Structural Reliability
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Lecture –09
Review of Probability Theory (Part -01)

As I listed in the introduction a basic undergraduate probability and statistics course is a prerequisite for this one. So, we will go over some of the key concepts in today's lecture that will be useful later. Since probability is a measure defined on the sample space of events and all its relevant subsets we will start with a quick review of set theory. The next 15 or so, slides will be rather dense.

There will be a lot of text and symbols I will not go through them verbatim but if you need to please pause the video for the details you will find all these in the course material of course.

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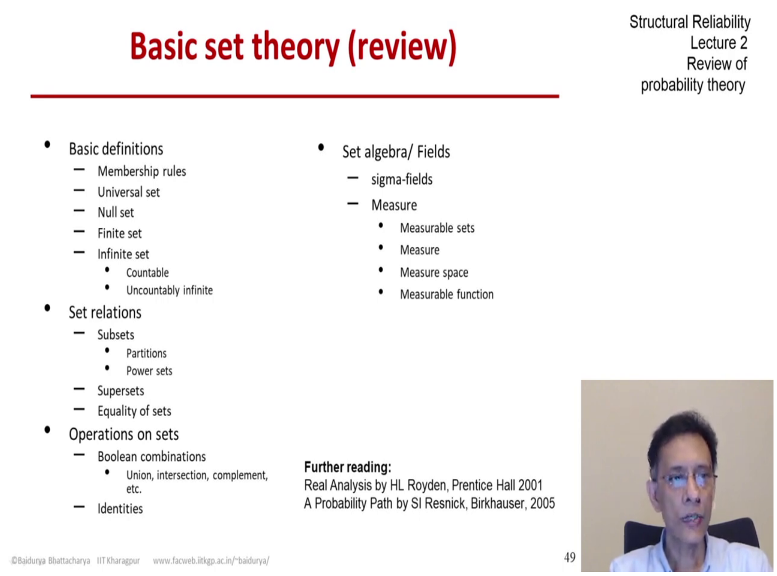

The topics that we would cover are the basic definitions; the basic set relations, basic set operations and a few words on the algebra of sets. For further reading these are the two excellent texts the real analysis book by Royden and a probability path by Sydney Resnick.

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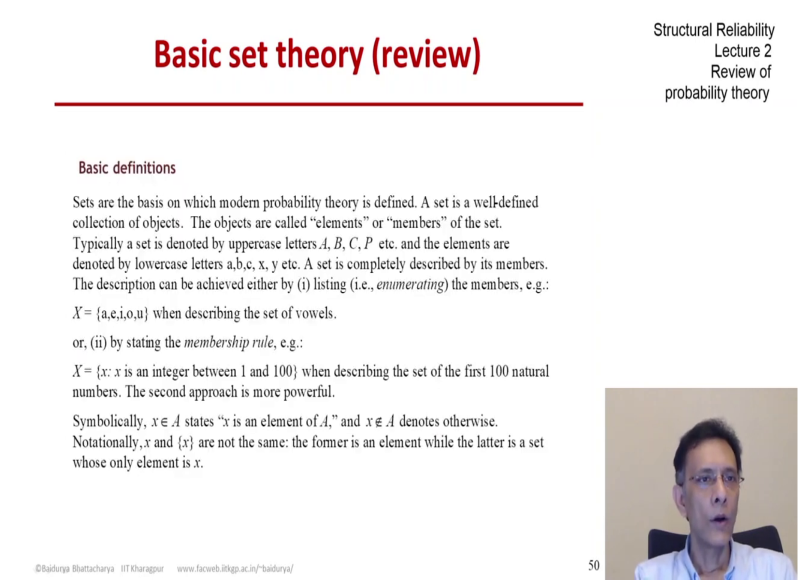

Now a set is completely defined by its members we typically use uppercase letters to denote sets and lowercase letters to denote its members. You can either list the members or you can state the membership rule if possible and which is obviously preferred for large sets. Now in probability theory we call the universal set the sample space everything that can happen in the context of the problem. And then we assign probabilities to the relevant collection of eight subsets.

Now one set that must be present in that collection is the null set the impossible event if not the null set then a set is either finite or infinite.

