Structural Reliability
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Lecture –51
Joint Probability Distributions (Part - 02)

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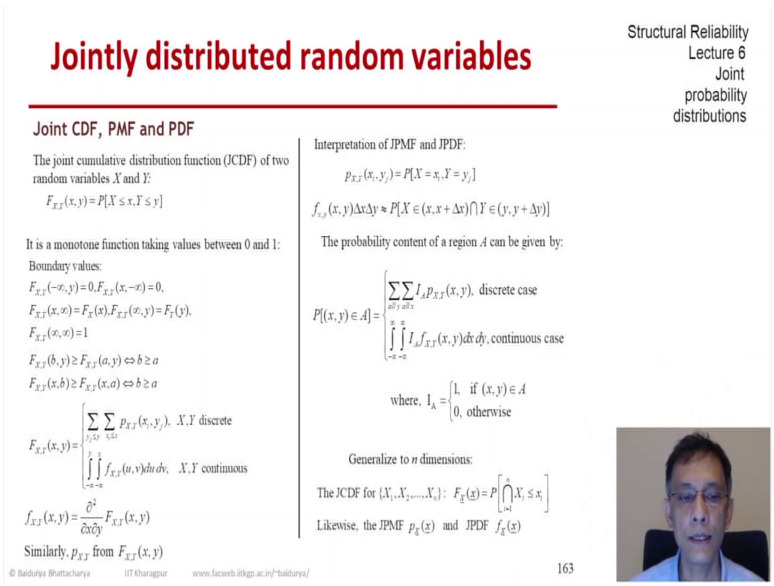

Let us start with the joint cumulative distribution function of two random variables X and Y. It is defined as the probability that the random variable X takes on values little x or less and the random variable Y takes on values little y or less. It is a monotonically increasing function between zero and one with values at the boundaries as we know very well at negative infinity for x the distribution function is zero whatever the value of y may be.

Likewise F(xy) at x, minus infinity is also zero the function reaches one at the limits infinity for both of them and when you set one of the variables at infinity what you get back which we will see in the next slide is the marginal distribution function for the other. So, F(xy) evaluated at x, infinity is nothing but F X at little x similarly for F Y at little y. Also for any *b* greater than *a*, the joint distribution function keeping the y fixed would be greater for b, y than a, y.

And likewise for x, b and x, a as long as b is greater than or equal to a the interpretation could also be given in terms of the probability masses or densities. So, if we want to evaluate the distribution function at fixed values of x and y little x and little y we would add all the probability masses are the joint probability masses for x's and y up to the respective values. And if the random variables are continuous then we would integrate the density function up to the limits little x and little y.

This also tells us that we could invert the relationship and show the density function as the differentiation of the distribution function and likewise we could get a similar expression for the mass function in terms of the distribution function we. Now move on to the mass function and density function in a little more detail. So, the mass function is the joint probability that the random variables take on the particular values.

So, p of x, y is the probability of intersection that X is equal to x i and Y is equal to y j and that would be the interpretation of the joint probability mass function. Likewise the density function would be approximated as an elemental probability if I take a little area around that delta x and delta y and multiplying the density function with that elemental area would give me the probability of finding x around that point and y around that point whose size is respectively delta x and delta y.

If I now expand this idea over a region, so, if I want to find x and y within a region defined by capital A then I could sum them in the discrete case for all those values of x and y that belong to the region A or I could integrate the density function in the continuous case for all those values that belong to the region A where the indicator function I conveniently takes on the value 1 if the point is within the desired region A otherwise 0.

This all this discussion we cast in the format of pair of random variables x and y but we could generalize this to n dimensions. So, we could define the join distribution function for n random variables x_1 up to x_n in terms of the joint probability. The probability of the intersecting events that capital X_i is less than or equal to little x_i for all the i's. And likewise we could define the joint probability mass function and the joint problem density function for the group of n random variables.

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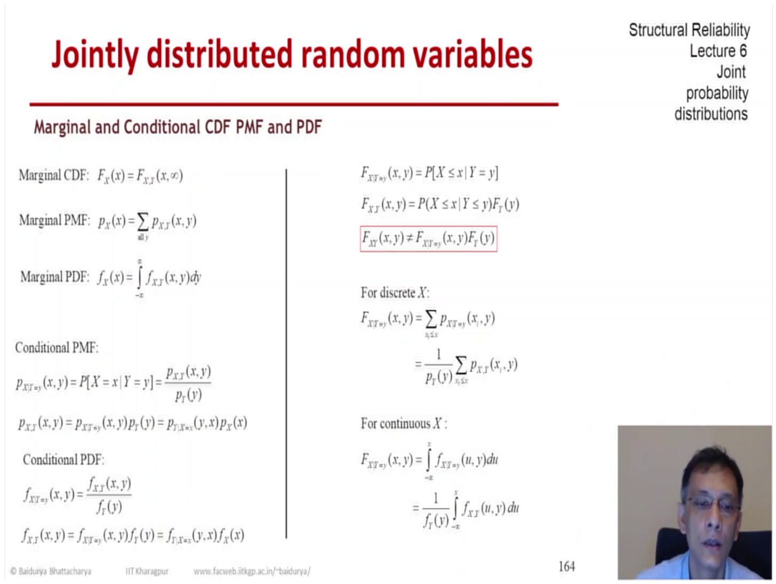

The marginal and the conditional probabilities are defined similarly as we did in the case of the univariate random variables. So, the marginal CDF as I said is simply when you put the argument of the other one at infinity the marginal PMF is obtained by integrating out by I am sorry by summing out the other variable the marginal PDF is obtained by integrating out the other variables. So, these are the equations that you see on the screen.

We could define the conditional mass function similarly just as we define probability of a given b. So, here it is the same approach p of x given y has taken a particular value little y would be the conditional PMF of x given y and that would be the ratio of the joint mass function divided by the marginal mass function. We could just as we did in the case of p of *a* intersection *b* we could describe the joint mass function as the product of the marginal and one of the conditionals.

So, p of x, y would be p of x given y times p of y at the appropriate arguments. We could do something similar for the density function the density function of x given y at a particular value would be the ratio of the joint density function and the marginal density function. And like we did in the discrete case the joint density function is the product of the conditional density function times the marginal of the other.

So, f of x, y is f of x given y times f of y or equivalently f of y given x times f of x we have to be a little careful when we come to the joint CDF or we come to the conditional CDF. So, if we want to fix the value of y and then find the probability of x would be less than or equal to little x. So, that is the definition of my conditional CDF now we can expand it just by using the definition but we have to be careful that we cannot write the joint distribution function as the product of the conditional and the marginal you have to be a little careful for that and why it should be clear from the above two equations.

We could continue in this line and describe the cumulative distribution function of x given a particular value of y. So, that conditional CDF of x we could in the discrete case define as the ratio as you see on the screen. And for the continuous case defined as the ratio as you see on the screen in the first case the numerator is a sum and in the second case the numerator is integration.