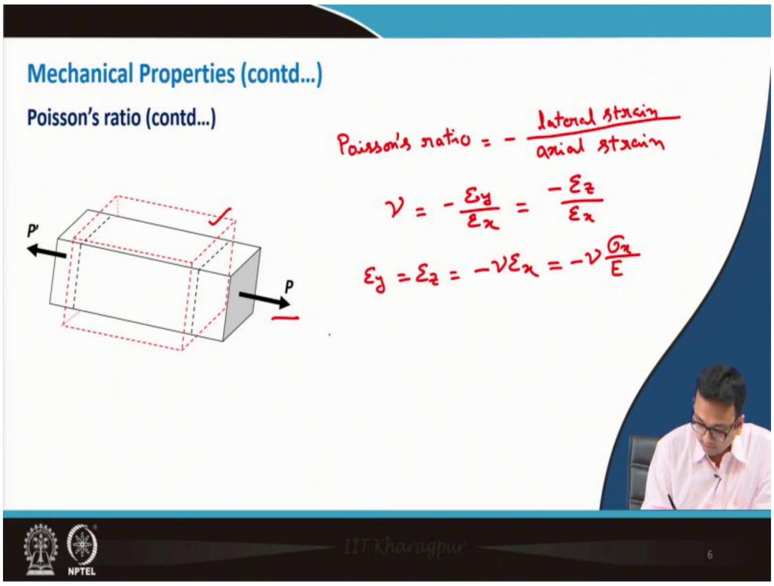

Now, let us see next diagram. Yes, so, this is what actually what I was telling, if I apply a force P here so then means, if it is initially suppose of this shape, then because of the application of load, what will happen it will elongate and it's in y and z direction it dimension will reduce. So, that is very well understood. So, that is nothing but called as the Poison's effect and from there this Poison's ratio comes to the picture.

So, the Poison's ratio it is also well known to us, Poison's ratio is nothing but equal to the minus lateral strain by actual strain or longitudinal strain. So, this Poison's ratio generally present using this term nu and nu can be written as then minus y by x or minus epsilon z by epsilon x because epsilon y and epsilon z both are nothing but our lateral strains. So, just you have written remember, here for lateral strains, they are these are the lateral strains. So, we can write it as like this very easily.

Now, if we fought the tried to get the as we have seen here that the actual strain is nothing but sigma x by E. Now, if we are interested in obtaining this epsilon y and epsilon z in terms of the stress what will happen, that we can very easily right from this we can get epsilon y is equal to or even epsilon z is also equal to both we can write as equal to nothing but minus nu epsilon x, no confusion. So, from here we are writing this.

So, now, what we know epsilon x is equal to sigma x by E. So, this is equal to minus nu sigma x by E. So, that means, if again if we look at this one this block, so because of the application of this nonzero stress, though here these are 0 stress some change in its lateral dimension is happening, because of this force. So, that is nothing but the Poison's effect.

So, what we can see, if we consider this scenario, where we have applied only force in the x direction not in y and z direction, then what we can write, we can write the strain if I want to write down the strain, so, like my strain maybe, let me go to the next slide.

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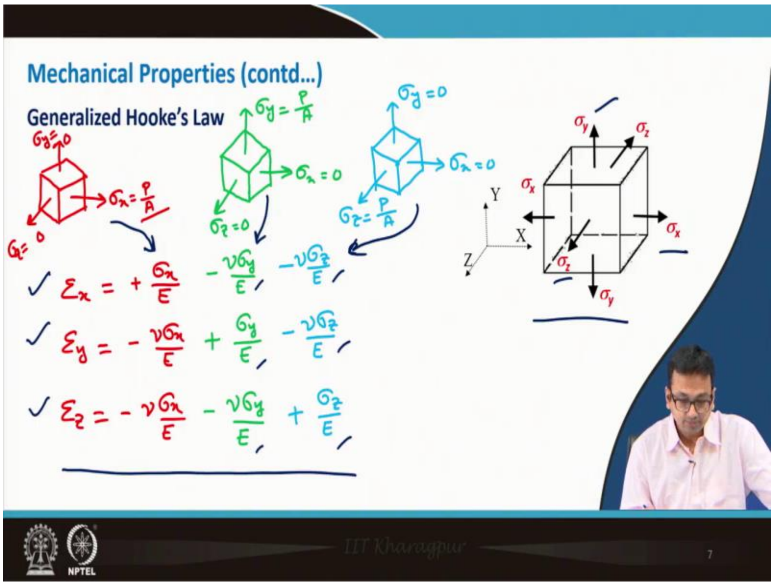

