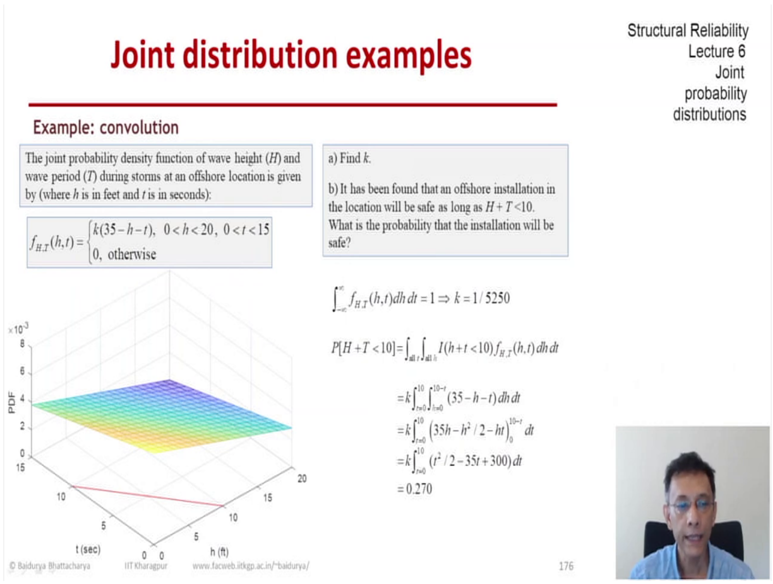
Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –57
Joint Probability Distributions (Part - 08)

(Refer Slide Time: 00:27)



In this example we compute the failure probability by performing a double integration. So, let us take a minute to read the problem and then we will go about solving it. I am actually I will also plot the density function and then let us discuss it after completing the reading of the problem. So, we have on the plane of h and t the units of h being feet and for t being second the third dimension being the density we have the joint density of h and t the height and period defined the limits on h are 0 to 20 feet and the limits on t are 0 to 15 seconds.

$$f_{H,T}(h,t)=\begin{cases}k(35-h-t), & 0<h<20,\ 0<t<15\\ 0, & \text{otherwise}\end{cases}$$

The first is just to ensure that we understand the normalization property of the density. So, we need to find k and by just diligently performing the double integration between the limits 0 to 20 and 0 to 15 of this linear density function we can find the value of k which is 1 over 5250. The second question is interesting. So, the safe region is defined as you can see the the red line on the ht plane the line demarcates the safe from the unsafe region. So, that is the limit state equation and the area towards the origin left of the red line that is the safe region and on the other side of the red line away from the origin is the unsafe region.

$$\int_{-\infty}^{\infty} f_{H,T}(h,t)dh\,dt = 1 \Rightarrow k = 1/5250$$

So, we need to find the probability content under the density function in that over that triangular area. So, the triangular area being 0, 0 to 10, 0 and 0, 10 so, these are these are the three vertexes of the triangle and we need to find the volume over this triangle under the density plane. So, um how do we do that h + t is less than 0. So, that is our safe event. So, we need to find the probability of that and again we use the indicator function just to keep things clean.

$$\begin{aligned}
P[H+T<10] &= \int_{\text{all } t}\int_{\text{all } h} I(h+t<10)\, f_{H,T}(h,t)\,dh\,dt \\
&= k\int_{t=0}^{10}\int_{h=0}^{10-t}(35-h-t)\,dh\,dt \\
&= k\int_{t=0}^{10}\left(35h-h^2/2-ht\right)_0^{10-t}dt \\
&= k\int_{t=0}^{10}(t^2/2-35t+300)\,dt \\
&= 0.270
\end{aligned}$$

Now if we get rid of the indicator function the limits have to be properly defined and so what we are doing is we are letting t run from 0 to 10 and h run from 0 to 10 - t. So, that would be the correct limits on the integration which would let us cover the entire triangular area. And if we carefully complete the integration we should reach the answer of 0.270. So, the probability that the installation will be safe is about 27 for this given problem.