

Structural Reliability
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Lecture –94
Reliability Problem Formulation (Part - 06)

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Structural reliability problem formulation

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Lecture 11
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Example: failure of a ductile frame
From Thoft Christensen and Murotsu, Springer-Verlag, 1986

Fundamental mechanism 1:
 θ_i = rotation of hinge i , R_i = resistance of hinge i , δ_i = displacement of hinge i
 By symmetry, $\theta_1 = \theta_3 = \theta$ (say).
 By compatibility, $\theta_2 = \theta_1 + \theta_3 = 2\theta$ and $\delta_2 = \theta_1 \times l / 2$
 By energy balance, $P_1 \delta_1 = R_1 \theta + R_2 2\theta + R_3 \theta$
 i.e., $R_1 + 2R_2 + R_3 - P_1 l / 2 = 0$ at limit state
Failure: $R_1 + 2R_2 + R_3 < P_1 l / 2$

Fundamental mechanism 2:
 Failure: $R_1 + 2R_3 + R_2 < P_1 l / 2$

Fundamental mechanism 3:
 Failure: $R_2 + R_3 < 0$

System failure:
 $\{R_1 + 2R_2 + R_3 < P_1 l / 2\}$
 $\cup \{R_1 + 2R_3 + R_2 < P_1 l / 2\}$
 $\cup \{R_2 + R_3 < 0\}$

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We next look at our small frame structure this example is taken from the excellent book on system reliability by top Christensen and Murotsu. So, this frame has two elements one being one column and there are two loads two point loads acting on them now there are many different ways in which we could define failure for such a structure it would also depend on what kind of material behaviour it has if brittle is a ductile or is it something in between.

For example we could define failure as loss of stability in terms of softening by a certain degree in terms of excessive deflection in terms of manufacture. If we want to talk about member failure then there are several failure theories that we could invoke for example maximum principle stress maximum octahedral shear stress and so on and so forth. Here we take a popular approach of defining failure for a frame in terms of the maximum work that the maximum plastic work that can be absorbed by the system.

So, we have to define a set of fundamental mechanisms this structure can have 6 potential hinges the locations are at nodes 1, 2, 3, 4, 5, and 6 and it is 3 degree redundant 3 degrees statically redundant. So, there could be $3, 6 - 3, 3$ fundamental mechanisms. Now obviously these fundamental mechanisms are not unique but here we have a tree that I have taken from the book uh. So, let us go to the steps for the first mechanism.

So, let us define θ_i as the rotation of hinge i going from 1 to 6 let r_i be the plastic moment resistance at hinge i and let δ_i be the displacement of hinge i . So, if we can by inspection we can clearly see that θ_1 and θ_3 have to be equal and we can also impose compatibility and conclude that θ_2 has to be the sum of θ_1 and θ_3 and the displacement under point node 2 δ_2 will be $\theta_1 l$ over 2.

So, with these things are defined we now invoke energy balance. So, the work done by P_1 if it equals the work done by the hinges we have this equation and if we now express δ_2 in terms of θ we get the limit state where the two the work and the energy are equal in terms of the equation that you see on this screen. So any excess work would imply failure. So, we can define an imbalance.

So, we define failure as the external work done being more than the maximum work that can be dissipated by the system. So, the failure in the first mechanism is given by the inequality that you see on the screen. Now if you want to work through the other 2 mechanisms. Mechanisms 2 and 3 please pause the video otherwise let me go and present the solution. So, for the mechanism 2 the answer would be given in terms of R_4, R_5 and R_6 as you see on the screen.

And mechanism 3 does not involve any work done by the forces as they have been defined here. So, the mechanism 3 has a limit state given in terms of only R_3 and R_4, P_1 or P_2 is not involved. So, now here we have a system reliability problem because any one of these would cause system failures. So, I have a series kind of arrangement a union of the 3 individual mechanisms. So, this would be the problem formulation the failure description for a failure structure like this.