Structural Reliability
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Lecture –49
Common Probability Distributions (Part - 20)

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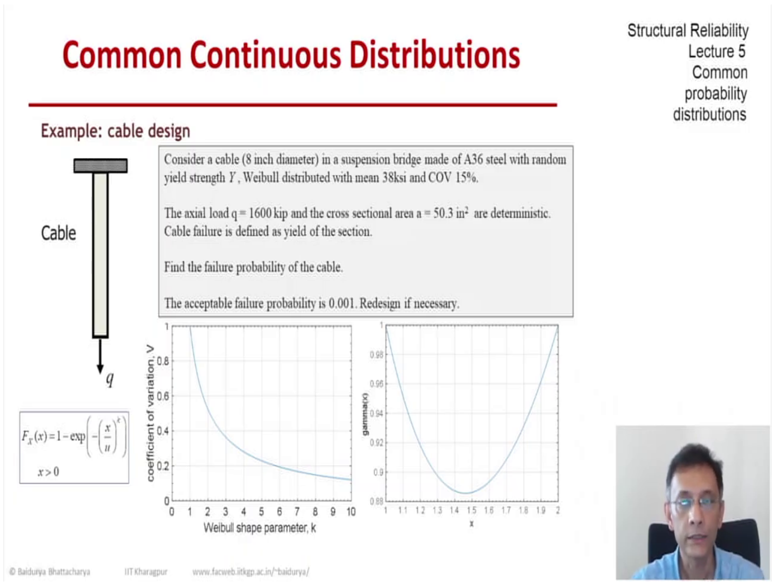
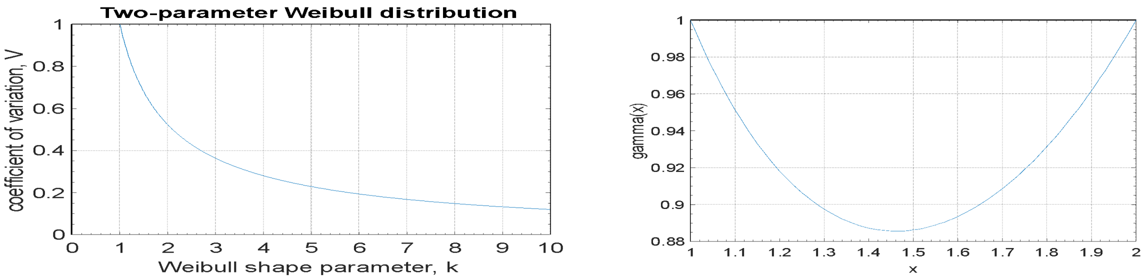


Let us solve one example involving the viable distribution or the type 3 distribution for minima let us take a minute to read the problem and then we will start solving it. So, the first part of the problem wants us to find the failure probability for which we would need to use the CDF of the viable distribution which as you can see is defined in terms of two constants u and k. So, given the mean and COV of y we need to find u and k for y.

$$F_X(x) = 1 - \exp\left(-\left(\frac{x}{u}\right)^k\right)$$
$$x > 0$$



$k = 8$ from graph, $u = m/G(1+1/8)=40.4$ ksi

And um we need to solve k in terms of v which I have given in the course material and also presented earlier in this lecture. And once we can find k given v we can use the gamma function at 1 + 1 by k to find u and the answer is for k it is 8 and for u it is about 40.4 ksi. So, with this information we are first going to find the failure probability of the cable.

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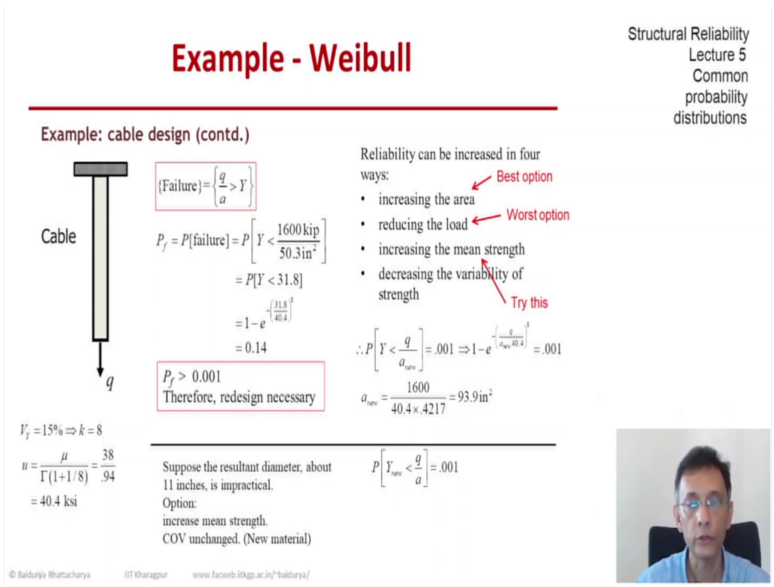