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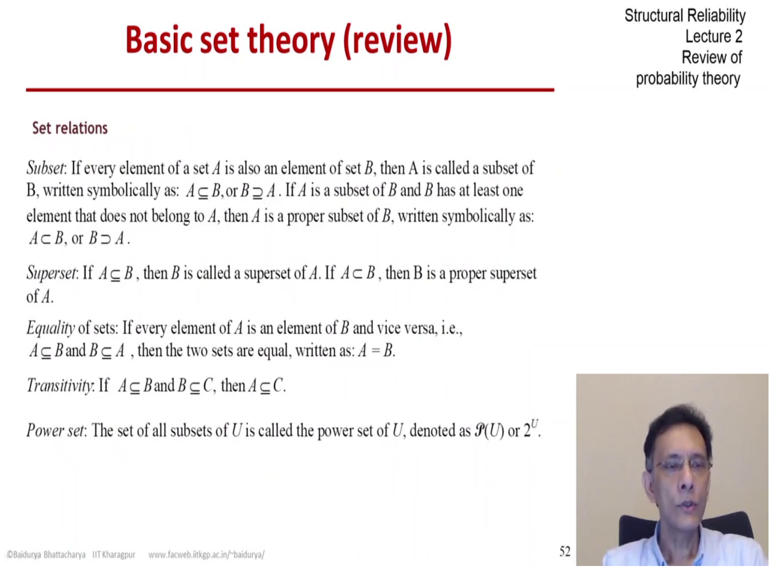

If there are two sets A and B they may have some common members or none at all if none at all we call them disjoint or mutually exclusive in some situations all elements of a could also belong to B then we call a a subset of B if A is a subset of B and B is a subset of A then A and b are equal. Let us solve a simple example on the next slide.

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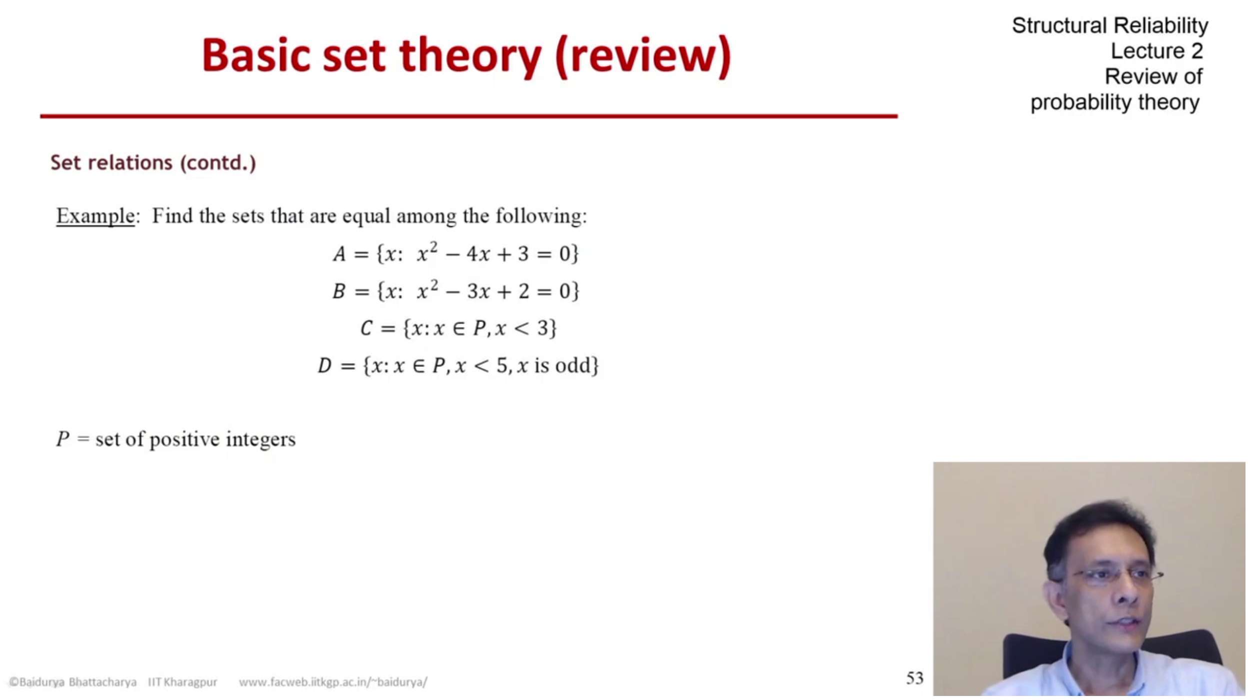