So, here just first you consider only the keys of only you have applied the force in the x direction. So, suppose just one by one let me take I think that will be easier for you to understand. So, if this is the case where only I have applied I have this only so it is sigma x is equal to suppose P by A average stress and here and here they are maybe 0 and 0, then if I want to write down my epsilon x that is definitely nothing but this plus sigma x by E.

But, because of this some Poison's effect due to the Poison's effect what is happening some strain is developing in y direction and some strain is developing in z direction. So, the strain is developing in the y direction is nothing but if we go back here it is nothing but this and as well as in the z direction also this. So, we can write it as this, because of this case we have this scenario, so, minus nu sigma x by E minus mu sigma x by E, no confusion very simple.

Now, if I want to think another scenario in the same line that we found to now think about it, we take another color maybe, so, maybe take green. So, if I consider now the case where we have applied only force in the y direction and in x and y direction, we have not applied any for suppose that is the scenario. So, suppose this is the case.

So, this is equal to 0, this is equal to 0 whereas, this is probably let us consider is equal to P by A (average stress). So, then what will happen, because of this force we have applied in the y direction what will happen there will be definitely a actual strain will develop in y direction. So, that is nothing but sigma y by E.

And now, because of the Poison's effect what will happen similar to this just if you look at here, because of like these, here presently we have sigma y we have applied nonzero values of sigma y. So, we have some axial strain that is sigma y by E, but lateral strain that is epsilon x and epsilon z will be nothing but minus nu sigma y by E. So, same similar way we can write there, so, minus nu sigma y by E and minus nu sigma y by E, no confusion.

Now, similarly, if you want to let me here also write z equal to 0 and sigma y equal to 0. Now, the third case let me use some another color, maybe this one so, where we will apply the force in the z direction, but in x and y direction there will not be any force applied. So, as a result of the average stress will be definitely equal to 0, but, so, this is the case sigma y is equal to 0 where but suppose in this direction we applied the force. So, sigma z is equal to now suppose P by A, the average stress.

So, if this is the scenario, then what will happen, our axial strain in z direction will be definitely our sigma z by E and now, due to Poison's effect we will have minus nu sigma z by E and minus mu sigma z by E. So, now, basically what we have now obtained here, so, this is from this, this from maybe let me so, basically what we can notice here that this is the case which is giving this column, this is giving us this column and this one is giving us this column.

So, now, together this is called as the generalized Hooke's law. Now, this diagram is the case that generalized case means, what we have done basically, we have individually considered 3 cases; first case only x direction force not in y, not in z. Second case is only y direction force, not in x not in z and similarly, third case where we have considered force is applied only in the, axial force applied only in the z direction not in x and not in y.

Now, because of the first case, we will get these strains will develop in x y and z direction. Now, for the second case, these stresses or strains will develop. So, this is the axial strain, these are the lateral strains. And similarly, for the third case, where we have applied axial force only in the z direction noting in y and x direction there is no force in that case strain will develop definitely in y and x direction.

So, together if we superimpose 3 cases, we will get this one which is nothing but my generalized Hooke's law and in this figure after super position, all the 3 directions, my normal stresses are acting as you can see, and non-zero normal stresses are acting and because of that, what will happen to my equation.

Now, obviously, if your other two in one direction it is some non-zero other two 0, then obviously, accordingly your things will also change and it will give us the in generalized form what will be the strain in x direction, strain in y directions, strain in Z direction. So, this is very important for any solid ones, if we are going to do any mechanics related what the understanding of generalized hooke's law, , it should be very clear to all of us, so, that is why we have taken this and have discussed about this in detail and spend a good amount of time. So, this is our generalized Hooke's law.

