Structural Reliability
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Lecture –55
Joint Probability Distributions (Part - 06)

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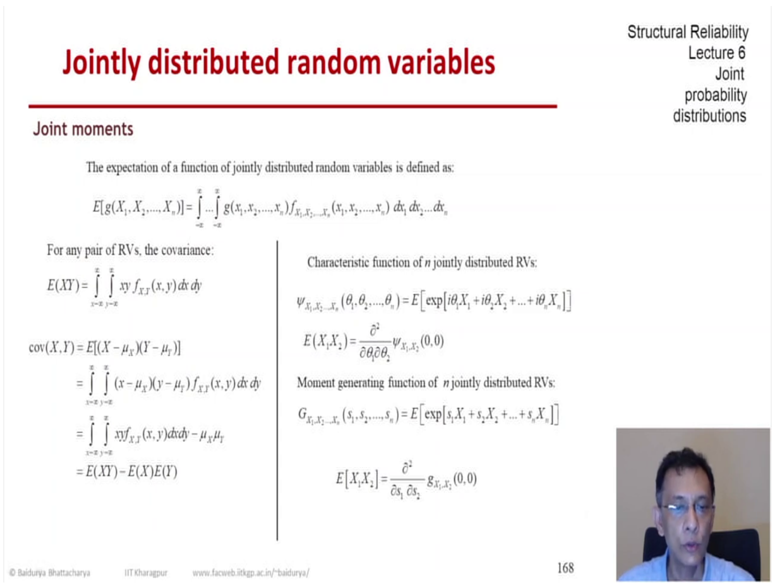

Joint moments, when discussing single random variables we defined the expectation of a function g of the random variable X as the; weighted sum or the weighted integral of the function with respect to the mass function of the density function as appropriate. So, we can extend the same idea when we have a function of n jointly distributed random variables g as you see on the screen. So, it is the n-dimensional integration if the X's were discrete then it would be the appropriate sum.

Now how we define g gives us some well-known expectations and let us start with two random variables the bivariate random variable X and Y. And let us say that we define g as the product of x and y. So, this product moment E of X times Y is what you see on the screen it is the integration of x, y with respect to the joint density function or if x and y were discrete double sum. You could also define g as a product of the deviations of x from its mean and y from its mean and that would be given as the integral as you see.

And which also happens to be the expectation of the product minus the product of the expectation. So, that is the covariance of x and y and if x and y were discrete then we would replace the integrals with sums. Now if this way of defining moments seems familiar to you it is no surprise because we have done this in mechanics when finding properties of sections various area moments. So here x and y are the distances either from the origin or from the centroid and the mass density is now replaced by the probability density.

So, there is a lot of parallel here with mechanics. Now we could continue defining g in other ways and here is the way we would define the joint characteristic function of the n random variables we could recover the moments if say for example we have x 1 and x 2. So, the expectation of x 1 x 2 would be the second derivative of the joint characteristic function evaluated at 0, 0. Likewise we could define the moment generating function by an appropriate function g as you see in terms of s 1 s 2 up to s n.

And we could like we did for the characteristic function we could take the second derivative of g at the origin and we could recover the expectation of x 1 and x 2. Let us move on now to conditional moments.

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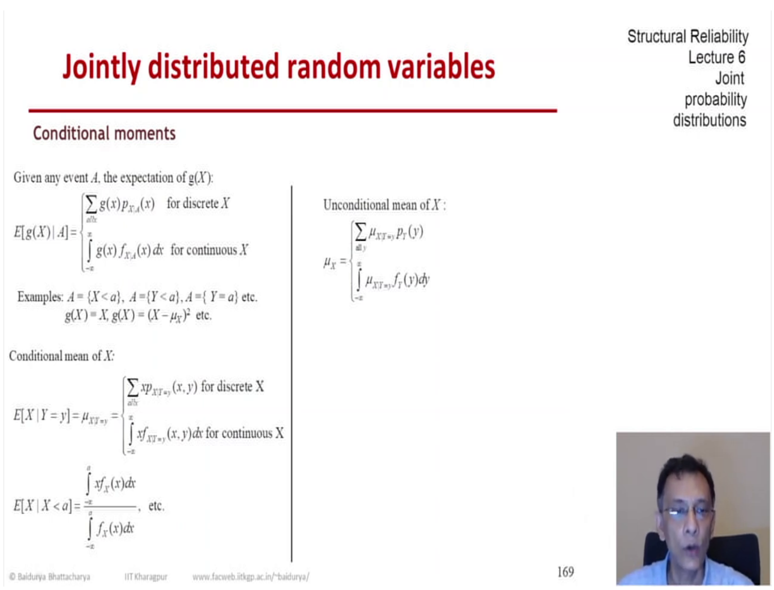

