Structural Reliability
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Lecture –82
Monte Carlo Simulations (Part - 14)

The last topic I would like to discuss in these lectures on Monte Carlo simulations is to introduce the idea of variance reduction let us introduce that through an example.

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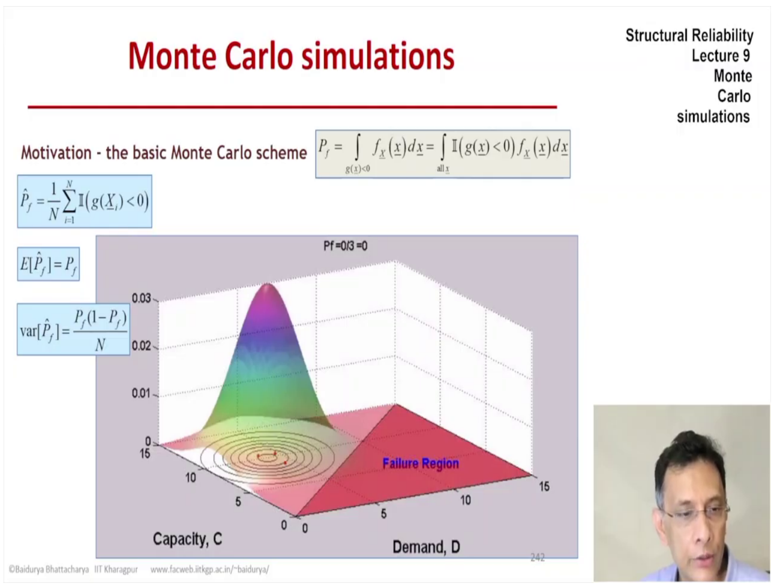

Let us say we would like to estimate the failure probability which we will do a lot in the coming weeks whose mathematical formulation we have already seen in different ways is that we find the probability content over a certain region in the space of random variables x and integrate the probability content over that region which can be expressed differently the last part of that equation on the right is using the indicator function.

So, now we integrate over the entire region but only those points that are in the failure region defined by g is less than 0 is counted. So, the indicator function is 1 when the argument is true and 0 otherwise. So, this gives me the idea that is what is done when estimating failure probability in Monte Carlo simulations is to estimate the failure probability as the average the expectation of the indicator function.

So, we generate the x vector n times and find the truth of g less than zero every time and I just take the average of those counts. So, if we have to show this graphically through a small movie we would look at it this way. While the movie is running let us also look at the mean of the estimated Pf and the variance of the estimated Pf on the right. So, we have seen this a few times in this lecture in the past that the variance of the estimate is proportional to 1 over n.

So, with number of samples the variance does go down. So, now let us see what happened here on the movie let us run the movie again what we are doing is we are generating points in the space of C and D which here x is just two random variables C and D. So, we are simulating points according to the joint distribution of C and D and counting those that are in the failure region.

We have done 1000 such counts and running the movie again only about 11 or so, of them are falling in the failure region because failure after all is a rare event in fact in most problems it will be much rarer than 0.01 that you see in this example. So, what should strike us is that most of these samples that have been generated have in some sense been wasted they did not contribute to failure.

They were very much in the safe region and in so, far as computing failure probability is concerned they did not really help. So, can we do something in order to make more efficient use of these samples? And in other words when we are trying to estimate a very rare probability whose variance now is only going to improve uh inversely with not n can we make it even better can we make the uncertainty go down even faster.

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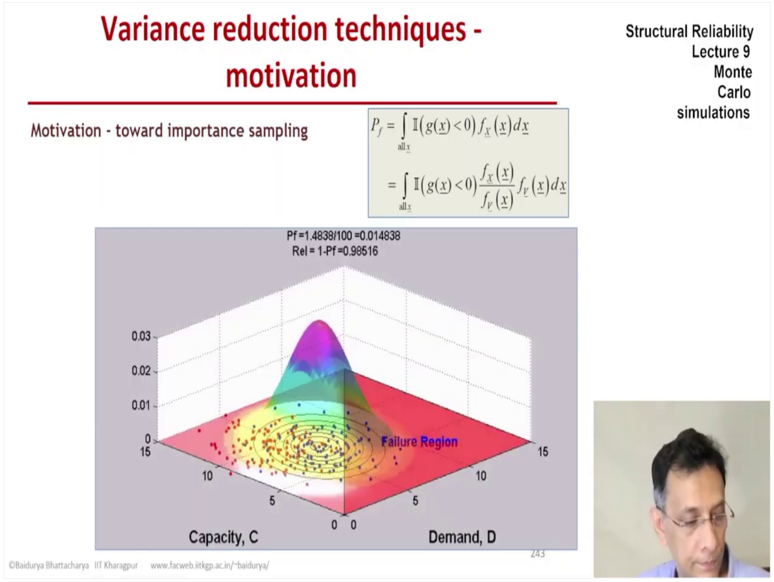

That brings us to the idea of important sampling where we do not so called waste the sampling points. Now if it is done carefully we can achieve the same accuracy with a lot less number of samples. And while generating random numbers may not be that expensive but what we do with those random numbers in every trial can be very demanding. For example each of those trials may involve one finite element run of a large structure which might take minutes or hours to solve. So, we do not have the luxury of doing millions of trials.

So, that is where this variance reduction technique this more efficient sampling becomes almost a necessity. This would be the mathematical description of what we are trying to do we basically no longer sample from x but we introduce a bias and we sample from V. So, again graphically speaking what we are doing we are bringing the sampling distribution closer to the failure region.

So, we are now counting many more points towards failure obviously that is a biased situation. So, we need to correct that bias in some manner. And if we do it right then we can kind of get the same sort of answer here as you see in 10% of the simulations is a 1000 we are only simulating 100 points and we get a failure probability of the same order of 1%. So, this is something we will come back to later and we will look at it in much more detail.