

Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –93
Reliability Problem Formulation (Part - 05)

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Structural reliability problem formulation

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Lecture 11
Reliability
problem
formulation

Example: Failure of a cable made of multiple strands

n strands in a cable
Elastic brittle in nature

X_i = strength (failure stress) of strand i ,
mutually independent

a = common area of all strands

E = common elastic modulus of all strands

ϵ = common strain in all surviving strands

Cable resistance at strain ϵ , $\sum_{i=1}^n E a \mathbb{I}(X_i > E \epsilon)$

Cable strength, $\sigma_f = \frac{E}{n} \max_{\epsilon} \sum_{i=1}^n \mathbb{I}(X_i > E \epsilon)$

σ_{app} = applied stress
(Cable failure) = $\sigma_{app} > \sigma_f$

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Let us take up the problem we alluded to at the beginning of this lecture that is a cable made up of a number of strands whether we look at it at the details of all the strands and then find out its capacity and then compare it with the applied demand the applied load that would be obviously up to the analyst but let us go through the steps. So, there are n strands in the cable each such strand is elastic brittle in nature it would be interesting to see what would be the behaviour of the aggregate the cable itself even though the strands are very simple elastic brittle in behaviour.

So, let X_i be the failure stress of strand i and all these X 's are mutually independent as a matter of simplification let us say that the cross-sectional area of all the strands are equal and let that be a let the elastic modulus of the strands also be equal and let that be capital E and let ϵ be the strain. So, these strands are connected in a manner or they behave in a manner that as the cable is stretched.

Each of these strands undergoes the same strain provided the strands are surviving. So, all the surviving strands have the same strain ϵ which is also the strain exhibited by the cable. Once we have the problems set up this way then let us see how the cable resistance would look like at any arbitrary strain ϵ . So, first it would be the contribution we need to add the contribution of all the strands.

So, each strand has the strain ϵ . So, multiplied by E capital E would give me the stress multiplied by A would give me the force provided the strand has not fractured. So, the indicator function I gives me that condition. So, as long as X is greater than $E \epsilon$ I would count that strand. So, that would give me the force in the in the total cable. Now what would be the strength of the cable? I need to find out the maximum value of that force which I looked at in the previous step and divide that by the total area of the cable which is the nominal area which is n times a .

So, once we go through one or two steps what you see on the screen is the cable failure strength in terms of the elastic modulus the number of cables and the sum the contribution coming from all the all the strands. So, if I wanted to go through the trouble of describing the cable in terms of its constituent elements then this is the level of detail that I would need to go to and then my failure of the cable the gross cable would be whether the applied stress in the cable exceeds the σ_f determined in the previous step or not.

If I did not want to do that I would just get by some other means the description of σ_f maybe through testing maybe by some other means and I would come to the definition of failure. But let us see what these look like through a simple example. So, let us say that we have each of the strands having a log normal strength with mean 1000 megapascals and a rather large COV of 40% and let us say the cable is made out of 1000 such strands.

So, this histogram is the distribution of the individual strand strengths. Once the strands are stretched we can see how the gross cable starts to behave. So, on the X axis in the figure below is the strain and the cable strength even though the individual strands are brittle in nature the cable strength actually looks much more interesting it rises linearly in the beginning and then we have five samples here.

So, it kind of starts to become a little non-linear and then reaches a peak and then has a considerable amount of softening flow as you may wish to call it until strain becomes larger and larger I have stopped the computation of the strain of 2% but we have taken the stiffness at about 100 gigapascals for each strand and one square millimeter just for sake of completeness the cross-sectional area of each strand and this gives me a cable strength peak cable strength of roughly about 550 or so, 525 in that range the peak strength of the cable.

If I do that many times this would be the distribution of the cable strength that I would get. So, it seems that the mean is somewhere in 540 megapascal range and whereas the value of the individual strands which was much higher but the behaviour also changes from brittle to something approaching a ductile behaviour. So, this would be an example of how to approach the behaviour and failure condition of such a cable depending on the available information the interest and the purpose of the problem at hat.