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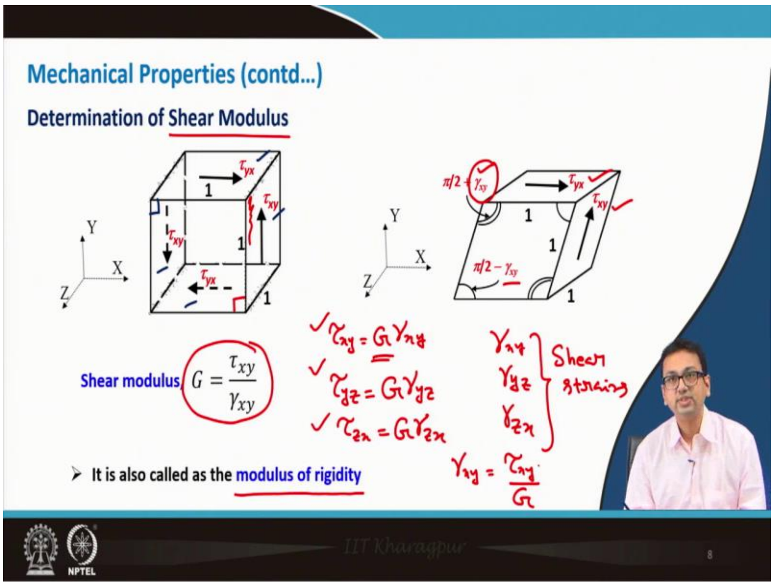

Now, we have discussed here about the actual force applied and because of that, some normal stresses are developing though mainly we are considering everything in terms of average normal stress as we know. Now, there is another type of stress may develop as we know that is nothing but my shear stress.

So, if it is a like a cube, so, you need cube if you consider like these and now, if you apply this kind of shear stresses like these and these and these, then what will happen, we can visualize this obviously, this will take a shape like this rhomboid shape it will develop. So, because of that what is happening initially, these angles were so like pi by 2, now, because of the action of this shear stress what is happening this angle is reducing and this angle is increasing.

So, this quantity is nothing but my, this is nothing but my shear strain. So, now, basically what first we can write maybe tau xy is equal to G comma xy, note here this G is nothing but the shear modulus or modulus of rigidity. So, that is only here it is written so, G is nothing but our tau xy by gamma xy. Now, this equations for other two stresses what it will be it is very simple tau yz will be nothing but equal to G gamma yz and tau zx will be equal to Z gamma zx.

So, this gamma xy gamma yz and gamma zx are nothing but my shear strains. And these 3 are nothing but my shear stresses and G is my modulus of rigidity or shear modulus. So, it is also known as the modulus of rigidity as it is written. Now, one small thing like this x and y this first subscript and the second subscript what are they, I hope you know.

Mainly the first subsequent justify in general for all if I want to say the first subscript like x is representing the means corresponding to the plane on which this shear stress is acting and next to y your second subscript is that related to the direction of the action of this shear stress. So, basically you see this is a plane which is basically perpendicular to my x axis. So, tau this first x is for that and next y is you see the direction of this shear stress is in the y direction.

So, y so, first subscript is related to the plane on which the stress is acting and second subscript is related to the direction of the that act, the action of that in the which direction the stress is acting. So, the plane means the plane perpendicular to here perpendicular x axis. So, tau x y likewise, if it is like tau yz that will indicate y will indicate the plane, but when will the y axis it is acting and the direction is in the z direction. So, this is I hope you already knew, but just discussed a little bit.

So, these 3 are the equations now, these strains obviously, similarly, the strains can also be written in the form of as we have seen the strain earlier case we have written in terms of epsilon like epsilon x or sigma x by E likewise, here what will happen , if I want to write gamma xy, it is nothing but equal to tau xy by G. So, likewise for other 3 cases or two cases also it will be similar.

