

Structural Reliability
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Lecture –96
Reliability Problem Formulation (Part - 08)


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Reliability problem formulation

Structural Reliability
Lecture 12
Representation
of systems

Recall: System vs component/element

- Is the item of interest:
 - made up of two or more units
 - Logically or physically connected
 - so that the item's performance can only be described in terms of the units' performance
- Yes \Rightarrow System reliability problem.
Each irreducible unit is an element (or, component).
- No \Rightarrow Element reliability problem



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In the previous lecture we looked at how to set up the reliability problem and one of the things we discussed was how to identify whether a problem with a system reliability problem or an element reliability problem. So, these are the questions that we asked is the system of interest made up of two or more units that are logically or physically connected. And the key point here was that the items performance whether it is working or not working can only be given in terms of those of the constituent units.

So, if the answer was no then we have an element reliability problem and if the answer was yes. Then we have a system so, that is what we are going to look at today in a more formal manner. We have looked at how to describe systems in terms of very simple combinations in some intersections in the previous lectures. But today we are going to look at various ways in which we can represent a system in terms of its constituent elements.

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System representation – series and parallel

A system is composed of interconnected components (or, equivalently, "elements").

The successful performance of the system requires that all or at least some of the elements, depending on the design of the system, perform satisfactorily.

System performance can be represented in terms of the performance of the elements.

Let F_i denote failure of the i^{th} element.

The two simplest representations of a system made up of binary components is the series system, and the parallel system.

Unfortunately, real systems are rarely this simple!

Take for example the dependent parallel system:

$$P_{\text{sys}} = P(F_1 F_2 \dots F_n) \\ = P(F_1 | F_2 \dots F_n) P(F_2 | F_3 \dots F_n) \dots P(F_1 | F_1) P(F_1)$$

For a series system, every element must work for the system to work:

$$\text{Series: } F_{\text{sys}} = F_1 \cup F_2 \cup \dots \cup F_n$$

$$\bar{F}_{\text{sys}} = \bar{F}_1 \bar{F}_2 \dots \bar{F}_n$$

$$\text{Series: } R_{\text{sys}} = P[\bar{F}_{\text{sys}}] = P[\bar{F}_1 \bar{F}_2 \dots \bar{F}_n]$$

$$\text{Series: } R_{\text{sys}} = \prod_{i=1}^n R_i \quad (\text{mutually independent})$$

For a parallel system, the system works as long as at least one element works:

$$\text{Parallel: } F_{\text{sys}} = F_1 F_2 \dots F_n$$

$$\bar{F}_{\text{sys}} = \bar{F}_1 \cup \bar{F}_2 \cup \dots \cup \bar{F}_n$$

$$\text{Parallel: } R_{\text{sys}} = 1 - P[F_{\text{sys}}] = 1 - P[F_1 F_2 \dots F_n]$$

$$\text{Parallel: } R_{\text{sys}} = 1 - \prod_{i=1}^n P[F_i] = 1 - \prod_{i=1}^n (1 - R_i) \\ (\text{mutually independent})$$



So, the system is considered constituted of its elements and its performance can be given in terms of those of its elements. So, that is our starting point. Let F_i be the failure of element i and we have looked at these two cases already simplest representation of a system made up of binary elements of the series system and the parallel system that what binary is important and which basically means that it can either be in upstate or down state safe state or failed state working state or not working state.

And that is the most common representation that we take in systems reliability both for elements and for the system and what we will do for our structural systems as well. So, for a series system every element must work for the system to work. So, that is the logical representation of the series system. So, the failure of the system for the series system is the union of all the individual failures all the elements.

Looking at the; system survival or F_{sys} complement that is the intersection of all the individual complementary events. So, all elements must work in order for the system to work. So, the system reliability for the series system then is the probability of the intersection of all the element survival nodes. So, P of F_{sys} complement of the reliability of the system is P of F_1 complement intersection F_2 complement all the way up to F_n complement.

