

Structural Reliability
Prof. Baidurya Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture –46
Common Probability Distributions (Part - 17)

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Common Continuous Distributions

The normal CDF


$X \sim N(\mu, \sigma^2), \quad \text{CDF: } F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$

Table A.1. Values of the standardized normal distribution (Continued)

The cumulative distribution function, $F(z) = \int_{-\infty}^z f(x) dx$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50400	0.50801	0.51200	0.51600	0.51996	0.52399	0.52799	0.53199	0.53599
0.1	0.53999	0.54399	0.54799	0.55199	0.55599	0.55999	0.56399	0.56799	0.57199	0.57599
0.2	0.57999	0.58399	0.58799	0.59199	0.59599	0.59999	0.60399	0.60799	0.61199	0.61599
0.3	0.61999	0.62399	0.62799	0.63199	0.63599	0.63999	0.64399	0.64799	0.65199	0.65599
0.4	0.65999	0.66399	0.66799	0.67199	0.67599	0.67999	0.68399	0.68799	0.69199	0.69599
0.5	0.69999	0.70399	0.70799	0.71199	0.71599	0.71999	0.72399	0.72799	0.73199	0.73599
0.6	0.73999	0.74399	0.74799	0.75199	0.75599	0.75999	0.76399	0.76799	0.77199	0.77599
0.7	0.77999	0.78399	0.78799	0.79199	0.79599	0.79999	0.80399	0.80799	0.81199	0.81599
0.8	0.81999	0.82399	0.82799	0.83199	0.83599	0.83999	0.84399	0.84799	0.85199	0.85599
0.9	0.85999	0.86399	0.86799	0.87199	0.87599	0.87999	0.88399	0.88799	0.89199	0.89599
1.0	0.89999	0.90399	0.90799	0.91199	0.91599	0.91999	0.92399	0.92799	0.93199	0.93599
1.1	0.93999	0.94399	0.94799	0.95199	0.95599	0.95999	0.96399	0.96799	0.97199	0.97599
1.2	0.97999	0.98399	0.98799	0.99199	0.99599	0.99999	1.00000	1.00000	1.00000	1.00000
1.3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.6	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.7	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.8	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.9	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.2	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.3	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.4	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
3.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
3.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
5.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
5.5	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

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Lecture 5
Common
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distributions



Taken from: Benjamin and Cornell, McGraw Hill, 1970

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Now evaluating the normal CDF, so, as I said that we come back to the standard normal CDF which is extensively tabulated in all the textbooks and reference books. This particular page is from one of my favorite books by Benjamin and Cornell. So, let us just take a few seconds. For example if we want to find out the normal CDF at one and you will see that these tables because of trying to save paper they take advantage of the symmetry of the normal distribution and they start at zero and higher values of the standard normal variance z.

So, it starts with 0.5 as you can see. So, at z equals one or what Benjamin Cornell called here u the CDF is about 0.84. So if I wanted to find out the CDF of -1 the standard normal deviate of -1 I would subtract that from 1. So, it would be 1-0.84. So, roughly 0.16. I could also be interested in finding that particular value of the standard normal variant whose CDF is say 0.9. So, I would carefully look at the increasing values and see that somewhere around 1.2728 I am getting closer to the CDF of 0.9 and somewhere between 1.28 and 1.29.

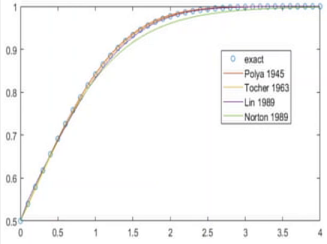
I have the CDF of 0.9. So, the CDF of 0.1 would be the negative value of that. So, somewhere around negative 1.29 I would get the CDF of 0.1 and so, on and so, forth. In the next example we are going to look at the CDF of 0.95 or actually 0.05. So the deviate at which the CDF is 0.95 is roughly about 1.6. So, the CDF of 0.05 would be negative of that value. So, at phi of -1.645 or something we are going to get the CDF of 5%.

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Common Continuous Distributions

The normal CDF (contd.)

The normal CDF is not available in closed form.
 The standard normal CDF is extensively tabulated. For others, use: $z = (x - \mu) / \sigma$
 Numerous closed form approximations to the standard normal CDF have been, and continue to be, proposed.
 Here we list a few well-known ones:



$$\Phi_{\text{Polya}}(z) = 0.5 + 0.5\sqrt{1 - \exp(-2z^2/\pi)}$$


$$\Phi_{\text{Tocher}}(z) = \frac{\exp(2z\sqrt{2/\pi})}{1 + \exp(2z\sqrt{2/\pi})}$$

$$\Phi_{\text{Lin}}(z) = \begin{cases} 1 - 0.5 \exp\left(-\frac{z^2 + 1.2z^{0.1}}{2}\right), & 0 \leq z \leq 2.7 \\ 1 - \frac{\phi(z)}{z}, & z > 2.7 \end{cases}$$

$$\Phi_{\text{Norton}}(z) = 1 - 0.5 \exp(-0.717z - 0.416z^2)$$

Further reading: Choudhury and Roy 2009
 DOI: 10.1080/03610910903009344

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Since these normal distribution values are valid numerically there are many available approximations and some of them are well known some of them continue to be derived or proposed. And here I have listed some of the well known ones and as you can see these are also starting from zero and higher values of the standard normal deviate. And as you can see most of them are quite satisfactory.

And some of these can be you know used in a very convenient manner when you are trying to solve problems involving the normal distribution. Let us look at one simple example involving the normal random variable it involves cube strengths of concrete.

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Common Continuous Distributions

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Normal distribution - examples

Cube strength from a batch of concrete is normally distributed. If the 5th percentile characteristic strength must be no less than 90% of the mean value, what is the maximum permissible COV of the distribution?

Characteristic value, x_k :

$$F_X(x_k) = 0.05$$

$$\Rightarrow \Phi\left(\frac{x_k - \mu}{\sigma}\right) = 0.05$$

Inverting the normal CDF:

$$x_k = \mu + \sigma \Phi^{-1}(0.05)$$
$$= \mu - 1.645\sigma$$

Required:

$$x_k \geq 0.9\mu$$

$$\Rightarrow \mu - 1.645\sigma \geq 0.9\mu$$

$$\Rightarrow 0.1\mu \geq 1.645\sigma$$

$$\frac{\sigma}{\mu} \leq \frac{0.1}{1.645}$$

$$V \leq 6\%$$

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So, let us take a few seconds to read the problem. So, the characteristic value is defined here as the fifth percentile value. So, x_k corresponds to the CDF of 0.05 and I could now write that in terms of the normal CDF making use of the mean and standard deviation of x and I could invert that as I we already found out in the previous slide that phi inverse of 0.05 is something like negative 1.645. So, x_k the fifth percentile value in terms of μ and σ is $\mu - 1.645\sigma$.

Now the problem wants that that x_k should not be anything less than 90 of μ so, let us proceed and see what we come to that gives me a relationship involving μ and σ and since the coefficient of variation COV is σ over mean this condition restricts σ over mean or V to about 6%. So, clearly this requires a good amount of quality control to keep COV restricted to only 6%.