

Structural Reliability
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Lecture –68
Joint Probability Distributions (Part - 19)

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Structural Reliability
Lecture 7
Joint
probability
distributions

Law of large numbers

Preliminaries

Consider the case of partial sum S_n of n RVs that are mutually independent, but not necessarily identically distributed:

$$S_n = X_1 + X_2 + \dots + X_n$$

with $\mu_k = E(X_k)$, $\sigma_k^2 = \text{var}(X_k)$ if they exist

Define $Y_n = S_n/n$, i.e. the average.

Does the sequence Y_n converge to anything?

Say the sequence Y_n “converges” in some sense to μ .

The nature of convergence will depend on the probability structure of the X_k 's.

And the nature of convergence determines which Law of Large Numbers governs – the strong type or the weak type.


Strong law and weak law

Strong law: If the convergence is in L_2 norm, or almost surely, then we have the strong law of large numbers.

- The convergence of the series $\sum \sigma_k^2 / k^2$ is a sufficient condition for the Strong Law to hold for the sequence of mutually independent RVs (Kolmogorov criterion).
- Also, if the sequence is IID and the mean exists, the Strong Law holds.

Weak law: If the convergence is only in probability, then we have the weak law of large numbers.

- The Weak Law holds whenever the X_k are uniformly bounded, i.e., whenever there exists a constant A such that $|X_k| < A$ for all k .
- Another sufficient condition for the Weak Law to hold is $(1/n^2) \sum \sigma_k^2 \rightarrow 0$



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The law of large numbers in the previous slide we gave an example of where a sequence of random variables can naturally occur and give the example of a sequence of sample means with increasing sample size. So, we broke that concept further here consider the partial sum of n such random variables with the means and variances as you see on the screen and then define the average. So, define the sum divided by the number of such terms.

And the question we already posed is does the sequence of these averages converge to anything. And now if the sequence Y_n converges to some number μ then the nature of the convergence depends on the probability structure of the individual random variables and that determines in term which law of large numbers governs whether it is the strong law or the weak law. So, the strong law states that if the convergence of that average to that particular number μ is in L_2 norm in mean square or almost surely then we have the strong law of large numbers.

Now there are two very useful sufficient conditions for the strong law the first one is the Kolmogorov Criterion and the second law is that the sequence has to be IID and the means the mean must exist which is the common for all the random variables then the strong law would hold the strong law of large numbers. The weak law is relevant when the convergence is only in probability.

So, which we discussed in the previous slide that it is a much weaker condition of convergence. So, that is the weak law of large numbers and there are also two sufficient conditions for the weak law the first one is that the random variables are uniformly bounded and the second one involves the sum of the variances. Now in statistical estimation theory which we did mention an estimator is said to be consistent if it approaches the true the population value as the sample size grows.

So, μ here would be the population mean now the estimator itself is a random variable obviously because of the sample size being finite. Now in that context of estimation and an estimator being consistent weak consistency implies the convergence is in probability and strong consistency it implies that it is almost sure converges.