Structural Reliability
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Lecture –53
Joint Probability Distributions (Part - 04)

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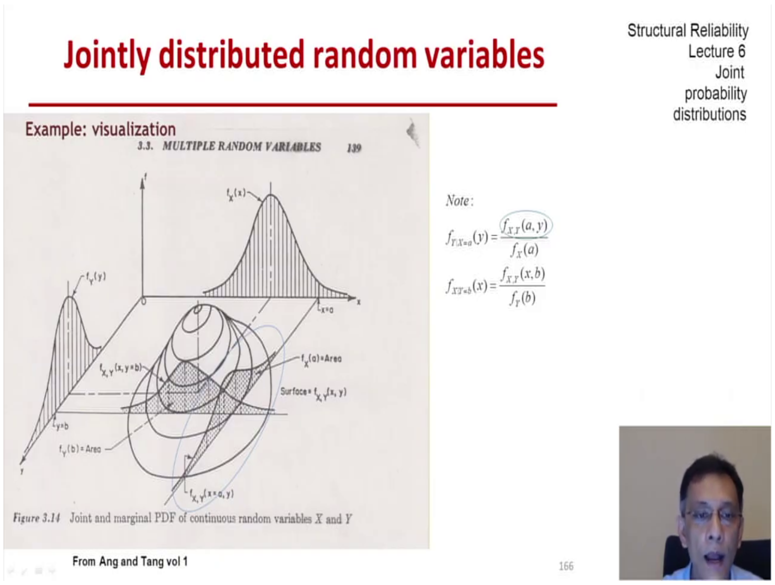

We next have another example from Ang and Tang's book the picture that you see is actually taken from the first edition of Ang and Tang's text and this is a beautiful picture of the bivariate density function. So, let us spend a few minutes looking at the quantities that you see here the marginal density functions of x and y the joint density function of x and y with x fixed at a particular value a or y fixed at a particular value b and so on.

Note:

$$f_{Y|X=a}(y)=\frac{f_{X,Y}(a,y)}{f_X(a)}$$

$$f_{X|Y=b}(x)=\frac{f_{X,Y}(x,b)}{f_Y(b)}$$

We will also look at the conditional densities of x given y and y given x. So, this dome that you see this probability surface it is the volume under this its integration over the entire plane has to be one just as in the invariant case the area under the density function is 1. So, let us see how this density function of y that I just circled in red on the wall how that is obtained it is not the projection of the dome on the wall it is actually integration section by section of the of the dome by fixing values of y.

So, for example if I fixed the value of y at b which means I took a section of that of the dome and I integrated the area of that section that would be. So, which basically means I integrate out x from the picture what I would be left with is the density function of y at b. So, it would give me the ordinate of the red circled figure at y equals b. Likewise the green circled figure is the joint density function of x and y when x is about to vary but y is kept fixed at b which I just discussed.

I have also circled as you can see on the right equation just to show that which figured that density function corresponds to. Now if I divide that density function that cross section by its area it becomes a legitimate density function it is the conditional density function of x given y is fixed at b. So, that would be the pictorial description of the conditional density likewise we could look at the other direction where x has been fixed at a.

So, we take the cross section accordingly what you see in the blue circle and the area of that would be the density function of x at a. And again if I divide the joint density function of f x comma y at that particular section with the density of f x at a I would be getting the conditional density function of y given the fixed value of x at a. So, this would be the nice pictorial interpretation of all these concepts we just discussed.