So, as you see I have defined four sets in terms of membership rules. Now if you want to work through this example. Now please pause the video otherwise here is the answer. So, if you solve the quadratic equation for A then A is composed of the integers 1 and 3 likewise B is 1 and 2 C is also 1 and 2 and D is 1 and 3. So the answer is that the sets A and D are equal and D and C are equal.

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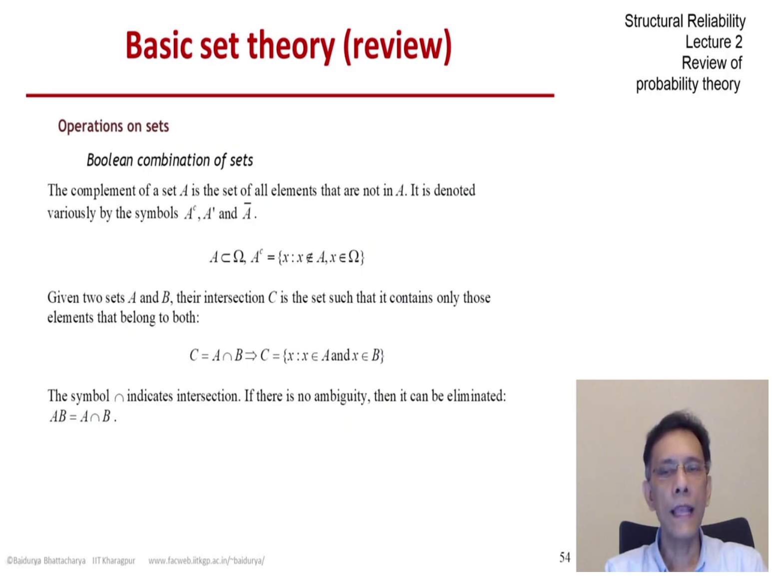

Now if we define an event A in the sample space then we automatically define what occurs when A does not occur. So, that is the complement of A and we denote the complement of either by an over bar or a superscript C next to A. For two events in the sample space we need to define the intersection and assign probability to it and we will often skip the intersection symbol and just denote the intersection by AB.

Now for the same two events A and B we can be interested in the union the difference and their symmetric difference the symmetric difference is saying that A alone occurs or B alone occurs they do not occur together.

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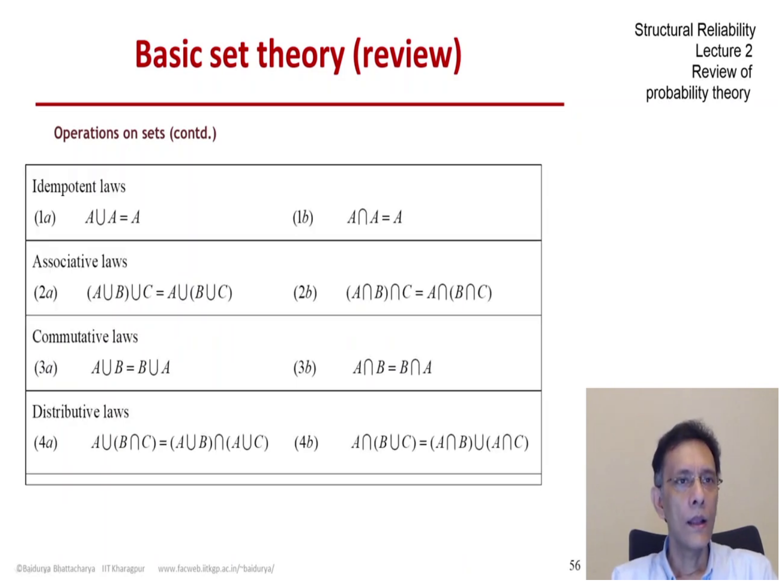

Now these identities are useful the associative laws the distributive laws the de Morgan's laws and. Now let us solve one example using some of these identities in the next slide.

