

Structural Reliability
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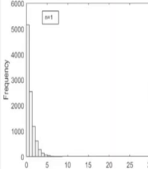
Lecture –75
Monte Carlo Simulations (Part - 07)

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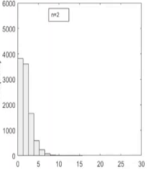
Structural Reliability
Lecture 9
Monte Carlo
simulations

Monte Carlo simulations

Example: Illustration of CLT

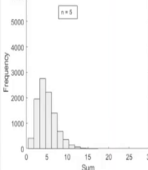


n=1

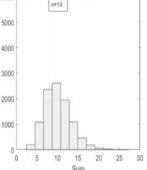


n=2

Illustration of CLT by summing n IID exponentials ($\lambda = 1$). Four values of n are considered with 10000 samples each. Clearly, it does not take too many members for the sum to approach normality.




n=4



n=10

```
nmc=10000;  
n=10;  
lambda=1;  
for i=1:nmc  
    sum(i)=0;  
    for j=1:n  
        u=rand;  
        x=(1/lambda)*log(1-u);  
        sum(i)=sum(i)+x;  
    end  
end  
histogram(sum)
```

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Let us continue with our example of simulating exponential random variables and see if we add a few of them if central limit theorem seems to be kicking in. So, this is what we did; we generated some samples here we have 10000 samples from the exponential distribution with lambda equal to 1. And if we draw the histogram of those 10000 in this case it is clearly going to look like the exponential density.

And this is this is one of those distributions that are you know quite unnormal or non-normal to look at it has a peak at zero and then drops monotonically. So, let us see if we what happens if we just sum two of these. So, and then we will sum maybe you know four of them five of them and then ten of them and see what the distribution what the histogram starts looking like. So, this would be the code.

let us say let us look at the inner loop the for loop on j . So, we run that n times and for each j we simulate one exponential as you can see and then sum those values. So, we sum those n values of exponential deviates and then we look at the outer loop which is for loop on i . So, we repeat that $nmct$ time. So, $nmct$ here as I already said is 10 000. So, let us see what would happen for different values of n .

So, what you see here is n equals 1 let us make n equals 2. So, we are summing 2 exponentials and we can see some interesting things already obviously since we are not normalizing the distribution is shifting a little bit to the right. So, the mean was one. Now the mean is two because we are adding two exponentials each with mean one. So, obviously the distribution has shifted a little bit to the right.

But interestingly the peak the quite sharp peak of the single exponential has kind of blunted. Let us do that for five. So, we are summing five and importantly all these n these five exponentials are all IID. So, and that is clear from the way that we are simulating this. Every time we simulate a rand it is a an independent sample from the uniform distribution. So, clearly when we sum five exponentials the distribution does shift more to the right.

But it is becoming more and more kind of rounded with a central peak and falling off on both sides although it is a bit more it is still a bit skewed to the right. But the inevitable nature of the normal is clearly becoming apparent. So, now when we do 10 it is looking more and more like the classical bell shape of the normal. Obviously this is this is no proof this is very non rigorous but if we want to make it a little make it more rigorous obviously we would do a goodness of fit test.

But that is not the purpose of this example we just wanted to illustrate pictorially graphically what happens once we start adding a few independent random variables. And by simulation what the distribution what the histogram of the sum would look like. And at least graphically it seems that central limit theorem is kicking in pretty early. We do not have to wait for n equals a very

large number even with 5 or 10 the result the strength is becoming very clear.