Structural Reliability
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Lecture –77
Monte Carlo Simulations (Part - 09)

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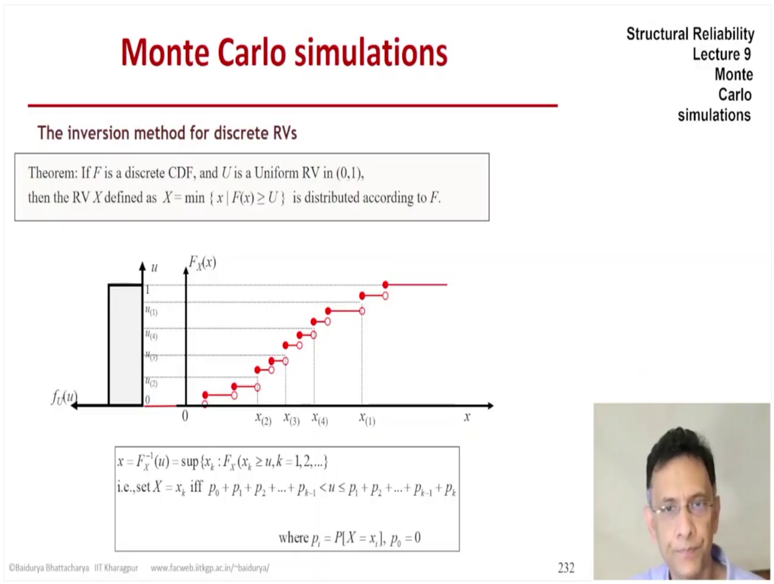

Generating discrete random variables is a little more involved than generating continuous random variables that are because the discrete CDF is not a one-to-one function. So, we need to keep in mind the step function nature of the discrete CDF and rewrite the inversion theorem accordingly pictorially speaking what we do is as we had before in the continuous case on the left we have the uniform distribution from which we are sampling the uniform deviates.

And once we come to the red CDF function we need to remember that the function is left continuous. So, for example when we generate u1 we need to read off x1 from the step function likewise we read of u2 u3 u4 and obtain x2 x3 x4 from the CDF remembering every time that the step function is left continuous. So, which basically means is that we need to make a finite number of comparisons to obtain the discrete deviate from the uniform number that has been generated.

The uniform distribution is still the continuous distribution taking values between 0 and 1. So, in the block below the figure you see the comparisons that need to be done the very first time that the cumulative probability exceeds the random number generated the discrete the uniform deviate generated that gives me the value of the discrete deviate.

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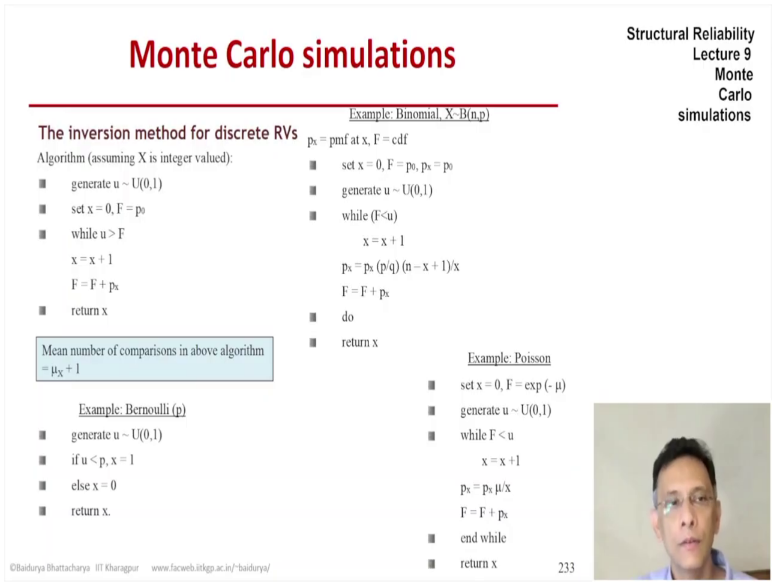

Let us look at the algorithm the general algorithm we keep adding the step heights the PMFs until for the first time we exceed the uniform deviate generated. And the average number of comparisons is mean x + 1 provided the deviates are the discrete distribution is integer valued. We can look at several examples the Bernoulli random variable is generate u and compare it with the probability of success.

We can look at the binomial and do the same sort of comparison and exit the while loop as soon as we hit the right value we can do the same with the Poisson PMF and obtain the Poisson deviate. There are equivalent ways where we can avoid this inversion process the cost that we bear is we need to generate a lot more devious uniform deviates for generating just one value of the discrete random variable.

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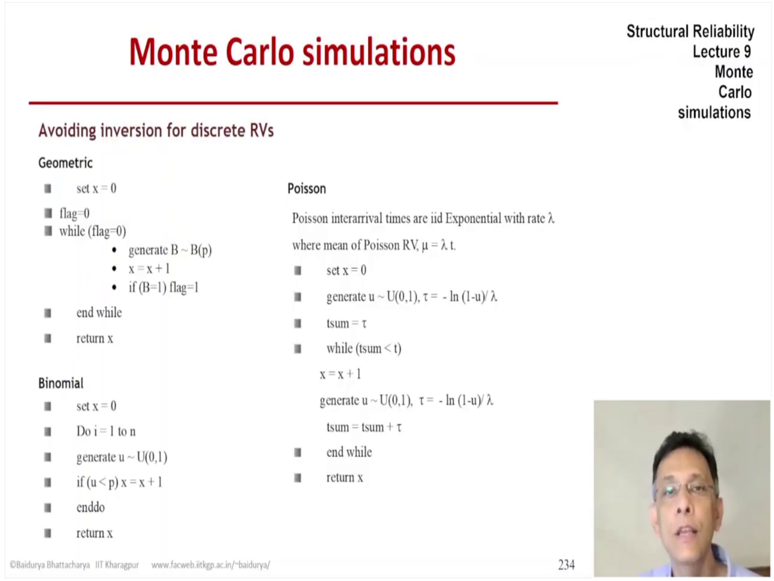

So, for example to generate the geometric random variable if we wanted to avoid inversion the comparison we keep generating the Bernoulli random variables with parameter P until the first success occurs the first time that B is equal to 1 then we exit the while loop and return the value. We can take such first principles approach for the binomial also uh. So, we generate n Bernoulli random variables with parameter P and then just sum them until we exhaust all the n Bernoulli random variables.

The Poisson to generate without the inversion we need to make sure that the Poisson random variable is basically the count of events in a Poisson process where each inter-arrival time is an exponential random variable. So, we can generate an exponential which we already know how to do and then keep adding them until we exceed the time interval in question. And as soon as we do that we return the number of events that have been generated.