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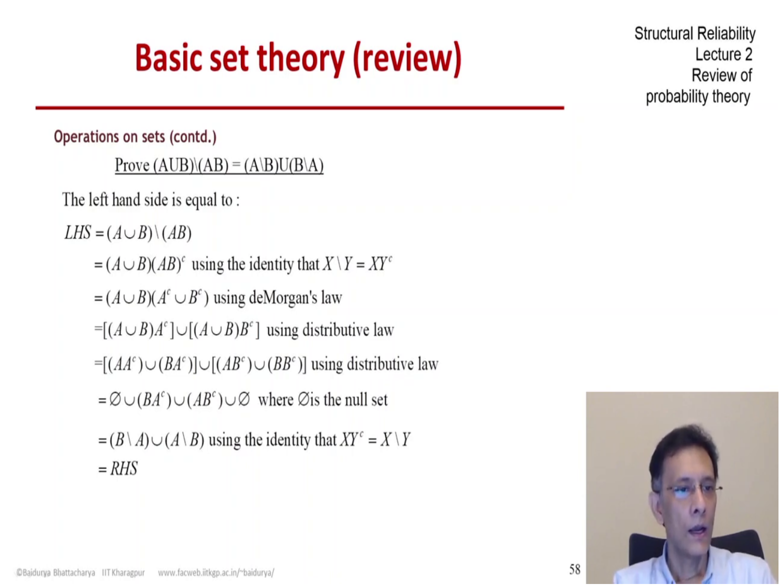

So, we need to prove that A union B difference AB is A difference B union B difference A. So, again if you want to solve this please pause the video otherwise I will proceed. So, the first step is to define what is meant by the difference of two sets. So, X difference Y is X intersection Y complement. So, that is what we are using to first expand the left hand side and then we use de Morgan's laws and then we use the distributive law once.

And then once more and then identify the null sets because A intersection A complement is the null set likewise B intersection B complement is also the null set and then we again use the identity in the reverse way that X difference Y is X intersection Y complement and which is nothing but the right hand side. So, it is proved.

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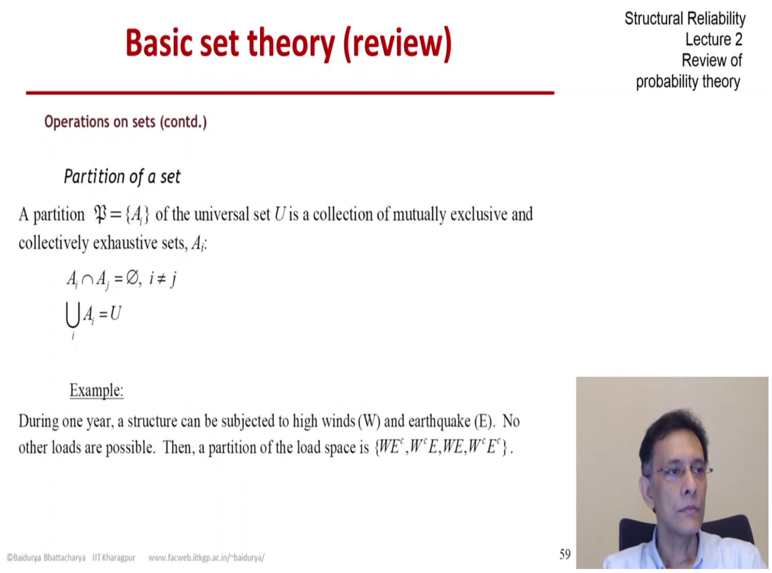

One more concept that's quite useful we often need to split the sample space into disjoint events that taken together make up the entire sample space. So, we call them mutually exclusive and collectively exhaustive or in other words a partition. For example if a structure can be subjected to high winds and earthquake and suppose these are the only two loads possible. So, then a partition of the load space would be E complement W complement EW and E and W complement and E complement. So, these 4 would constitute a partition.