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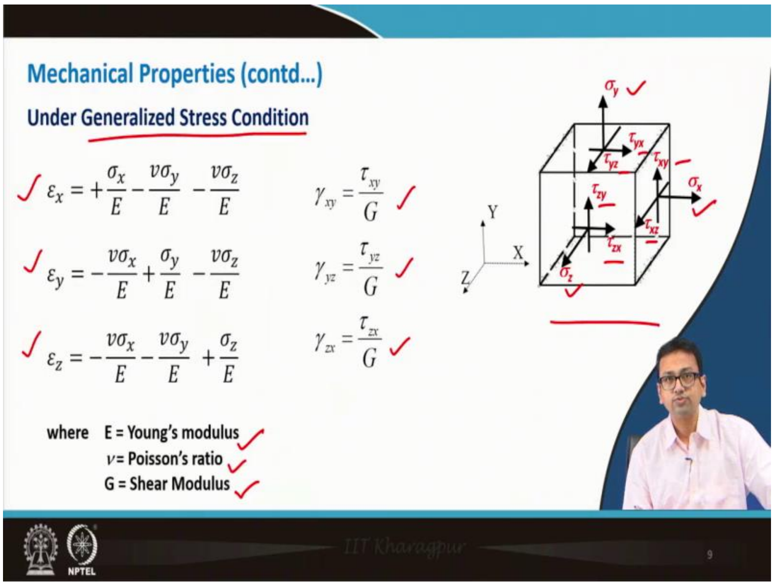

So, now, if I consider a case like this generalized case were not only the normal stresses but also the shear stresses are acting. So, that becomes actually in true sense of the generalized stress condition and under there not only these 3 equations what we have just have derived actually, along with this we need to also consider these 3 equations also.

So, under generalized stress condition, if you want to find out the strains like epsilon x, epsilon y epsilon z and as well as gamma xy, gamma yz, gamma zx, for that we have to use these equations. So, that is very important thing and here as we know E is nothing but the Young's modulus, nu is my Poison’s ratio and g is my shear modulus.

So, this is a case of generalized loading and generalized stress condition as it is shown over in this diagram for that, this state of equation we need to, so, this is also means it is in true sense generalized Hooke’s law and this equation like as we see well these are called also sometimes called as the Hooke’s equation or Hooke’s law for shear stress strain, the stress-strain relationship.

So Hooke’s law for shear stress and relationship very much similar to like sigma x is equal to E epsilon x likewise, here this is for the shear stress. These are the some few things which you already knew, but I think we should also discuss that so, you have discussed over here

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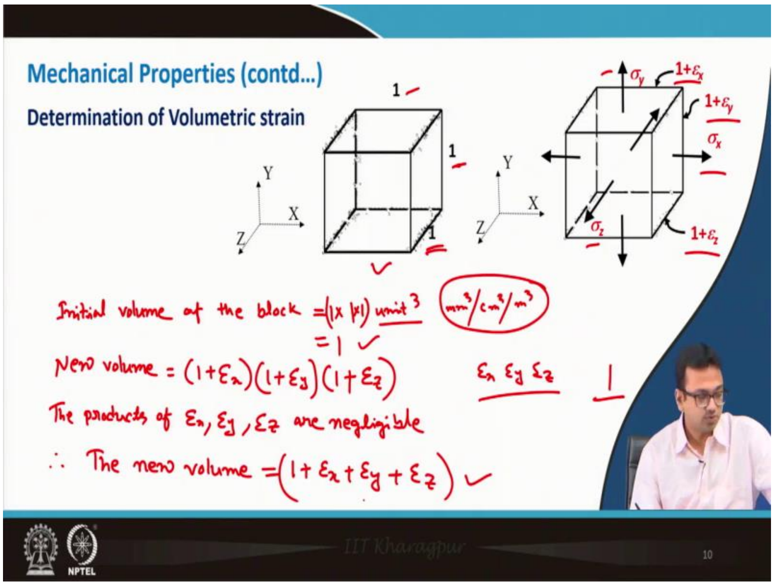

Now, another term is very important that is called the volumetric strain. So, we should have some knowledge about the volumetric strain also. So, basically I will try to basically derive the expression for volumetric strain. All of us or probably you know what is the expression but we can derive it in maybe different we can take different approaches. So, maybe here I will take a simple approach that is just consider first this is the initially, this is a cube, unit cube, so, all the x y c (directions) dimensions are 111 and now, in this diagram what we have shown just the normal stresses are applied over here.

