Structural Reliability
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Lecture –79
Monte Carlo Simulations (Part - 11)

So far we have discussed generating discrete and continuous random deviates from a sequence of IID uniform random numbers but these were all mutually independent. But we will have situations where a problem is defined by several random variables and some or all of them mutually dependent. So, we need to generate samples of such dependent deviates we still have a sequence of IID uniform deviates that the computer generates.

So, we somehow need to use them along with the joint probability information the dependence information to obtain the dependent samples that we need. So, we look at a few cases.

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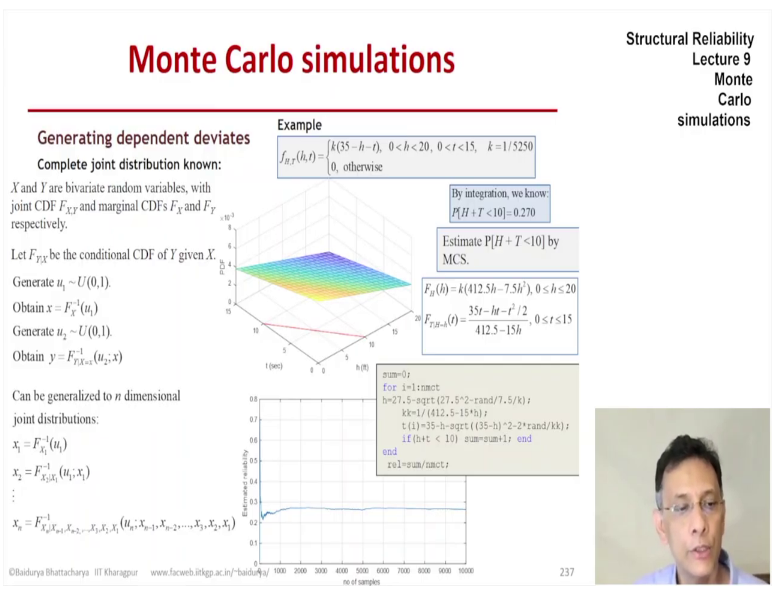

The first one is when we have the full dependent structures we have the complete joint distribution, no. So, let us take two random variables X and Y which are dependent we know the joint CDF we know their marginal CDFs Fx and Fy but most importantly we know their conditional CDF. So, F y given x or F x given y. So, this would be the scheme that we generate u1 from the uniform distribution for that u 1 we invert the CDF of x to get one value of x.

And then we generate another uniform u 2 which is independent of 1 of course but then we use that in the conditional CDF of y given x the x that we just generated in the previous step and invert that to obtain the deviate y. So, this way we preserve the dependence information while making use of the IID uniform sequence. It can easily be generalized to n dimensions start with x 1 then use that to generate x 2 and then use x 1 and x 2 generate x 3 and so, on until we have completed the entire n random variables in the n dimensional problem.

So, let us walk through an example this problem we have seen before we have looked at it twice. This was the joint distribution of h and t wave height and wave period and we actually one of the problems we solved was we calculated analytically the probability that h + t is less than 10. So, the probability content over the region defined by the red line and the triangle that contains the origin.

So, we found that probability is 0.270. Now our task is to do the same thing estimate the same probability but this time using Monte Carlo simulations. So, let us walk through the steps the first thing is to be able to generate h. So, we need the distribution the marginal CDF of h which we get by integrating the density of h which we get from the joint density of h and t by integrating out t.

So, we have the ability to generate h and then for that given h we need the conditional CDF of t. So, what we do is that we find the conditional PDF the density function of t given h which would be the ratio of f h, t and F H and then once we have that conditional density we integrate that with respect to t. So, to get the conditional distribution function of t and you see both those functions on the screen and then we can write a small code.

So, this is what we have we go through the loop I, 1 through nmct and for each search run through the loop we generate h from its CDF using rand which is one uniform deviate. And then use that value of h to generate the corresponding value of t it is paired by using another random deviate and keep doing this and every time to see to find out if H + T is less than 10 or not.

So, if it is then we increase a counter which is sum by 1 and once you are out of the loop then the probability of H + T less than 10 or which we soon will call reliability would be the ratio of the counter over the total number of trials. And if we do it enough number of times we actually should be able to come close to the answer that we already know and what you see here is such an exercise. So, with increasing number of samples we see that the estimated reliability does indeed convert somewhere like 0.27.