So, suppose we have the function g of X as we discussed in the previous slide and now we want to condition the whole thing on an event *a* and find the conditional expectation of that function g of X. So, just like we did when discussing single random variables we would take the weighted sum or the weighted integral of g with respect to the conditional mass function or the conditional density function defined when A is given.

Now there are various ways in which we could define that event *a* and the function g. So, the event *a* could be defined on the random variable x itself and we did something like that when we talked about proof loading. So, the random variable was restricted on the on the left it was greater than or equal to the proof load the event *a* could also be given in terms of another random variable so, which is what is more relevant here because we are discussing joint random variables.

So, the event *a* could be the other end of variable y less than a certain constant or equal to a certain value and the function g could be x itself it could be x minus mean of x whole square. So, I think you understand where we are going with this and so, the conditional mean of x given that y has attained a y is fixed at a certain value is the same approach fixing y. So, the conditional mass of X given Y of the conditional density of x given y and performing the sum of the integral as appropriate we could again as I said define the condition movement a in terms of x itself.

So, we would get the conditional mean in terms of the ratio that you see at the end of that column. We again we have looked at this in the case of something similar in the case of proof loading. Later on when we are going to look at the mean residual life of the time to failure a random variable we would define things similarly. Now given the conditional mean we could recover the unconditional mean provided we have enough information.

So, if we have the conditional mean of X given Y is fixed at a particular value and then if we know the mass function or density function of y we could recover the mean of x the unconditional v of x.

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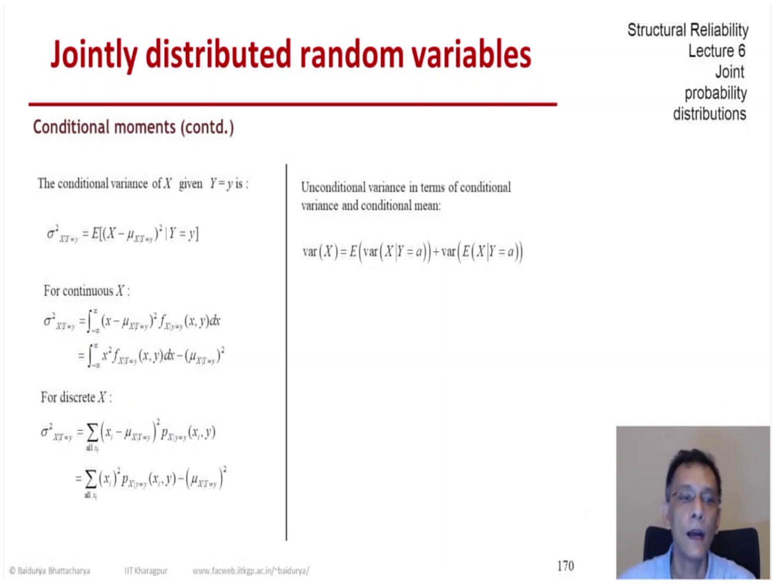

Now let us move on to x minus mean squared. So, we are trying to go towards variance. So, the conditional variance of X given again a certain value of Y is defined as you see as the expectation on the first equation now for continuous x that is how we would get it as you see on the screen for discrete x this is how we would get it. It should be straightforward except it is important to remember that not only is the density function of the mass function conditional on the particular value of y but so, is the mean the mean about which you are taking the deviation.

So, it is x - mu of X given Y. So, that is also the same conditional mean that we have to use. Now just like we got the unconditional mean back given the conditional mean of x can we get it for in case of the variance? As well yes we can. So, the variance of X can be obtained in terms of the conditional variance of X given Y and the conditional mean of X given Y. But you see there are two terms on the right.

So, the first is the expectation of the conditional variance and the second is the variance of the conditional expectation. So, which means that you have to remove the conditioning on y the proof is there in any standard textbook like the book by Ross, we have mentioned before. But before you look it up it would be quite interesting and to try to solve it. So, that would be an exercise that you could think of.