So, now, if we apply normal stress like this, so, what will happen epsilon sigma x if I apply so, this dimension initially was 1, but now it will become 1 plus epsilon x. Similarly, initially this dimension was 1 as we can see now, it is becoming this because of the application of these sigma x sigma y and sigma z and similarly, initially this dimension was 1 and now it has become like this.

So, what we can write the to derive the expression so, we can write the initial volume of the block if we consider is nothing but what, 1 so, maybe if it is centimeter cube one meter cube or maybe millimeter cube. So, you need to cube just some bits maybe millimeter like millimeter cube or centimeter cube etcetera or meter cube etcetera. So, simply writing unit cube.

Now, as I have stated initially this was 1 because it is nothing but how it is coming 1 by the way, it is coming as 1 into 1 into one 3 dimension so, it is becoming 1. Now, the new dimensions are these as we have marked, so, new volume will be 1 plus epsilon x 1 plus epsilon y 1 plus epsilon z.

So, now, if you simplify it, what we will get? We see that epsilon x is multiply, epsilon y or sometimes epsilon y you will multiply with epsilon z and epsilon makes epsilon z they will be multiplied with each other. Now, what is the thing like this epsilon x or epsilon y or epsilon z they are very small quantity as compared to this 1 what we have considered the unit dimension, means these are very small quantity as compared to 1.

So, what we can consider that since they are very small quantity and the multiplication of that means the product of this epsilon is x into epsilon y or epsilon x into epsilon z they will be even smaller. So, what we can do, we can simply neglect those times. So, what we can do in that case there we can write the products of epsilon x, epsilon y epsilon z are negligible. So, what we can write?

So, therefore, the new value can be simply written as 1 plus epsilon x plus epsilon y plus epsilon z. So, this is my original initial volume and this is my new volume, final volume. Now, what we can do let us go to next page. So, remember this is 1, initial volume 1 and new volume is this.

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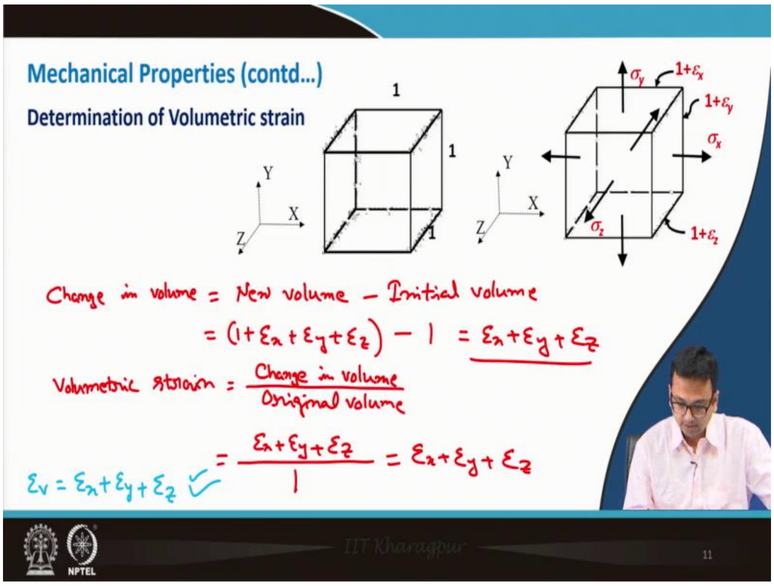

So, what we can write the change in volume is what then so, the change in volume is nothing but the new volume or final volume minus initial volume. So, this is equal to 1 plus epsilon x plus epsilon y plus epsilon z minus initial volume was 1. So, which is equal to simply 1 1 cancels so, epsilon x plus epsilon y plus epsilon z.

Now, what is volumetric strain or simply what is strain, then that is nothing but the change in length by origin length. Likewise, volumetric strain is change in volume by original volume. So, volumetric strain can we return as change in volume by original volume or initial volume.