If the elements were all independent are mutually independent then the reliability of the system

is simply the product of the individual reliabilities. This is also one of the drawbacks of the series system because even if an element or all the elements have high reliability by themselves but the product of each of those finally might lead to a number that is not high enough. So, that is one of the essential drawbacks of the series configuration.

On the other extreme for a parallel system the system works as long as at least one element works. So, that is a very benign configuration and we can express the system failure as the intersection of all the failure events or the element frameworks or conversely the complement the system survivor is the union of the element survivals. So the reliability of the parallel system is one minus the system take the probability which is simply the product of the which is simply the probability of the intersection of all the events.

So, system reliability is $1 - P(F_1 \cap F_2 \cap \dots \cap F_n)$ and if the elements were mutually independent then the failure probabilities are the product the system failure probability is the product of the element from the probabilities and the reliability of the system is one minus of the product of all the system for your probabilities. So, that would be the simplification for parallel systems if all the elements were mutually independent.

Now unfortunately real systems are really this simple. So, it is offering highly unrealized or too idealized to represent the system as a series or parallel configuration and that too with independent components. Take for example this particular case it is a parallel system but the elements are dependent. So, we can express the system failure probability as the product of the chain where on the right you see $P(F_1)$ and then $P(F_2 | F_1)$ and then so, on F_1, F_2 and so on all the way up to $P(F_n | F_1 \cap F_2 \cap \dots \cap F_{n-1})$.

Now if they were mutually independent then this would simply be the product of all the individual failure which if they are not mutually independent the problem is then these probabilities especially when we have several conditional events F_1, F_2 although $n - 1$ those terms quickly approach might approach one because of load sharing because of dependence and so on. So, in such an example the benefits of the parallel configuration would not be realized to the extent that we would if they were all mutually independent. So, these are some of the things

that we see in real systems.

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System representation - issues

For real systems, more complex system logic may be necessary.

- Element failures may be dependent events.
- Not all elements may be loaded at the beginning.
- An element may have more than one failure modes, e.g.
 - a diode that can fail either in open or short circuit mode
 - a pump that provide full power or half power
 - A truss member that buckles in compression but is able to take high tensile loads

Different ways of representing a system

- State space
 - Binary components
 - Multi-state components
- Structure function
 - Monotone
- Reliability block diagram
- Fault tree
- Event tree
- BBN
- Cut sets
 - minimal
- Path sets
 - Minimal
- etc.



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So, for real systems more complex system logic may be necessary in most cases because as we said element failures may be dependent events not all elements may be loaded at the beginnings. These are the different situations which would require more complicated modeling. An element may have more than one trillion; I mentioned binary elements but if an element is not binary with the multi-state failures.

I have given three examples of a diode that can fail either in an open or a short configuration. So it is a three-state system a pump that works fine then works at half power and then does not work at all. So, it is again a Free State system a trust member which is either working fine but then compression it buckles and then in intention it has a completely different behaviour. So, these are multi-stage systems.

In such cases we might need to look at different ways of modeling the system there are different methods and let me list some of them and then indicate which ones we are going to look at in greater detail in this lecture. So, one way is to look at simply the state space of the system and that is possible if the state space is finite in size and this works whether the system or whether the element whether the elements are binary or multistate in nature. So, this method can work in both situations.

Then we have the method of the structure function and especially if we can assume or we can conclude that a system has a monotone structure function then it becomes somewhat easier to model it. Then we have the reliability block diagram which is also very useful very popular method in marketing systems in terms of its elements the Fault Tree another very popular method the Even Tree, base basin belief network Cut Sets especially.

And then within the Cut Sets we really like to look at minimal fact sets and we will discuss that and it is complementing the path sets. So, the items that are marked in red are the RBD the derivative the block diagram the Fault Tree and Cut Sets we are going to look at more detail during this lecture.