

Structural Reliability
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Lecture –61
Joint Probability Distributions (Part - 12)

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Functions of random variables

Structural Reliability
Lecture 7
Joint
probability
distributions

Example: Function(s) of several random variables

Sum of two IID geometric RVs

$X_2 = G_1 + G_2$, where $P(G_i = m) = q^{m-1}p$, $G_1 \perp G_2$
 What is the PMF of X_2 ?

$P[X_2 = n] = P[G_1 + G_2 = n]$

Using the theorem of total probability, we can write:

$$P[X_2 = n] = \sum_{m=1}^{n-1} P[G_2 = n-m | G_1 = m] P[G_1 = m]$$

$$= \sum_{m=1}^{n-1} P[G_2 = n-m] P[G_1 = m]$$

since G_2 and G_1 are independent


Putting in the geometric PMFs,

$$P[X_2 = n] = \sum_{m=1}^{n-1} q^{n-m-1} p q^{m-1} p$$

Simplifying the algebra,

$$P[X_2 = n] = \sum_{m=1}^{n-1} q^{n-2} p^2$$

$$= (n-1) q^{n-2} p^2$$



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Our first example on functions of several random variables involves the sum of two discrete random variables. We are interested in the sum of $G_1 + G_2$ where G_1 and G_2 are IID geometric random variables with parameter P . So, we write the PMF of x_2 simply in terms of $G_1 + G_2$. And now say we fix the value of G_2 at any particular number m , so, then G_2 is equal to $n - m$. So, with this idea we invoke the theorem of total probability and express the PMF of x_2 as G_2 fixed at a particular m and G_2 equals $n - m$ and we sum this over m from 1 to $n - 1$.

Now G_1 and G_2 are independent of each other. So, that conditional probability simplifies to the products as you see and then we can just substitute the geometric PMF for G_1 and G_2 and express the PMF of x_2 simply as the sum that you see on the screen. And going through the algebra we come up to the solution that the PMF of x_2 is $n - 1$ times q to the power $n - 2$ times p squared which is the well-known Pascal PMF for x_2 .

So, here we showed that the sum of IID geometric random variables is the Pascal or the negative binomial distribution and here we had the example of the sum of two geometric random variables.