So, change in volume is epsilon x plus epsilon y plus epsilon z what we have seen over here and original volume is unity. So, volumetric strain is nothing but epsilon y plus epsilon z. So, generally it is written using this v epsilon v epsilon x plus epsilon y plus epsilon z. So, this is the expression for volumetric strain we have derived with just with the base fundamental considering a unit cube simply, means cube of having all the dimensions 111 and its volume is this 1 unity.

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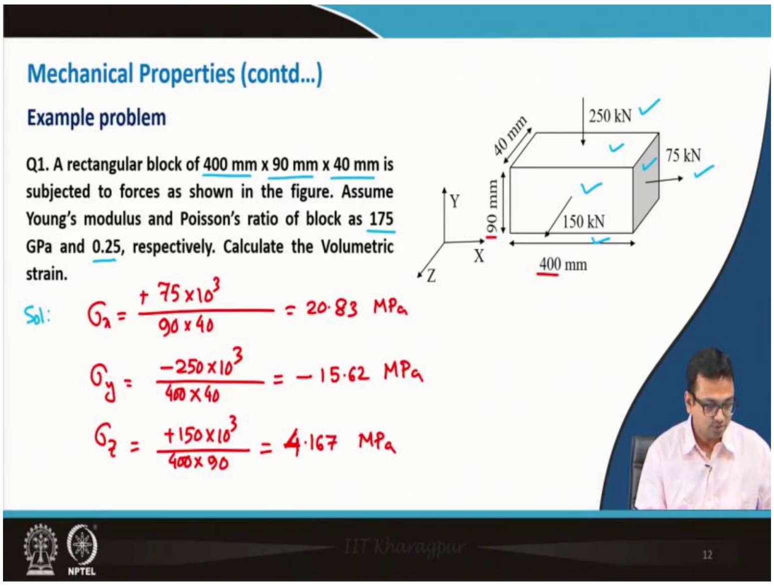

Now, with this idea, let us take a small problem the problem says the a rectangular block of 400 mm by 90 mm by 40 mm is subjected to forces as shown in figure. As you can see here 75 kilo newton, here tensile force there it is 250 kilo newton compressive force again here Z direction 150 kilo newyon tensile force is acting. Assume Young's modulus and Poison’s ratio of block as 175 Giga Pascal and 0.25 respectively calculate the volumetric strain.

So, very simple problem. So, what can be done let us see. First solution. So, first what we have to get see here I ultimately I have to obtain epsilon x, epsilon y and epsilon z we need to add them and we will get my volumetric strain, but here things would what are the informations available to us, we know the area of this area, this area, this area and the corresponding force what is acting.

So, that means, we can find out the stresses from there and from there what we can do, we can apply our knowledge of generalize Hooke's law that will obtain the strains and from there we can obtain the volumetric strain. So, let us do that quickly. So, first is sigma x is nothing but in the x direction 75 kilonewtons. So, 75 into 10 to the power 3 and the area is how much, you see it is 90 this and this is 40. So, 90 into 40 it is becoming 20.83 Newton per millimeter square which is nothing but mega Pascal.

y is equal to what, considering tension as positive actually we are doing so, here we have the tensor plus so likewise here should be minus 250 into 10 to the power 3 to make it in newton and area is how much, 400 by 40, 400 into 40. So, this is becoming minus 15.62 newton per millimeter squared, which is nothing mega Pascal. And Z is equal to what?

So, we have 150 kilonewton again tensile 150 into 10 to the power 3, kilonewton to Newton and the area is how much 400 and here 90 so 400 into 90 which is becoming now 4.167 Newton per millimeter scalar mega Pascal. So, this is these are the 3 stresses. Now, what do we know, we can we ultimately have to obtain the strain. So, let us go to our maybe the next page.