And we need to be able to define failure then and since failure is yield of the section. So, the force divided by the cross section area is the stress and if y the yield strength is less than the applied stress then we have yield failure. So, that clearly tells us that we need to evaluate the CDF of y at q divided by a which comes out to be about 31.8 ksi and plugging in the values of u and k the probability comes to about 14%.

$$V_Y = 15\% \Rightarrow k = 8$$

$$u = \frac{\mu}{\Gamma(1+1/8)} = \frac{38}{.94}$$
$$= 40.4 \text{ ksi}$$

$$
\begin{aligned}
P_f = P[\text{failure}] &= P\left[Y < \frac{1600\,\text{kip}}{50.3\,\text{in}^2}\right] \\
&= P[Y < 31.8] \\
&= 1 - e^{-\left(\frac{31.8}{40.4}\right)^8} \\
&= 0.14
\end{aligned}
$$

Now if you remember the limit that has been placed on the failure probability that it cannot be higher than 0.001 which is what we see here. So, this is not acceptable and redesign will be necessary. So, now what are our options in order to satisfy this failure probability that it has to be 0.001 or less. So, we can increase the reliability or we can decrease the failure probability in one of four ways in this problem one is we increase the cross sectional area two is we reduce the load and 3 and 4 would be to change the properties of the distribution.

So, we can either change the mean or decrease the standard deviation or both. Now it so, happens from practical point of view reducing the load is the worst option because then I would be limiting the functionality of this cable whatever it was going to be used for. It could be at least for the first case that we should look for is to see if we can increase the cross section area if that would solve our problem.

So, which basically means that we resolve this CDF problem this CDF with a new area a new and for which the failure probability would be limited to 0.001 and if you solve the equation the answer is about 94 square inches. So, going up from 50 square inches to about 94 square inches. Now this would be all fine but suppose this 94 square inch because it corresponds to a diameter of about 11 inches this might be impractical maybe that there is not enough space to accommodate a cable of 11 inch diameter.

So now I have to look at the other options that I listed above that we increase the mean strength or reduce the variability reducing variability you know is quite demanding because we might have to improve the quality of the manufacturing of fabrication process. So, let us go for option number three which is increase the mean strength and which would basically mean that we are going for a new material which might be expensive but let us see if that gives us some useful results.

So, now we have the same q the same a but now we are looking at a new y which means y with a new mean and we assume that the COV we are not touching. So, the coefficient variation is still the same as before. So, which basically means the value of k is the same as before. So that gives us an expression in terms of u and with that we can find the new mean which comes to about 71 ksi.

So, if I can find a material which gives me that strength I will be able to use it and keep the same diameter. Obviously there are questions like you know is the material ductile as before or is it too brittle and is that going to serve my purpose but those discussions are beyond the scope at this point.

$$\therefore P\left[Y < \frac{q}{a_{new}}\right] = .001 \Rightarrow 1 - e^{-\left(\frac{q}{a_{new}\,40.4}\right)^8} = .001$$

$$a_{new} = \frac{1600}{40.4 \times .4217} = 93.9\,\text{in}^2$$

$$P\left[Y_{new} < \frac{q}{a}\right] = .001$$

$$\exp\left[-\left(\frac{31.8}{u_{new}}\right)^8\right] = .999 \Rightarrow u_{new} = 75.4$$

$$\Rightarrow \mu_{new} = 75.4\,\Gamma\left(1 + 1/8\right) = 70.9 \text{ ksi}$$