Structural Reliability
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Lecture –54
Joint Probability Distributions (Part - 05)

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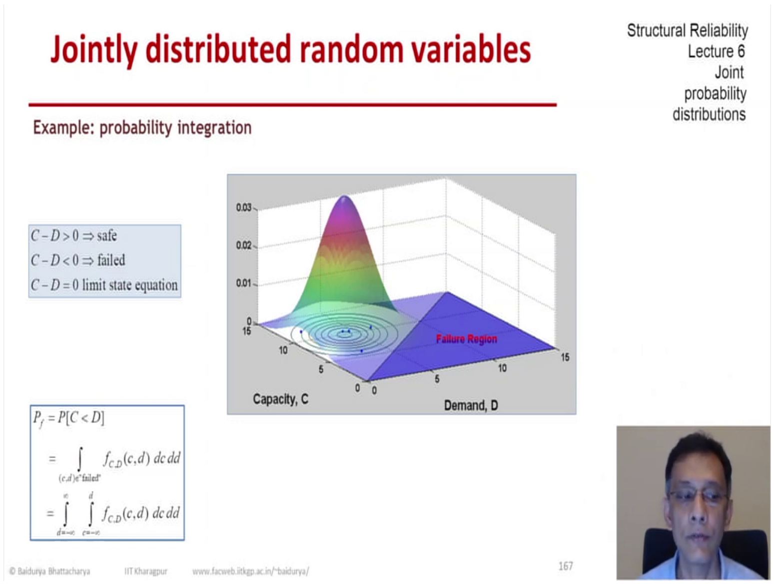

In this figure we get a preview of what is meant by failure the failure region in the space of the concerned random variables and how to compute the failure probability in terms of an integration. So, here you have 2 random variables C and D, C being capacity and D being demand and they are jointly distributed and the third dimension the Z dimension is the probability density.

So, this is something we will see a lot later that there would be some criterion which would separate the space of C and D into the safe set and the unsafe or failed state. So, here we have the very simple linear relationship that C greater than D is safe and C less than D is fade and the line separating the two regions is C - D equals 0 which is also known as the limit state equation. So, the failure probability would be actually to find the probability content of the failure region which you see in marked in a darker blue color on the CD plane.

$$
\begin{aligned}
&C-D>0 \Rightarrow \text{safe} \\
&C-D<0 \Rightarrow \text{failed} \\
&C-D=0 \text{ limit state equation}
\end{aligned}
$$

And what we would like to do is to integrate under this density function surface over the region defined by the failure condition. So, if I am able to integrate in two dimensions if i know the density function of C and D explicitly and if I know the limit state equation as we do here. So, here I would integrate the joint density function D going from minus infinity to infinity all possible values to demand and C because I am interested in failure going from negative infinity all the way up to D.

$$\begin{aligned} P_f &= P[C < D] \\ &= \int_{(c,d)\in\text{"failed"}} f_{C,D}(c,d)\; dc\, dd \\ &= \int_{d=-\infty}^{\infty} \int_{c=-\infty}^{d} f_{C,D}(c,d)\; dc\, dd \end{aligned}$$

So, that would be the definition of failure and if I can complete that integration I would have a numerical value of the failure probability.