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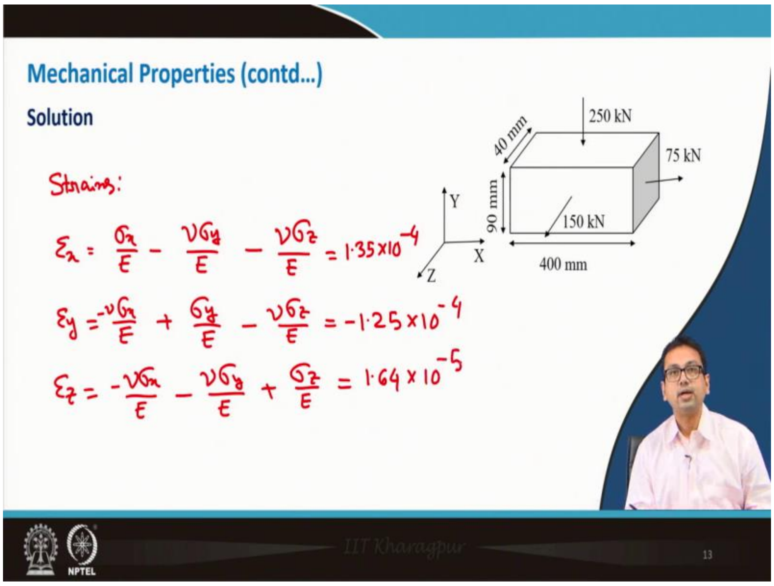

So, now, let us try to get the strains, so, the strains, so, first is epsilon x is nothing, but as we know sigma x by E minus what, nu sigma y by E minus nu sigma z by E. Just let us quickly verify we have it we have derived it ourselves only or maybe here we can see so, sigma x by E minus nu sigma y by E y minus nu sigma z by E or where you have derived this one thing these are the things you have derived yourself.

So, I just utilize that. Similarly, so, if I do that and we know sigma x we know E and nu are provided, E value is given as how much 175 Giga Pascal, GPa and a means, and this is Poison ratio 0.25 it is given. So, if you replace over here and just directly writing the value can cross check it 1.35 into 10 to the power minus four this will be the strain likewise, epsilon y will be likewise sigma minus nu sigma x by p plus sigma y by E minus nu sigma z by E.

This will give us minus 1.25 into 10 to the power minus 4 and z epsilon z is equal to minus nu sigma x by E minus nu sigma y by E plus sigma z by E. So, if we do that we will get it is 1.64 into 10 to the power minus 5.

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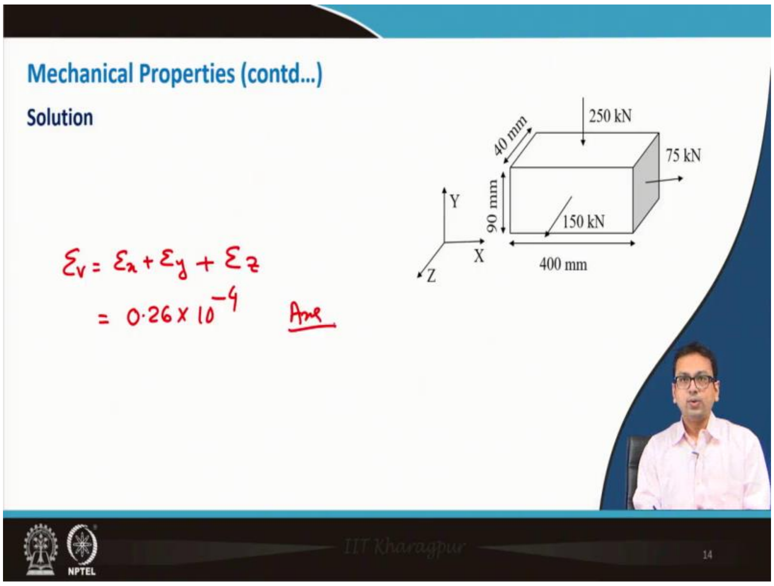

So, now, volumetric strain is what volume metric strain is nothing but my x plus epsilon y plus epsilon z. So, if we just simply add them you will get either 0.26 into 10 to the power minus 4. So, this is your answer. So, yes we have solved a problem also. So, and we have learned how to derive generalize Hooke’s law. So, with this let us conclude our today's lecture. Thank you.